Using Modal Analysis to Understand Transmission Line Speaker Enclosure Response Part 5 – Inside the Box and Outside the Box

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April 10, 2026

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Introduction

Several different software packages for designing transmission line loudspeakers became available to the DIYer beginning around 2000. Some of these programs have come and gone; others have come and evolved. Today there are probably three or four in regular use each with its own strengths and weaknesses. The software allows simulation of a wide range of quarter-wave geometries (TL, TQWT, or BLH for example) since the math and physics are the same for all these enclosures.

This presentation is intended to discuss how to characterize and understand the behavior of the air inside a quarter-wave loudspeaker systems using Mechanical Engineering based vibration analysis. Then once a deeper understanding of the air motions in the enclosure is achieved, the next step is to describe how this translates into a simulated measured response at a microphone in anechoic space (room interaction is a whole different topic). The extension from air motion in the enclosure to SPL in free space is missing from the currently available quarter-wave loudspeaker design programs.

Linear Analysis and Behavior

Linear systems have a few fundamental properties. First, if you double the input signal you double the output response. Second, if you excite a linear system at a given frequency the output is **only** at that specific frequency (you cannot excite at 50 Hz and get 50 Hz plus 60 or 100 Hz in the response). A linear system implies that superposition and conversion from the time domain to the frequency domain (or the reverse) are all repeatable and predictable. The downside of software based on linear analysis is that the user can dramatically increase the input, calculate a response exceeding the linear range of the speaker system, produce results that are very inaccurate, and all without any warning.

When measuring or designing speakers, the assumption is that you are working with a linear system model. Thiele – Small parameters are intended to be measured using small signals, so the derived properties are consistent with a linear system model. The speaker's equivalent circuit model, the crossover elements, the calculated or measured acoustic response, all post processing of measurement data, or any additional resonances seen in a measurements such as cone breakup or standing acoustic waves are all treated as linear responses. You cannot design a loudspeaker to be predictable, consistent, and repeatable using non-linear methods.

This presentation is intended to discuss how to characterize and understand the behavior of the air inside a quarter-wave loudspeaker system. A more in depth understanding of the air motions in the enclosure will allow the designer to make better geometric trade-offs and extrapolate the results to what is heard at a listening position in the room.

Currently the trend in speaker building is to jump right into taking measurements. Then use DSP techniques to correct any issues found with the design itself or the speaker's interaction with the room. The DSP tools are very powerful and sophisticated and can correct a lot of problems in a speaker build.

I tend to focus more on acoustic solutions and limit the quick DSP fixes; I have used both active and passive crossovers and filtering but in general prefer acoustic and passive solutions for improving speaker performance. Maybe this is driven by my mechanical engineering background versus having an electrical engineering or computer science background.

Either method can work, the choice is up to the speaker designer and builder.

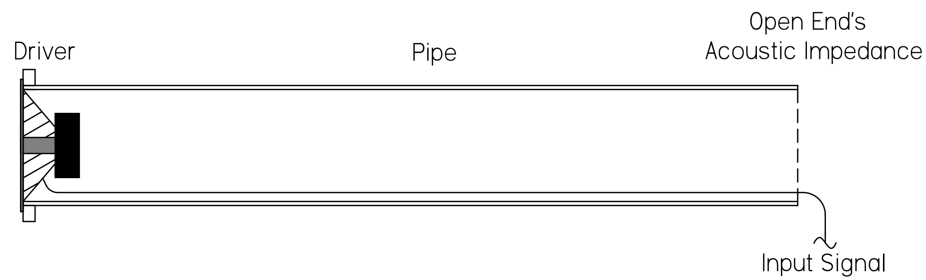
Sample TL Definition

To keep everything as simple as possible, and allow hand calculation verification of results, a sample problem was derived. Based on the Satori WO25P-4 woofer, mounted first at the closed end and then offset along the length, a straight constant cross-section TL was designed. I used the BR alignment tables, see link below, to determine the length and internal volume of the sample TL. All the following analyses will use this driver and TL geometry. Results for an empty undamped TL will be presented first, then the driver will be offset, and finally fiber damping will be introduced.

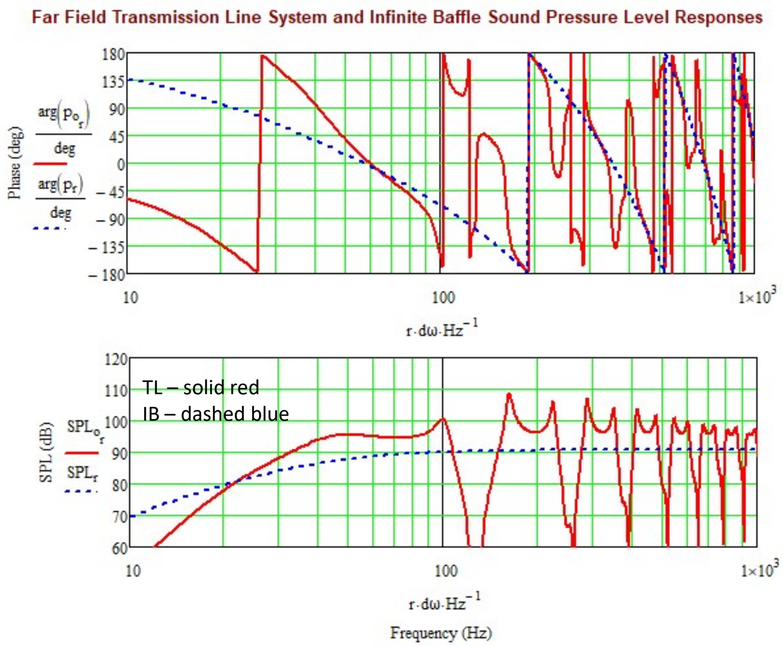
http://www.quarter-wave.com/TLs/TL_Alignments.pdf

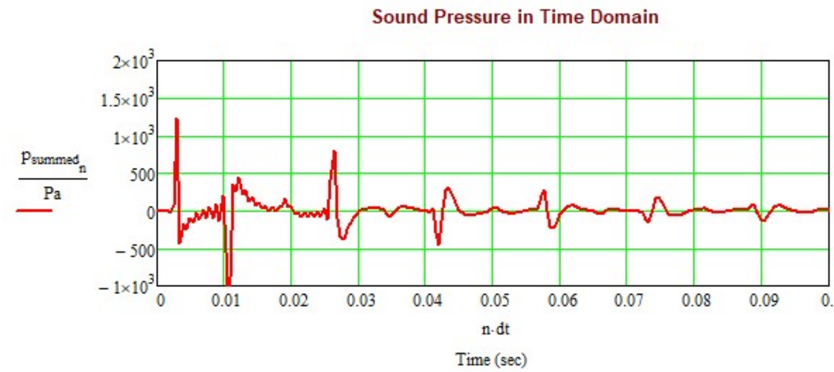
Enclosure Geometry Definition (1 : 1 Straight TL) $\frac{cycle}{4} \cdot \frac{c}{f_d} = 103.195\,in$

$L := 103.2 \cdot in$	(Length)	Driver Distance
$\xi := 0.0001$	(Driver Position Ratio : $0.001 < \xi < 0.999$)	$\xi \cdot L = 0.01\,in$
$H := 1$		
$\alpha := 0.6868$	$V_{ab} := V_{ad} \cdot \alpha^{-1}$ $\frac{V_{ab}}{S_d} \cdot \frac{1}{0.5 \cdot (1+1) \cdot L} = 1.052$	$V_{ab} = 70.341\,liter$
$f3\_fd := 0.8224$	$f3\_fd \cdot f_d = 26.983\,Hz$	
$S_0 := 1.0525 \cdot S_d$	(Area of the Closed End)	
$S_L := 1 \cdot S_0$	(Area of the Open End)	$S_0 \cdot L = 70.352\,liter$
$Density := 0 \cdot lb \cdot ft^{-3}$	(Stuffing density : $0\ lb/ft^3 < D < 1\ lb/ft^3$)	
$Power := 1 \cdot watt$	(Input Power) Applied Voltage Reference --->	$R_{ref} := 8 \cdot \Omega$



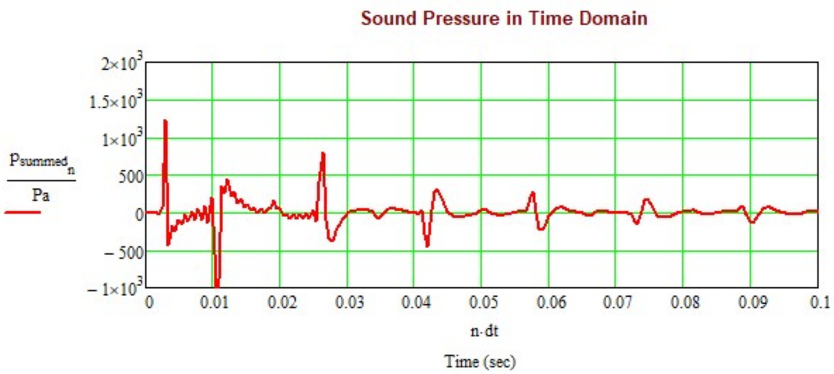
Frequency Domain and Time Domain





The plots above are derived from the same data and coupled through the Fourier Transform. On the left is the magnitude and phase of the sound pressure level as a function of frequency while on the right is the pressure transient as a function of time. The input signal is a 2.8284-volt pulse to the driver's voice coil, and the output is calculated at 1 meter on the driver's axis. The time domain input signal excites all frequencies equally in the frequency domain.

Time Domain



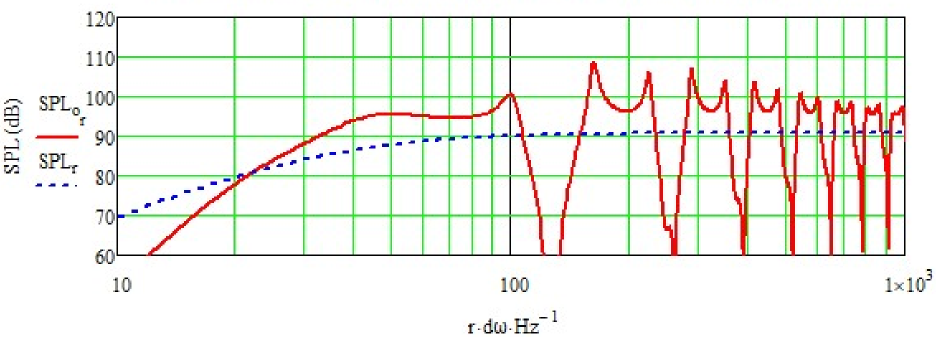
$\frac{1\cdot m}{c} = 2.907\times 10^{-3}\,s$	Driver
$\frac{1\cdot m + L}{c} = 0.011\,s$	TL
$\frac{1\cdot m + 3\cdot L}{c} = 0.026\,s$	1st Reflection
$\frac{1\cdot m + 5\cdot L}{c} = 0.041\,s$	2nd Reflection
$\frac{1\cdot m + 7\cdot L}{c} = 0.056\,s$	3rd Reflection

Simple TL models assume the driver and terminus are coincident and radiate from an infinite baffle, this is the assumption used in almost all free ware TL speaker design programs. There is a lot of information that can be unpacked from the time and frequency response plots.

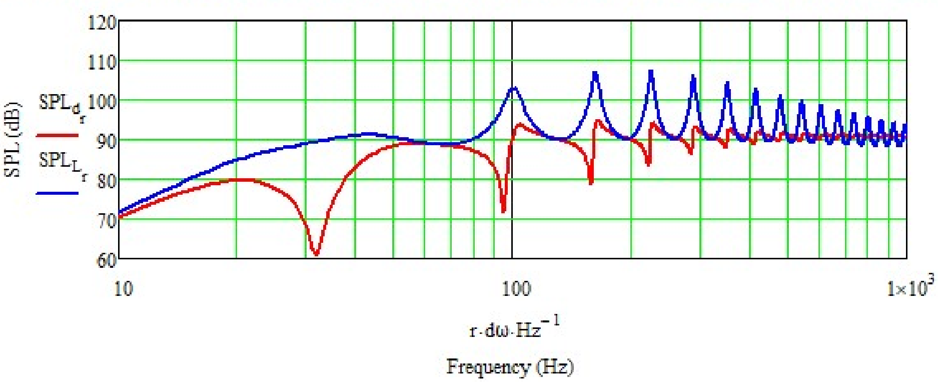
Starting with the time domain response shown on the left, this is the transient pressure that arrives at the microphone due to an input voltage pulse. The first pressure pulse comes directly from the front of the driver's cone after traveling 1 m. The second inverted pulse is from the rear of the driver's cone traveling the length of the TL and then the same 1 m. The following pulses are reflections inside the TL traveling odd multiples of the length and then 1 m to the microphone.

Frequency Domain

Far Field Transmission Line System Sound Pressure Level Response



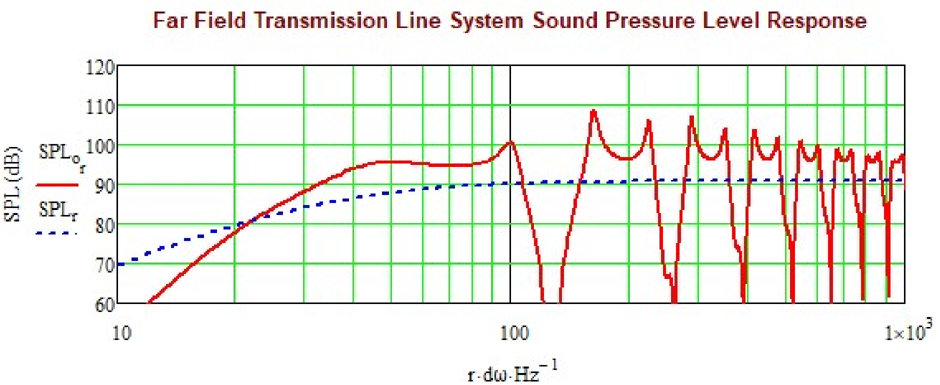
Woofer (red curve) and Open End (blue curve) Far Field Sound Pressure Level Responses

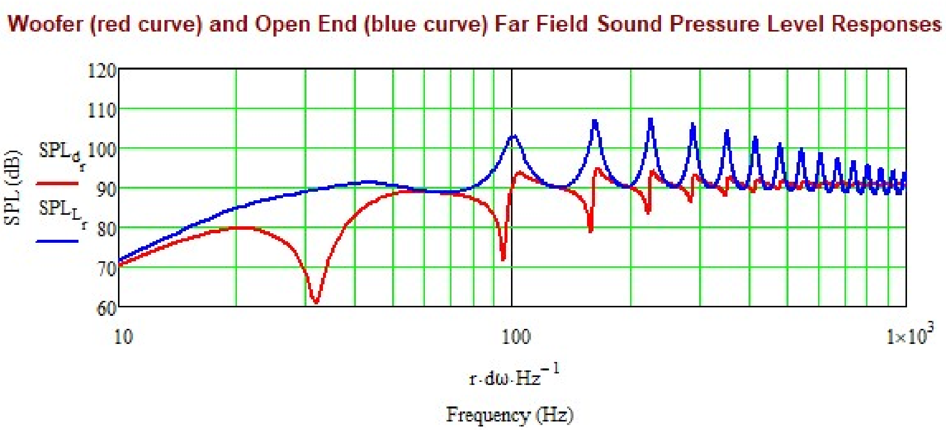


The summed frequency response is shown in the upper plot, the woofer and open-end frequency responses are shown in the lower plot. The phase plots are not shown, they are not needed at this point. These plots tell a story that will be covered in more detail later, but for now insightful observations can be made by comparing the components and the summed results.

Below 100 Hz, there are system resonances at 18 Hz and 50 Hz. The null in the driver response at 32 Hz is not a system resonance, it is a resonance in the enclosure subsystem but not in the combined driver plus enclosure speaker system.

Frequency Domain

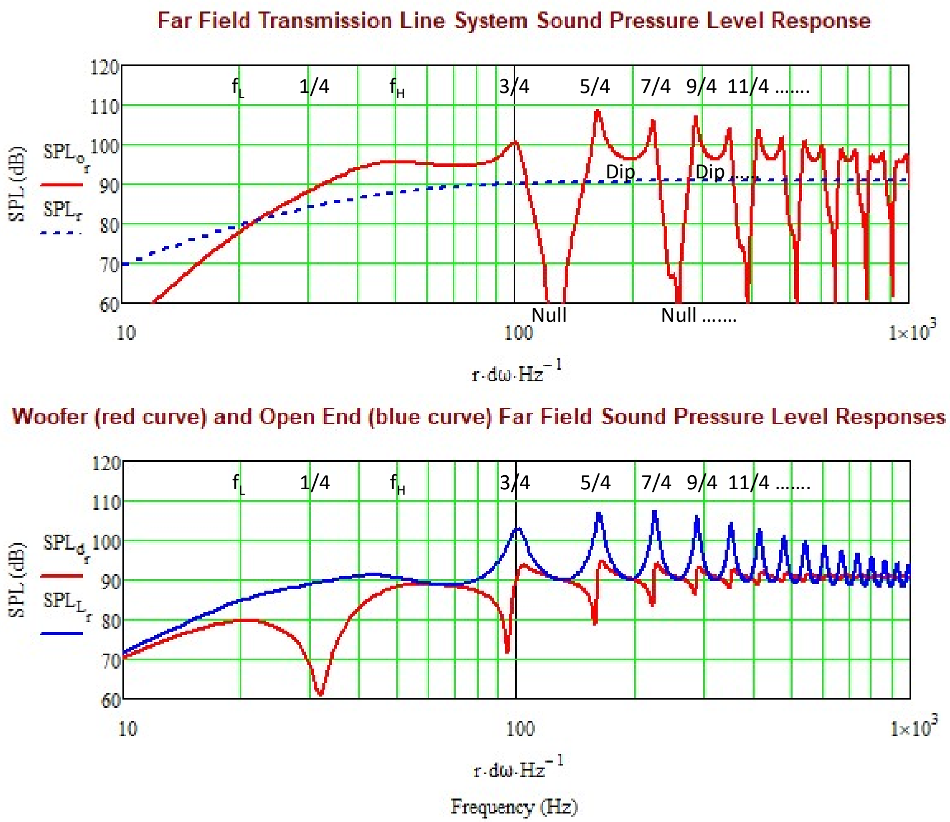




These plots should be very familiar to TL designers and are produced in all software simulations. Peaks and nulls appear in every undamped TL simulation and can be manipulated by the line's geometry and the driver offset.

Above 100 Hz is where things get interesting. There are a series of peaks in the open-end output that are also seen in the system response, these are the 3/4, 5/4, 7/4, … standing wave resonances. The driver's cone motion is suppressed, and almost all the system SPL output is generated by open-end of the TL enclosure.

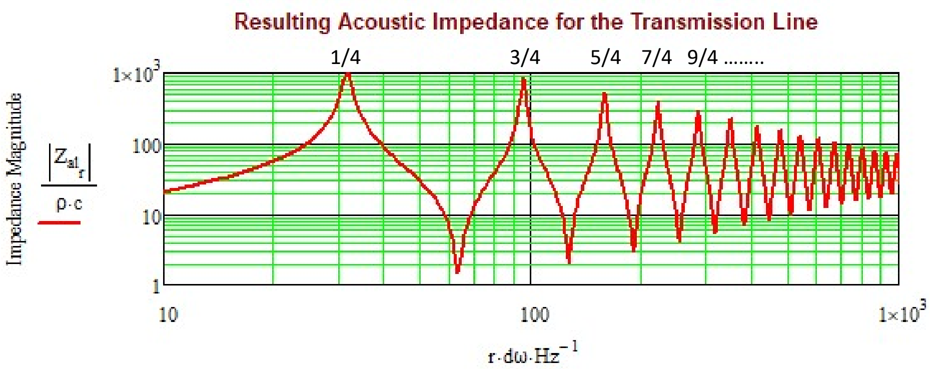
Frequency Domain

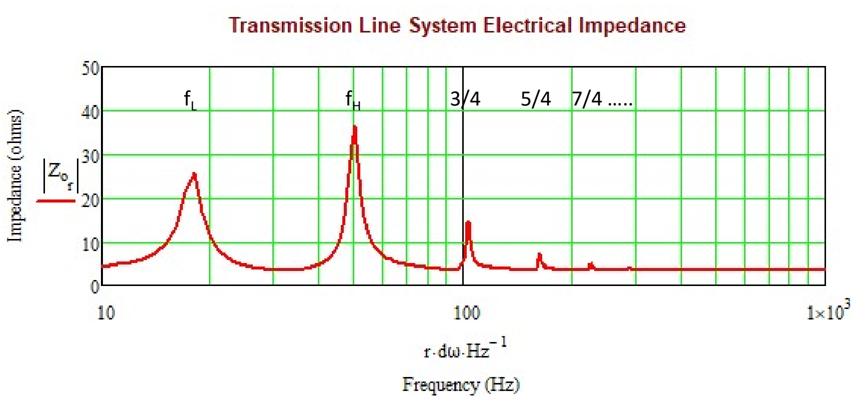


What happens between the peaks is not usually addressed. Starting with the lower plot, the SPL output from the driver and the open-end are equal midway between the peaks. This occurs because the sound waves generated from the back of the driver's cone are constrained by the walls of the TL and do not spread and attenuate with distance. The sound waves at the open end emerge equal in magnitude but with a different phase with respect to the front of the driver's cone.

The first null between the 3/4 and 5/4 peaks is due to the two signals canceling each other, the outputs are 180 degrees out of phase. Between the 5/4 and 7/4 peaks, the two signals are in phase, adding, resulting in +6db of additional SPL output. This pattern repeats for the higher modes.

Resonant Frequencies and Mode Shapes





There are three vibrating systems making up a TL speaker.

First there is the driver subsystem, which for this sample problem has a resonant frequency of ~33 Hz.

The second subsystem is the air column in the TL enclosure, which has a series of resonant frequencies starting with the quarter-wavelength mode and progressing up through the odd harmonics. This is clearly shown in top plot on the left, the acoustic impedance plot. For a straight TL the peak frequencies can be calculated by hand, f = n x c / (4 x L) where n = 1, 3, 5, … .

The final vibrating speaker system is the combined driver and TL enclosure shown in the electrical impedance plot, lower plot on the left. The peaks are the system's resonances.

Resonant Frequencies

Modal Map

Mode	MathCad Enclosure	MathCad Driver	MathCad System	Closed Form
fL			18	
1/4	32	33		32
fH			50	
3/4	95		101	96
5/4	158		162	159
7/4	222		225	223
9/4	286		288	287
	[Hz]	[Hz]	[Hz]	[Hz]

The resonant frequencies can be mapped, see the table on the left, for each subsystem as they combine to form the final speaker system. The Enclosure column was taken from the peaks of the acoustic impedance plot on the previous slide. These values can be checked by hand calculation as shown in the Closed Form column after rounding to the nearest Hz. The Driver resonance is shown almost equal to the enclosure's quarter-wave frequency.

The System frequencies all shift to bracket the two subsystems' results. The 32 and 33 Hz subsystem frequencies combine and split to form the dual peaks, at f_L and f_H, seen in the electrical impedance plot. The harmonics all shift up a few Hz. There is no system resonance at 32-33 Hz. The same combining and shifting to produce f_L and f_H occurs in a bass reflex speaker system.

Resonant Frequencies

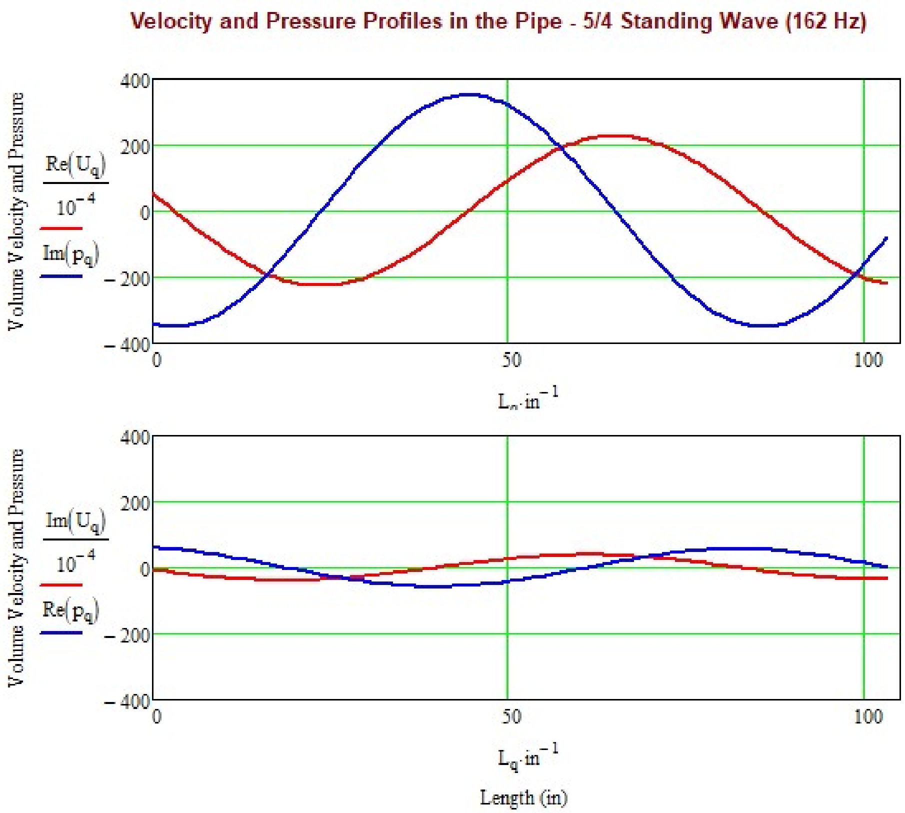
Modal Map

Mode	MathCad			Closed Form
	Enclosure	Driver	System	
fL			18	
1/4	32 →	33		32
fH			50	
3/4	95		101	96
5/4	158		162	159
7/4	222		225	223
9/4	286		288	287
	[Hz]	[Hz]	[Hz]	[Hz]

To verify the behavior seen in the combined system column, a second MathCad modeling technique was used. The air column in the TL enclosure was broken up into 100 discrete lengths, the masses and stiffnesses of each segment were assembled into matrices from which the eigenvalues (resonant frequencies) and eigenvectors (mode shapes) were calculated. This was done without the driver to verify the Enclosure column and then with the driver to verify the System column.

The continuous MathCad TL worksheet and the discrete MathCad eigenvector/eigenvalue worksheet correlated as a double check of the results in, and descriptions of, the modal map. The resonant frequencies and mode shapes matched in the two models.

Mode Shapes



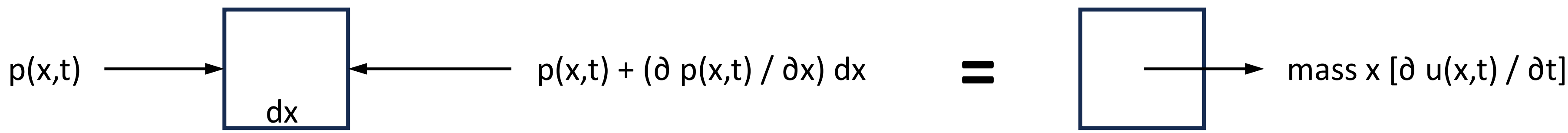
A sample TL system mode shape is shown on the left, the 5/4 wavelength standing wave at 162 Hz identified on slides 11, 12, and 13. The volume velocity U (red curves) and the pressure p (blue curves) have magnitude and phase which can are represented as real and imaginary components.

The plots depict the RMS magnitude on the y-axis. Since sound is a compression wave, not a transverse wave, the direction of oscillation is parallel to the x-axis, the vertical values plotted are only a representation. The driver is at the origin, and the open end is at 103.2 inches on the x-axis.

This is a frequency domain plot, so the wave oscillation is steady-state. At any axial position, the U and p oscillate between positive and negative values at a frequency of 162 Hz. The sound wave oscillation extends out into the environment, spreading, and eventually dying out at infinity. The sound waves do not shift or progress, no time propagation only steady-state cycling of the acoustic variables U and p.

Mode Shapes

Newton's 2nd Law : Summation of Forces = Mass x Acceleration



$$- \partial p(x) / \partial x = j \omega \rho u(x)$$

The equation of motion indicates that the rate of change of pressure p(x,t) with distance (the slope of the pressure profile) determines the velocity of the air u(x,t), increasing pressure pushes air back in the opposite direction. Stating it another way, increasing pressure with increasing distance to the right produces a net force directed towards the left in the picture above, resulting in an acceleration to the left. Also notice the "j" on the right side of the equation above indicating a 90-degree phase difference between pressure and velocity.

Mode Shapes

Separating the equation of motion into real and imaginary components produces the bottom two equations. When plotting the mode shapes the real part of U will be paired on the same plot with the imaginary part of p, if relevant a second plot will pair the imaginary part of U with the real part of p.

$$- \partial p(x) / \partial x = j \omega \rho u(x)$$

let $p(x) = Re(p) + j Im(p)$ and $u(x) = Re(u) + j Im(u)$

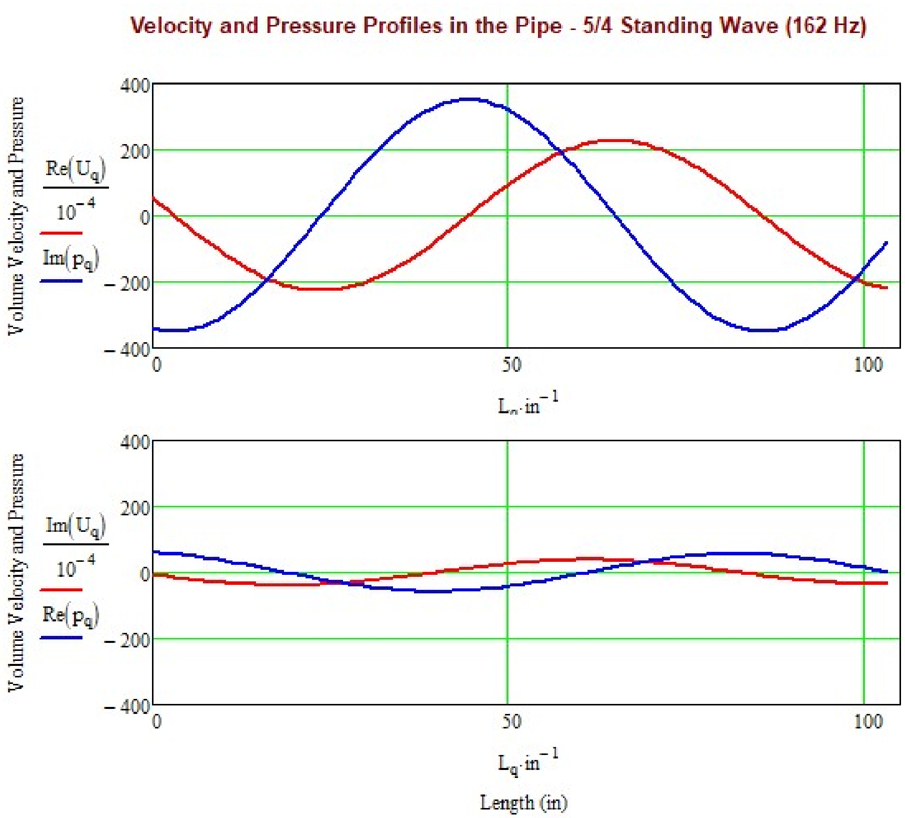
$$- \partial Re(p) / \partial x + -j \partial Im(p) / \partial x = j \omega \rho Re(u) + j \omega \rho j Im(u)$$

$$- \partial Re(p) / \partial x + -j \partial Im(p) / \partial x = j \omega \rho Re(u) - \omega \rho Im(u)$$

$- \partial Im(p) / \partial x = \omega \rho Re(u)$ <u>negative slope of Im(p) proportional to Re(u)</u>

$\partial Re(p) / \partial x = \omega \rho Im(u)$ <u>slope of Re(p) proportional to Im(u)</u>

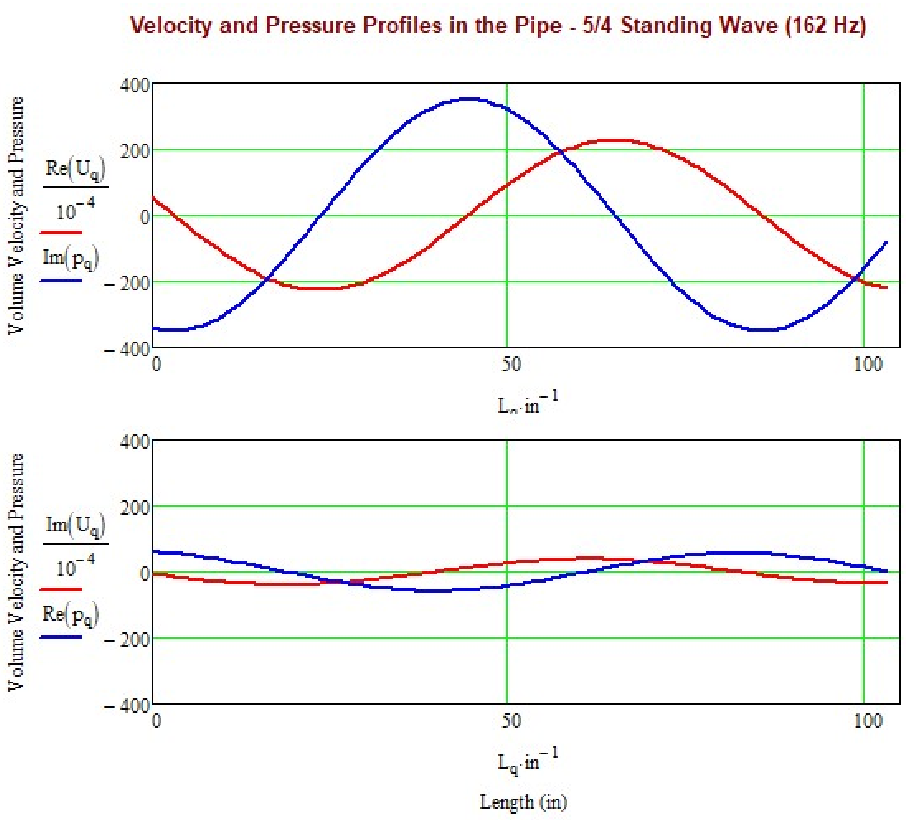
Mode Shapes



At the driver end, x = 0, the volume velocity (red curves) is almost zero. This is consistent with the lower plot on slide 11 where the open-end SPL output is peaking and the driver SPL output is a null. The pressure (blue curves) on the back of the driver's cone is high enough to control the cone's motion. This is typical of the standing waves in a TL at 1/4, 3/4, 5/4. 7/4 … frequencies as seen in the lower plot's red curve on slide 11.

Another feature easily seen in the top plot is that the wave's magnitudes do not decrease with distance along the TL. The walls of the TL do not allow the sound wave to spread like it would in free space. In free space, the SPL drops as a function of distance as sound waves spread spherically moving away from a source.

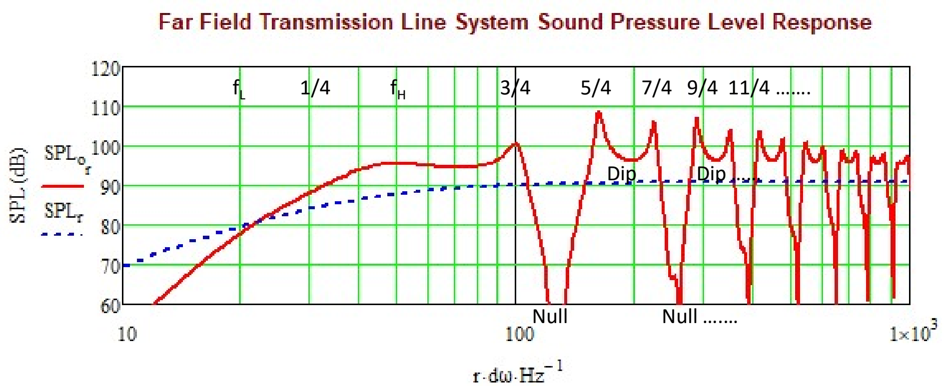
Mode Shapes

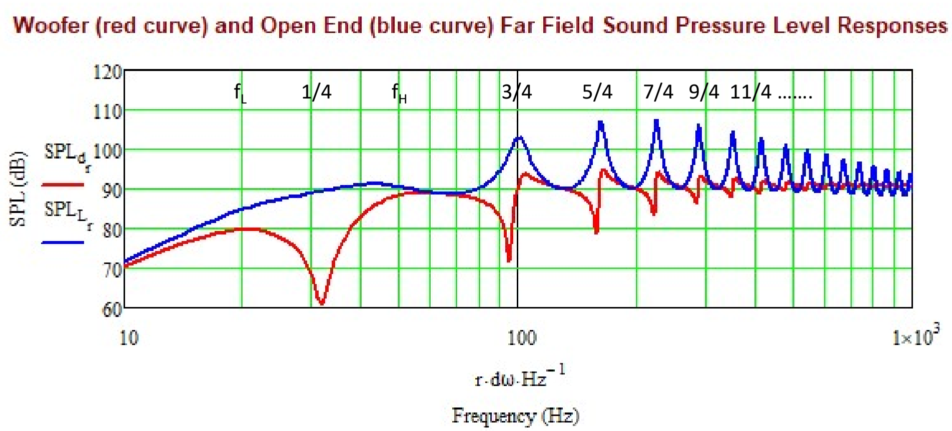


One other observation about the results shown in slide 11, there is an SPL contribution from the TL's open-end at all frequencies. We focus on the special quarter-wave frequencies which produce maximum output from the TL's open-end but tend to neglect these other frequency contributions. Every frequency produces sound waves from the back of driver that travel along the length of the TL, emerge, and combine with the front of the driver output at a listening position in the room.

For the rest of this presentation only one of these two plots will be shown if the real or imaginary part of the volume velocity dominants. In subsequent slides, for the 5/4 standing wave resonance only the top plot will be presented along with the RMS air displacement along the TL's length.

End Loaded Driver – Calculated Response



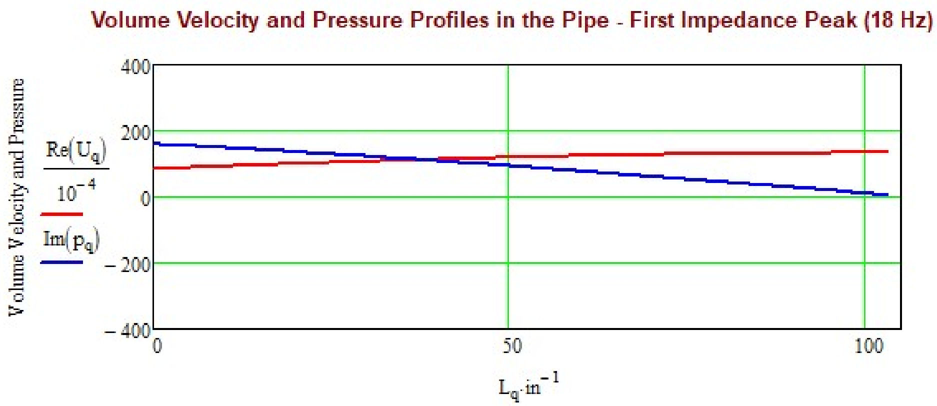


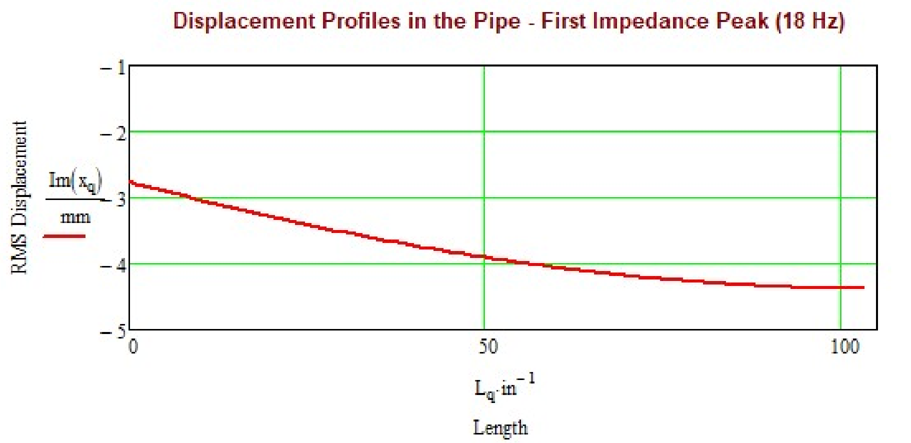
Both plots on the left are very typical of an empty TL's SPL response. If you model this speaker and TL geometry in any of the available TL simulation programs you should produce the same or very similar SPL curves.

The top plot shows the combined driver and TL open-end SPL response (red curve) and infinite baffle SPL response (dashed blue curve). The lower plot shows the separate driver (red curve) and open-end (blue curve) SPL responses. While some of the peaks in the upper plot are intuitively obvious, the deep nulls and the depth of the dips between the peaks are not.

Not every feature on the upper plot is a resonance (1/4, Nulls, and Dips are not). The following nine slides show the air vibration profile along the length and address each identified frequency between f_L and 7/4 to explain how and why the results produce this system SPL response curve.

Resonance and Mode Shape at f_L



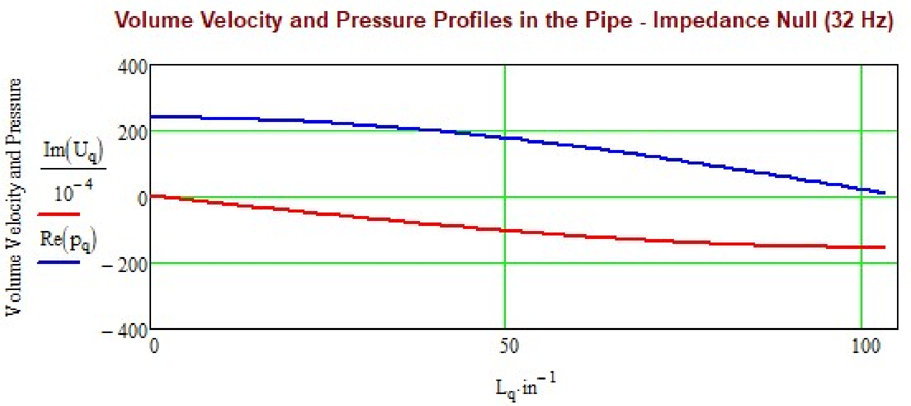


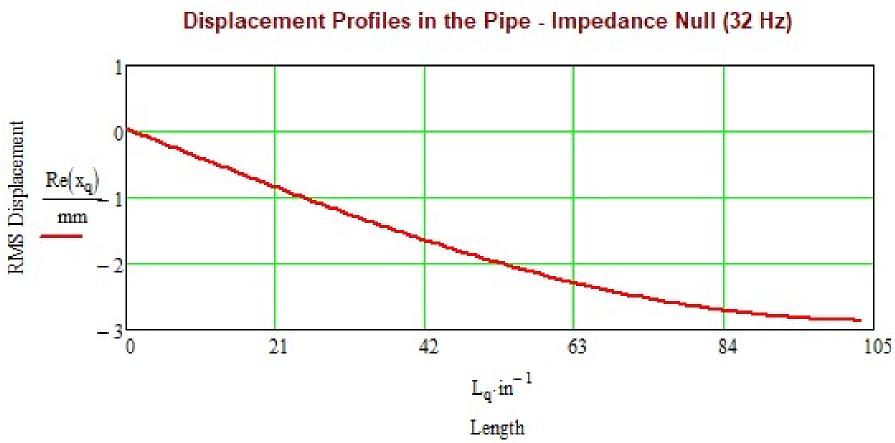
The top plot shows the real part of the volume velocity U (red curve) and the imaginary part of the pressure p (blue curve), as described on slides 17 and 19. The lower plot shows the RMS displacement of the air along the length of the TL.

The volume velocity at x = 0 is the back of the driver's cone. The volume velocity increases along the length reaching a maximum at the open-end. The air column adds a parasitic mass load riding on the back of the cone resulting in a drop in resonant frequency from 33 Hz for the driver to 18 Hz for the TL system (can be verified by hand).

The volume velocity at the back of the cone is in phase with the open-end volume velocity. This means the front of the driver's cone and the open-end are out of phase leading to cancelation and the low frequency 24 dB/octave system roll off seen on slide 20.

Standing Wave – 1/4 Wavelength



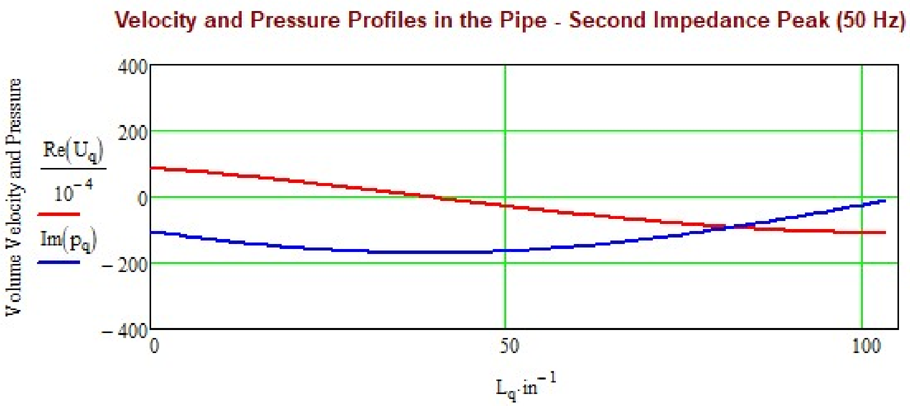


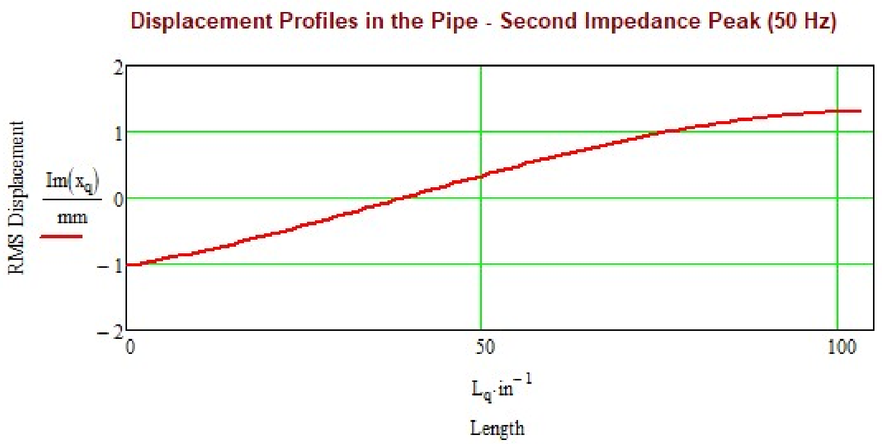
From the modal map, this is not technically a resonance. It is midway between two resonances f_L and f_H. When these mode shapes are scaled and combined the quarter wave volume velocity and pressure curves shown are produced, see Part 1 slide 23.

The volume velocity and displacement at the driver end are almost zero, this is typical of a standing wave result and is seen in the red curve in the lower plot on slide 20.

This standing wave produces the bass output of a TL speaker system (like a bass reflex system's port output). The maximum oscillating air displacement is at the open-end, in this example approaching an RMS value of 3 mm. In a bass reflex speaker system, the smaller port opening would cause this oscillating displacement, and hence the oscillating velocity, to be much higher.

Resonance and Mode Shape at f_H

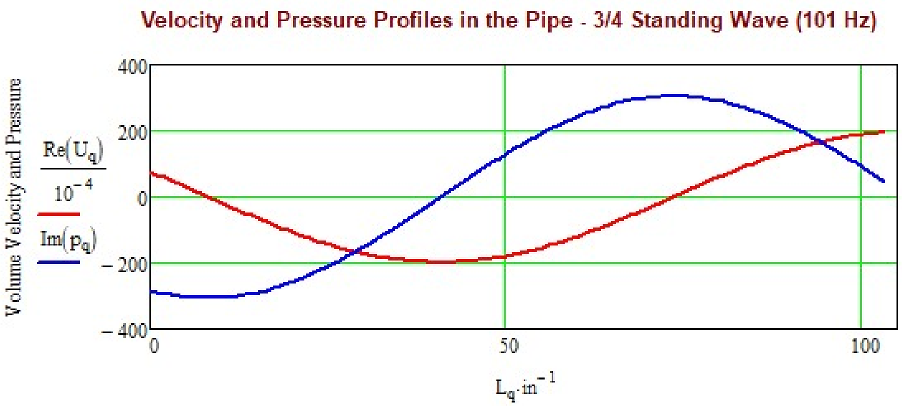


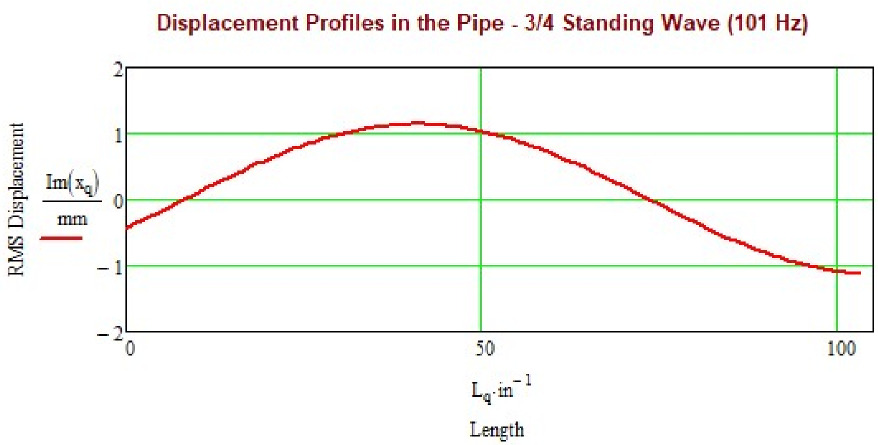


The volume velocity at the back of the driver's cone and at the open end are out of phase. The air in the TL is being compressed pushing on the back of the driver's cone, adding stiffness. The air column compression, resisting the movement of the back of the driver's cone, results in an increase in resonant frequency from 33 Hz for the driver to 50 Hz for the TL system.

The volume velocity at the back of the driver is out of phase with the open-end volume velocity. This means the front of the driver cone and the open end are in phase leading to reinforcement and improved low frequency performance, as seen on slide 20, compared to the driver in an infinite baffle. The volume velocity and displacement curves are almost half wavelength cosines. A bass reflex system also has a f_L, an enclosure tuning frequency, and a f_H just like a TL system

Standing Wave – 3/4 Wavelength Resonance



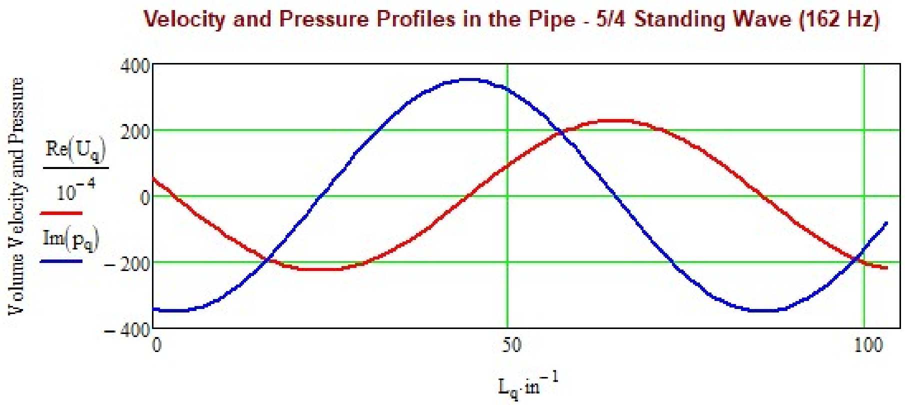


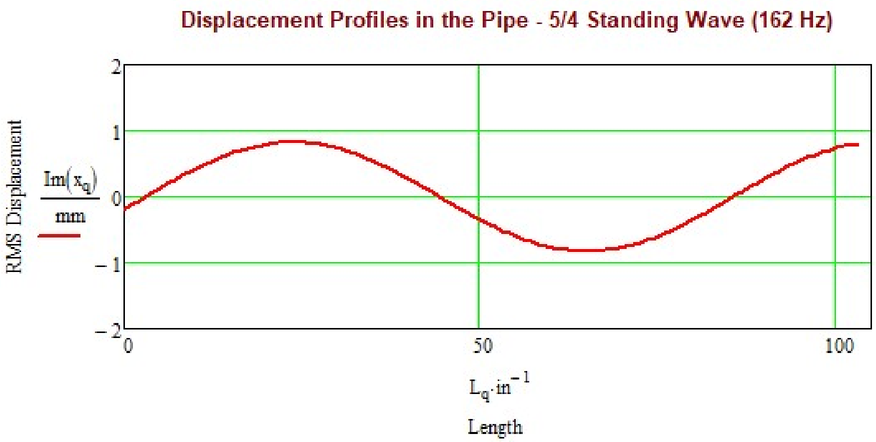
From the modal map, this is a resonance. Looking at the curves as a function of the length they do not form a complete cycle; they are three-quarters of a full sine or cosine cycle.

The volume velocity and displacement at the driver end are approaching zero, this is typical of a standing wave result and is seen in the red curve in the lower plot on slide 20.

While the volume velocity and displacement are almost zero at the driver end, they are at a positive and negative maximum at the open-end, respectively. Keep an eye on this relationship in the subsequent standing wave mode shapes.

Standing Wave – 5/4 Wavelength Resonance



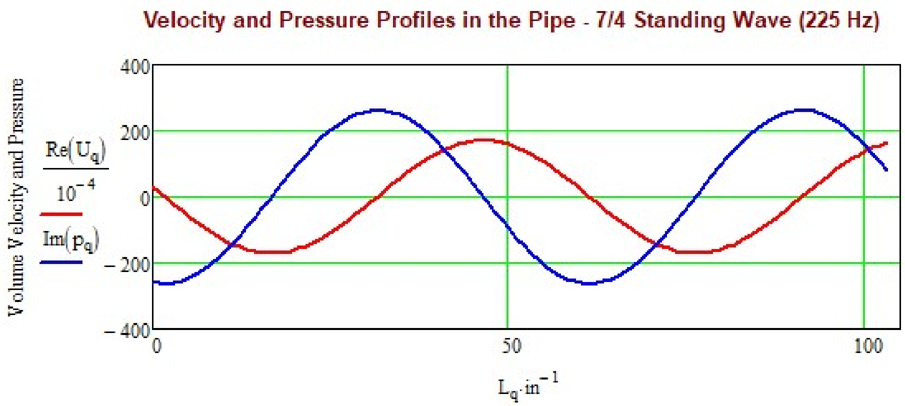


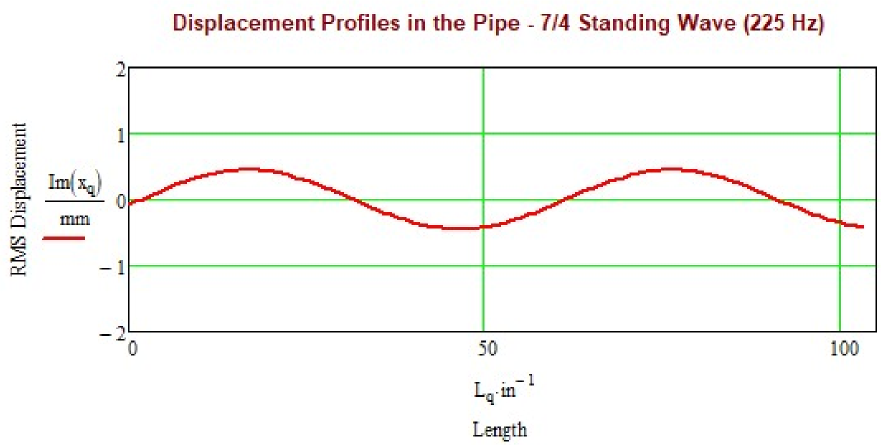
From the modal map, this is also a resonance. Looking at the curves as a function of the length they form a complete cycle plus a quarter cycle; they are five-quarters of a full sine or cosine cycle.

The volume velocity and displacement at the driver end are again almost zero, this result is seen in the red curve in the lower plot on slide 20.

While the volume velocity and displacement are almost zero at the driver end, they are now at a negative and positive maximum at the open-end, respectively. They have flipped sign, a 180-degree phase shift compared to the previous three-quarter wavelength resonance.

Standing Wave – 7/4 Wavelength Resonance



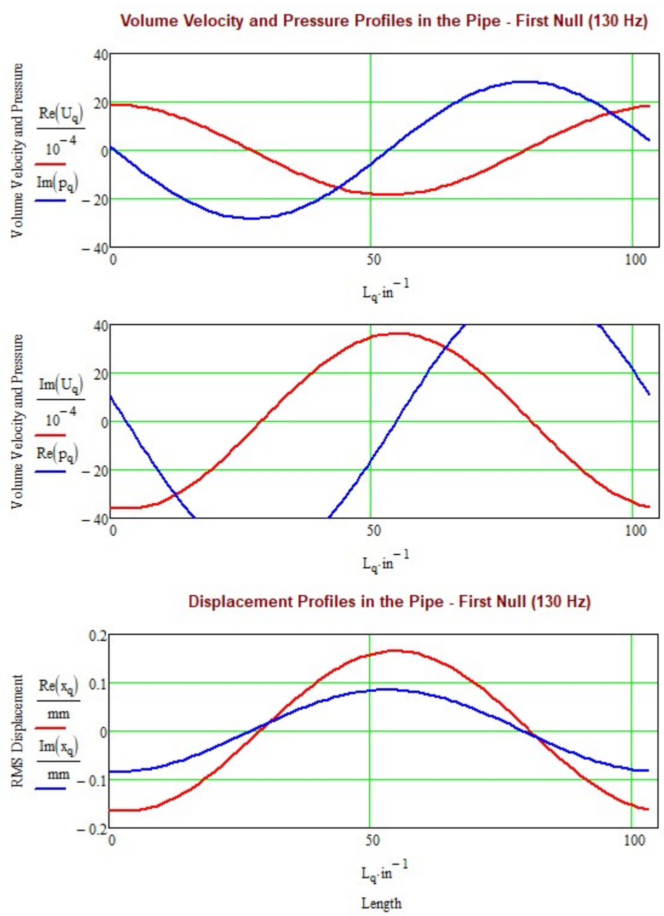


From the modal map, this is a resonance. Looking at the curves as a function of the length they form almost two complete cycles; they are seven-quarters of a full sine or cosine cycle.

The volume velocity and displacement at the driver end are almost zero, this result is seen in the red curve in the lower plot on slide 20.

While the volume velocity and displacement are almost zero at the driver end, they are now at a positive and negative maximum at the open-end, respectively. They have flipped sign again, a 360-degree phase shift compared to the previous three-quarter wavelength resonance. The open-end phases are continuously rotating as frequency increases. As frequency increases and the wavelength of sound decreases, the phase at the open-end continuously shifts.

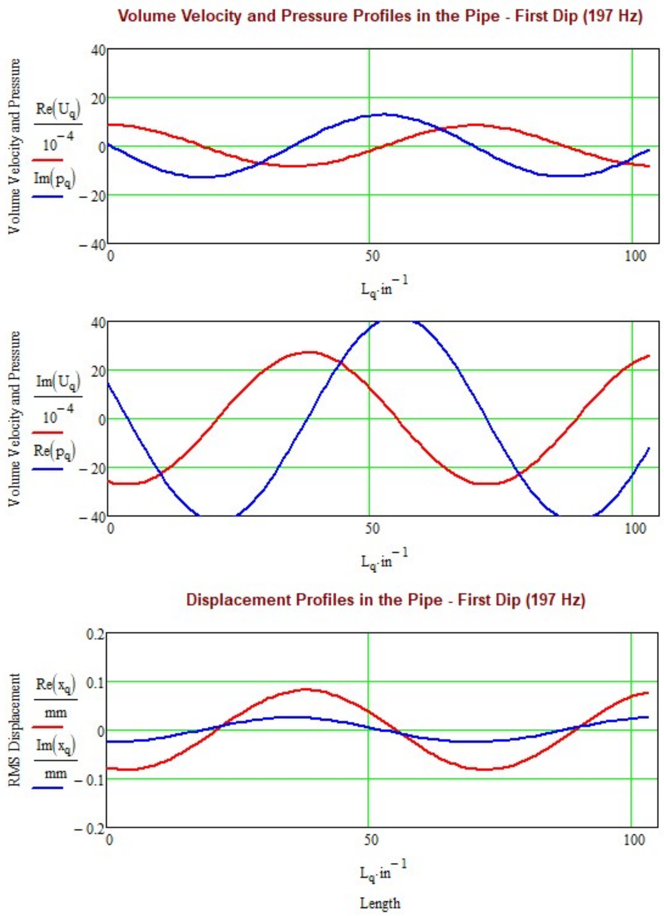
First Null



The first deep null in the upper plot shown on slide 20 occurs midway between the three-quarter and five-quarter system resonances. The volume velocity and pressure profiles shown on the left indicate that one complete cycle is produced along the length of the TL. Notice that the y-axes have been scaled up by a factor of 10, between resonances the TL enclosure is not nearly as responsive.

The volume velocity and displacement for the driver and the open-end are full cosine waves equal in magnitude and phase, this means that the front of the driver and the open-end are out of phase leading to the deep null in the upper plot on slide 20. Both the driver and the open-end are producing equal magnitude sound waves, but the summation adds destructively.

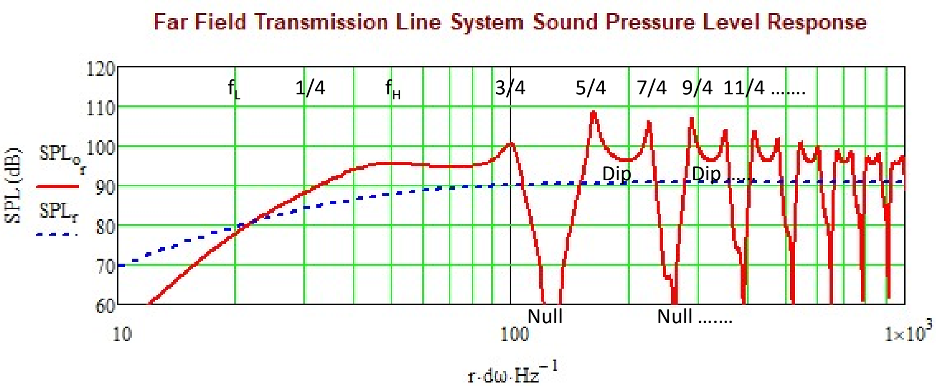
First Dip

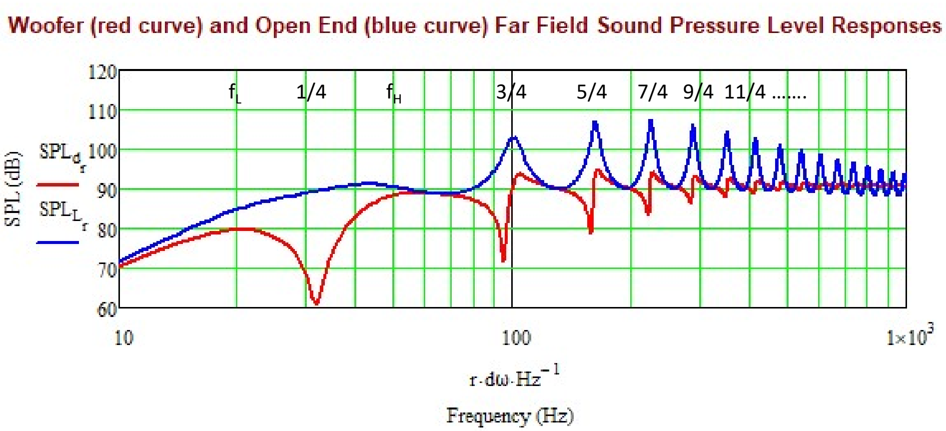


The first dip or valley in the upper plot shown on slide 20 occurs midway between the five-quarter and seven-quarter system resonances. The volume velocity and pressure profiles shown on the left indicate that one and a half complete cycles are produced along the length of the TL. Notice that the y-axes are still scaled up by a factor of 10, the TL enclosure is not nearly as responsive.

The cosine wave volume velocity and displacement for the driver and the open-end are equal in magnitude but out of phase, meaning the front of the driver and the open-end are in phase. This leads to a shallow dip in the upper plot in slide 20. Both outputs are producing sound waves that sum constructively producing +6 dB more SPL output compared to the same driver in an infinite baffle. This is also seen by inspecting and comparing the upper and lower plots in slide 20 at this frequency.

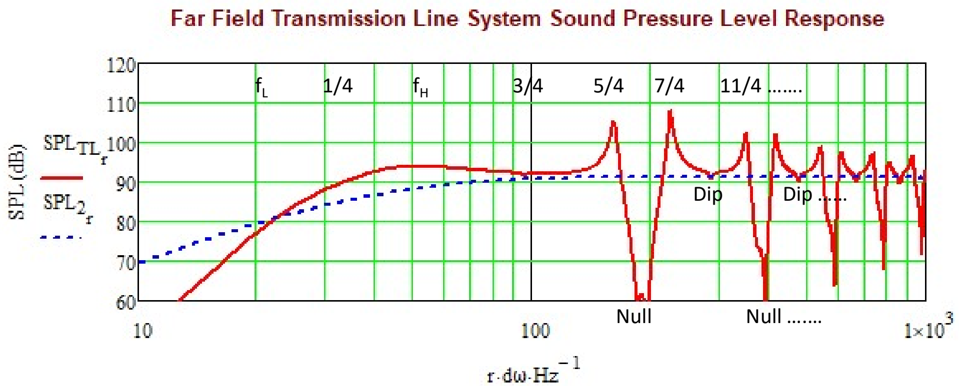
Higher Frequency Nulls and Dips

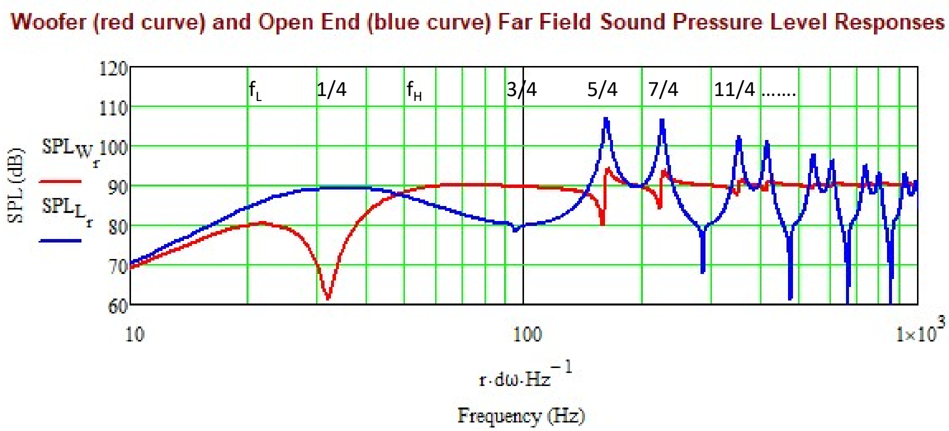




The first null and dip volume velocity and pressure distributions along the TL length were shown in the previous two slides. They are created by even harmonics of a quarter wavelength. Nulls occur at frequencies producing one, two, three and … wavelengths. Dips occur at frequencies producing one and a half, two and a half, three and a half and … wavelengths. Nulls result when the volume velocity at the back of the driver is in phase with the open-end and dips result when they are out of phase.

Offset Driver – Calculated Response

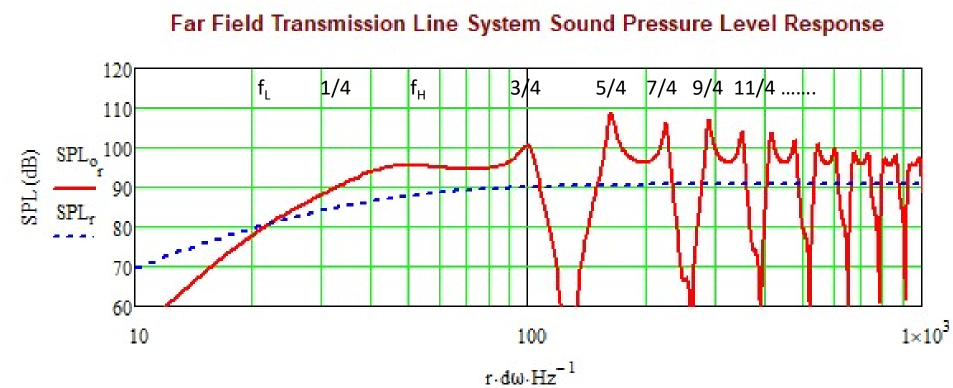


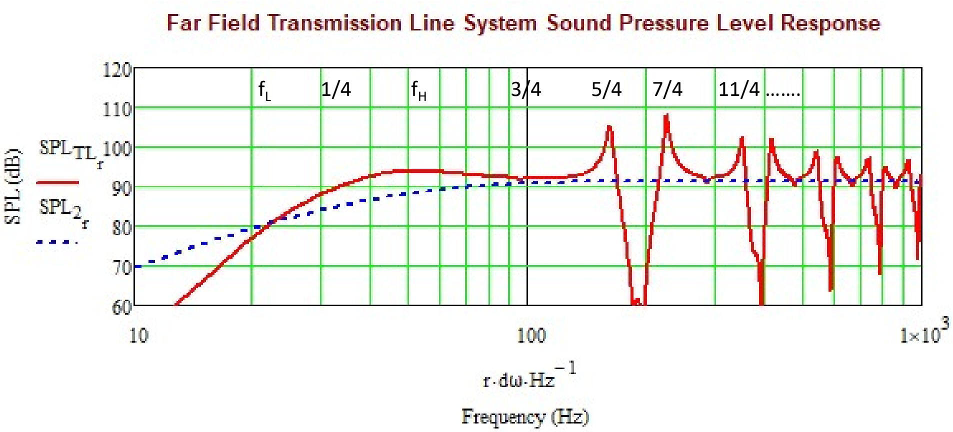


Both plots on the left are typical of an empty TL's SPL responses with an offset driver. The driver was offset approximately one-third of the length to eliminate the 3/4 wavelength resonance and mode shape.

The top plot shows the combined driver and TL open-end SPL response (red curve) and infinite baffle SPL response (blue dashed curve). The lower plot shows the separate driver (red curve) and open-end (blue curve) SPL responses.

Comparison of End Loaded and Offset Driver Responses

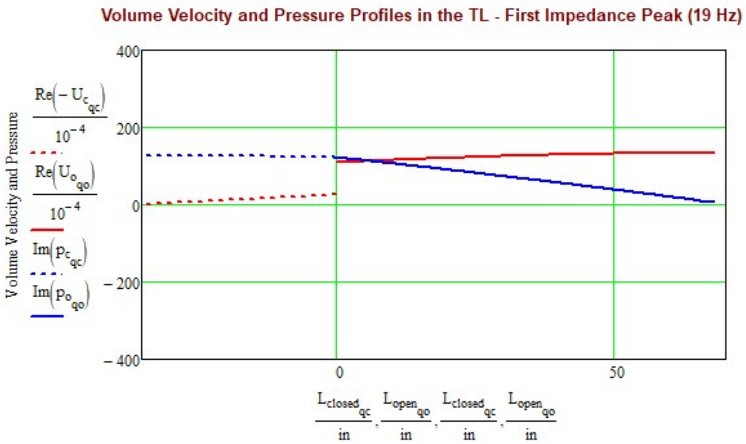


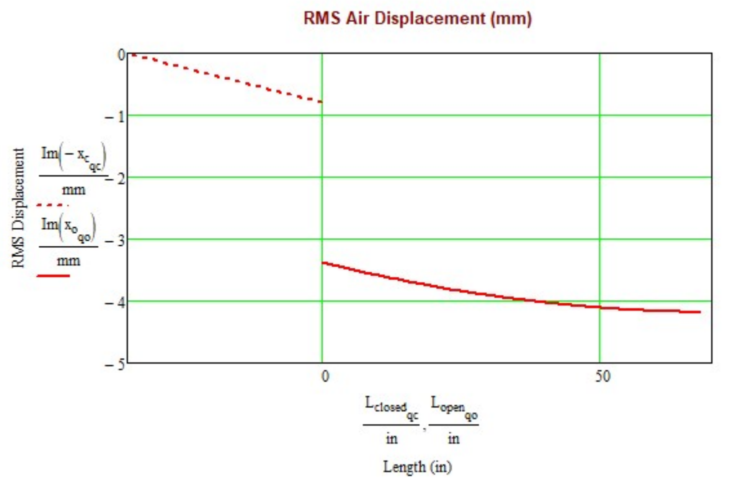


The top plot shows the combined driver and open-end SPL response and infinite baffle SPL response for the end loaded driver. The bottom plot shows the combined driver and open-end SPL response and infinite baffle SPL response for the offset driver.

Compared to the end loaded driver TL, some of the peaks are missing and the deep nulls and dips have shifted up in frequency for the offset driver TL. By offsetting the driver, peaks associated with the 3/4, 9/4, 15/4 and … resonances are no longer seen in the SPL response shown in the lower plot. For the offset driver, the deep nulls start at a higher frequency and the dips between resonant peaks do not add +6 dB more SPL output as seen for the end loaded driver. The following nine slides show the air vibration and address each identified frequency between f_L and 7/4.

Resonance and Mode Shape at f_L



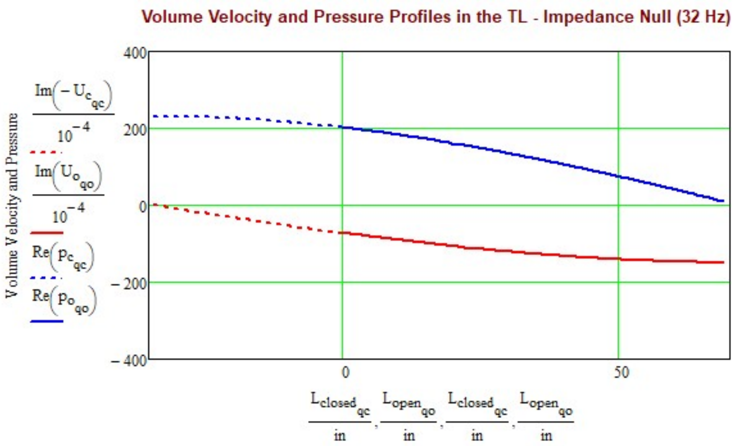


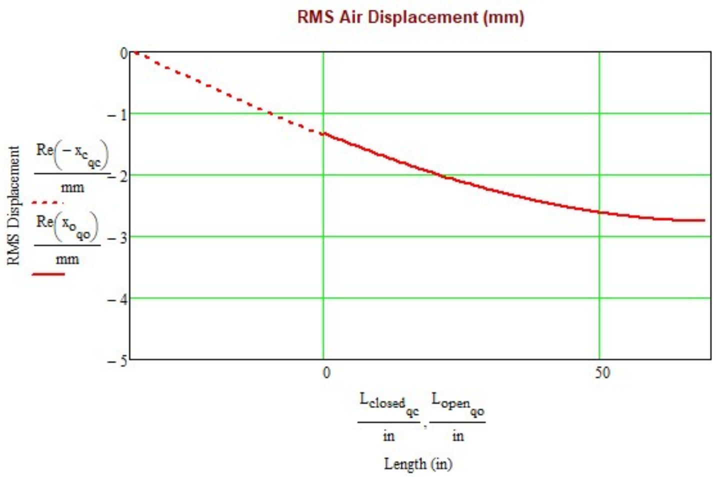
The top plot shows the real part of the volume velocity U (red curve) and the imaginary part of the pressure p (blue curve), as described in slide 17. The lower plot shows the RMS displacement of the air along the length of the TL.

The volume velocity at x = 0 is at the back of the driver's cone. The dashed curves are results for the driver to the closed-end and the solid curves are results for the driver to the open-end. The back of the driver "sees" two possible paths with different acoustic impedances. At x equal to zero the gaps between the red curves are the driver's volume velocity or displacement, the pressure is equal for both paths.

The same discussion as slide 21 is also applicable to these results.

Standing Wave – 1/4 Wavelength



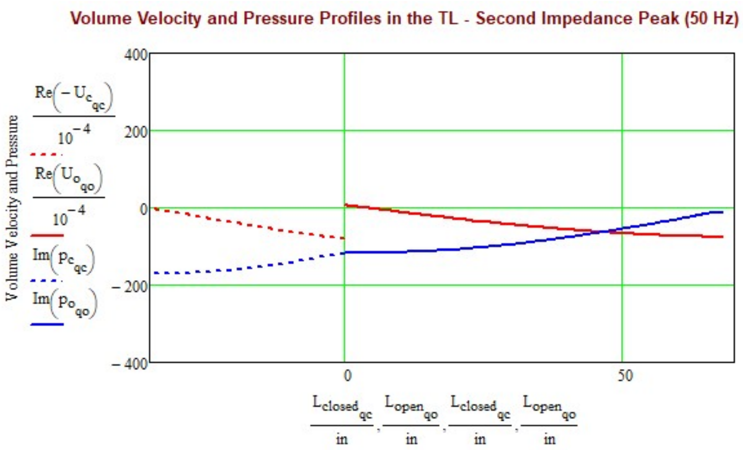


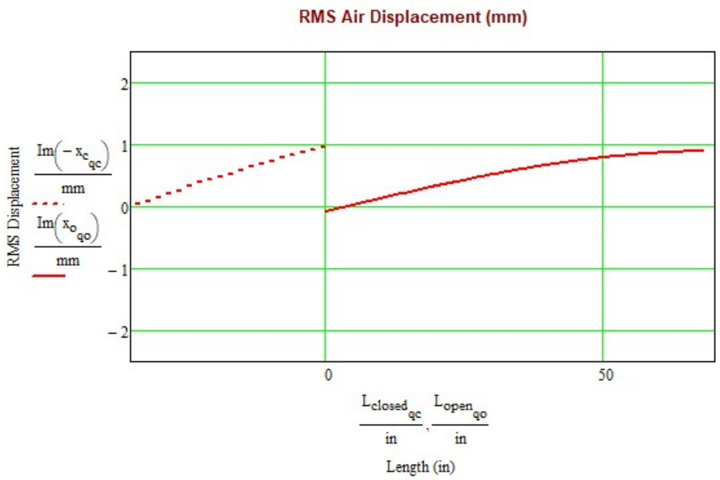
From the modal map, this is not technically a resonance. It is midway between two resonances f_L and f_H.

The volume velocity and displacement at the driver are now equal for the two paths and continuous, this means that the driver's cone motion is essentially zero. This is typical of a standing wave result and is seen in the red curve in the lower plot on slide 30.

This standing wave produces the bass output of a TL speaker system (similar to a bass reflex system). The maximum oscillating air displacement is at the open end and in this example is approaching an RMS value of 3 mm.

Resonance and Mode Shape at f_H



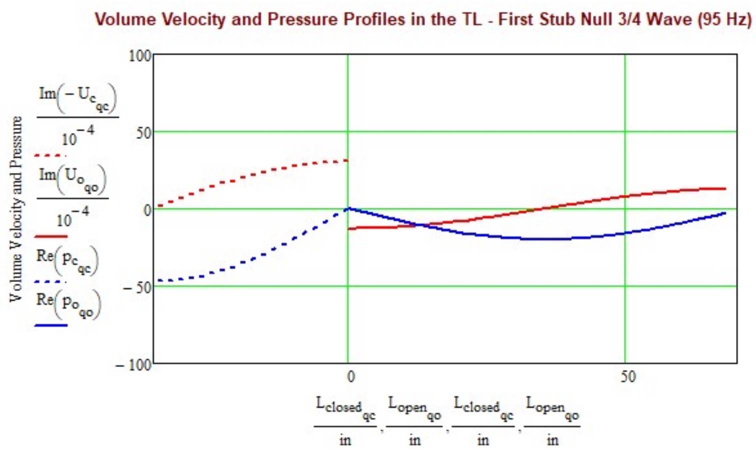


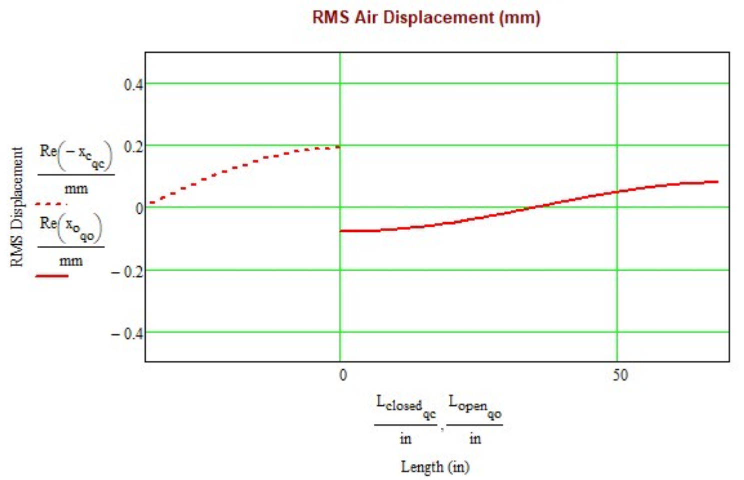
The volume velocity at the back of the driver's cone and at the open end are out of phase.

The air in the TL is being compressed and pushes on the back of the driver's cone. The air column compression, resisting the movement of the back of the driver's cone, results in an increase in resonant frequency from 32-33 Hz for the driver and enclosure subsystems to 50 Hz for the TL system.

The front of the driver's cone and the open end are in phase leading to reinforcement and improved low frequency performance, as seen in the upper curve on slide 30, compared with the same driver in an infinite baffle.

Standing Wave – 3/4 Wavelength Resonance

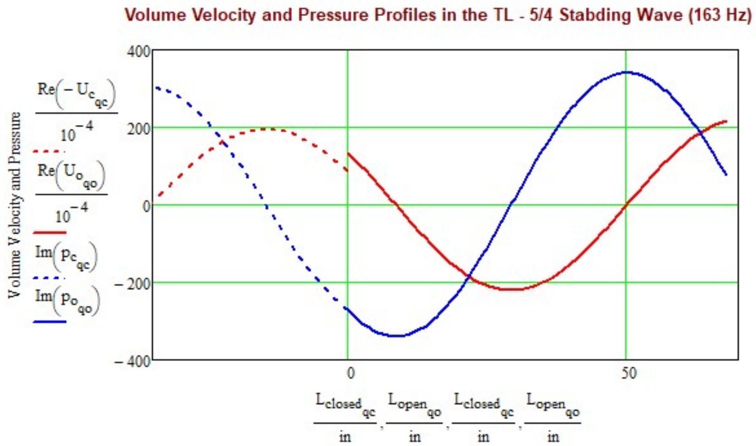


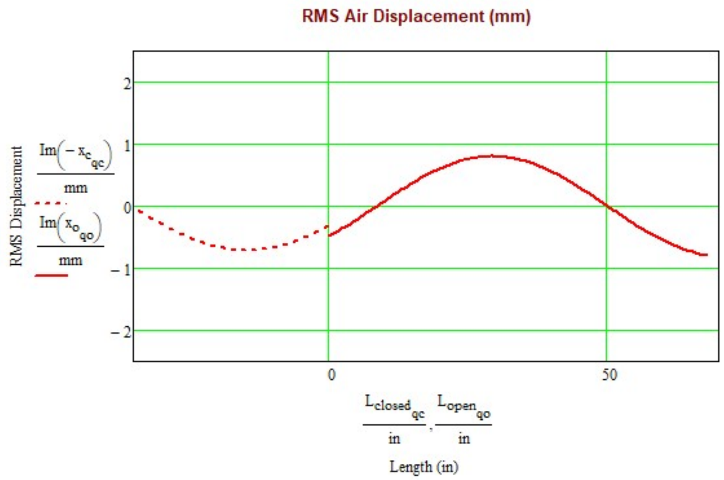


From the modal map this is a resonance, but it is weakly excited. The vertical scales in both plots have been magnified by a factor of 4 to more clearly show the results. Looking at the curves as a function of the length, the driver's volume velocity is primarily diverted towards the closed end. A quarter wave is seen between the driver and the closed end while the pressure on the back of the driver's cone is almost zero. The volume velocity and pressure curves between the driver and the open-end are very small in magnitude.

The SPL output from the open-end is not significant so almost all the system's SPL is produced by the front of the driver's cone.

Standing Wave – 5/4 Wavelength Resonance



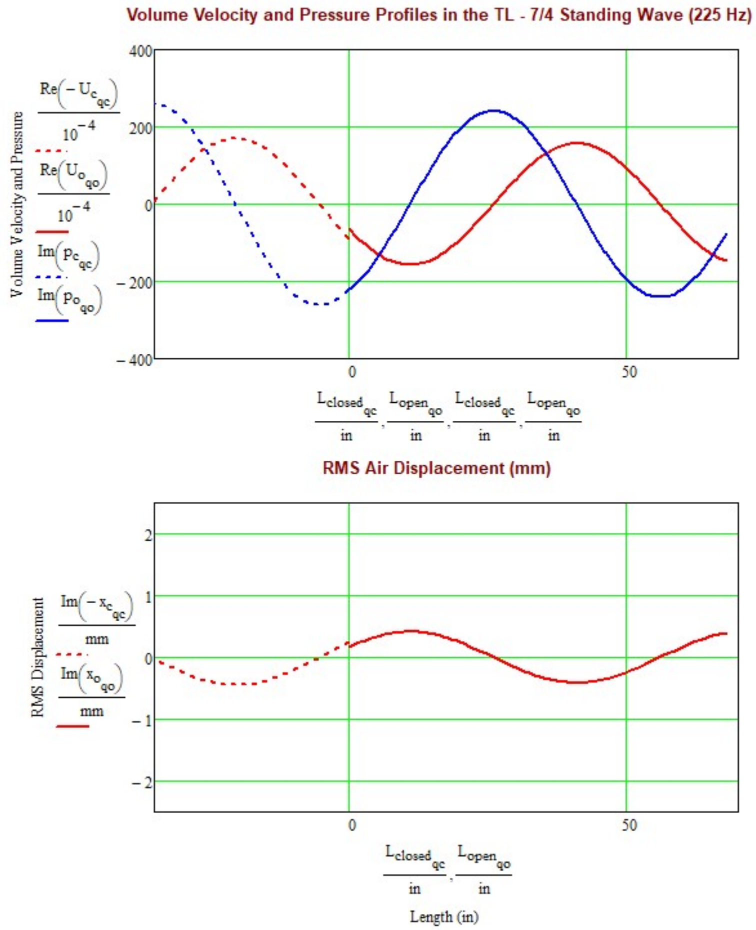


From the modal map, this is a resonance. Looking at the curves as a function of the length they form a complete cycle plus a quarter cycle; they are five-quarters of a full sine or cosine cycle.

The volume velocity and displacement at the driver end are again almost zero, the volume velocity and pressure curves are almost continuous at x equal to zero.

While the volume velocity is almost zero at the driver, it is positive and maximum at the open-end, respectively. It has not flipped sign compared to the suppressed three-quarter wavelength resonance. It has flipped sign with respect to the last resonance excited at f_H.

Standing Wave – 7/4 Wavelength Resonance

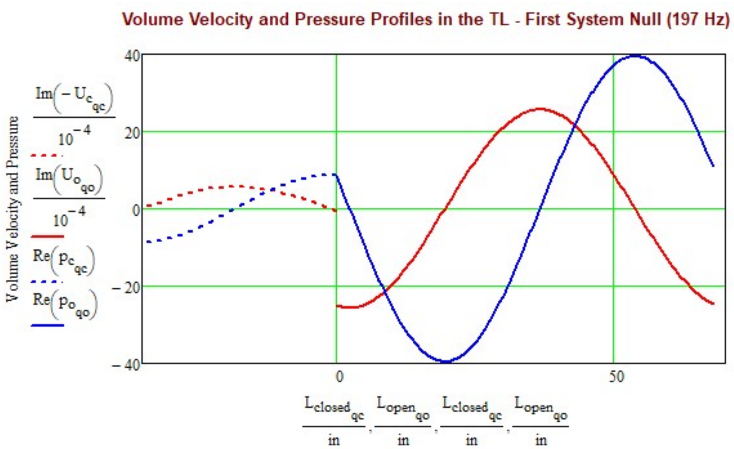


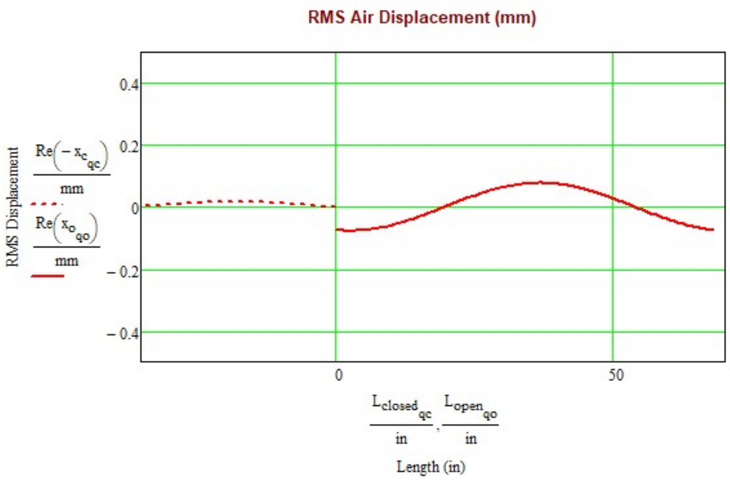
From the modal map, this is also a resonance. Looking at the curves as a function of the length they form almost two complete cycles; they are seven-quarters of a full sine or cosine cycle.

The volume velocity and displacement at the driver are again almost zero, this result is seen in the red curve in the lower plot on slide 30.

While the volume velocity is almost zero at the driver, it is now at a negative maximum at the open-end, respectively. It has flipped sign, a 360-degree phase shift compared to the previous f_H resonance. The open-end phases are still continuously rotating as frequency increases.

First Null

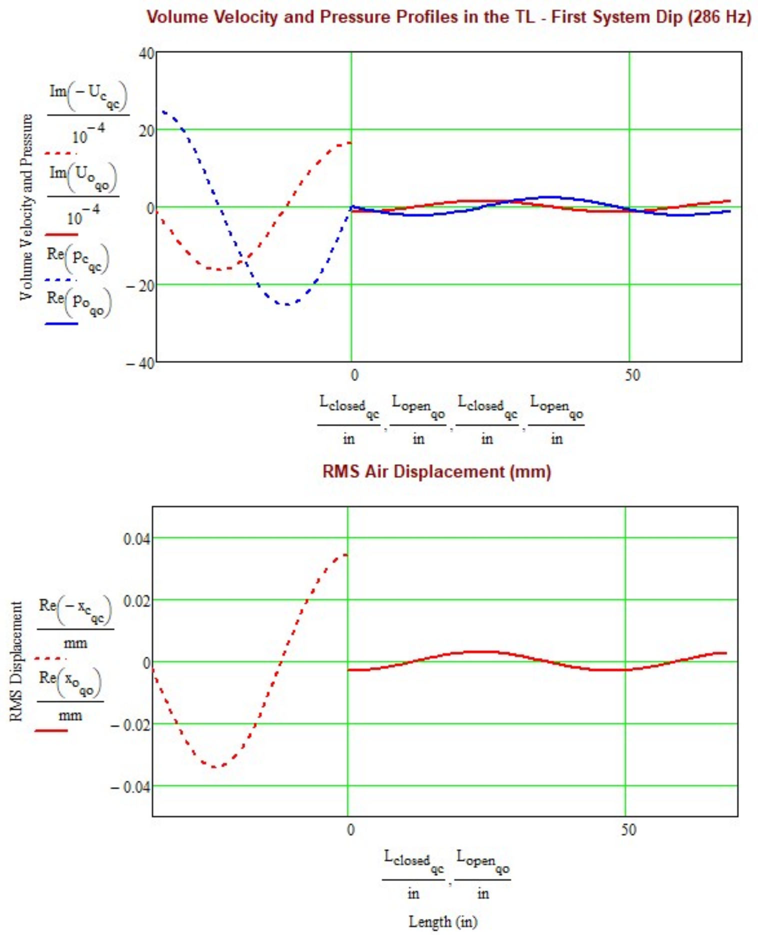




The first deep null in the upper plot shown on slide 30 occurs midway between the five-quarter and seven-quarter system resonances, higher in frequency than the end loaded driver results. The volume velocity and pressure profiles shown on the left indicate that one complete cycle is produced along the length between the driver and open-end of the TL. Notice that the y-axes have been scaled up by a factor of 10, the results are not as responsive as resonances.

The volume velocity and displacement between the driver and the open-end are full cosine waves equal in magnitude and phase, this means that the front of the driver and the open-end are out of phase leading to the deep null in the lower plot in slide 31. The increase in frequency of this null, compared to the end loaded TL, is due to the shorter distance between the driver and the open-end.

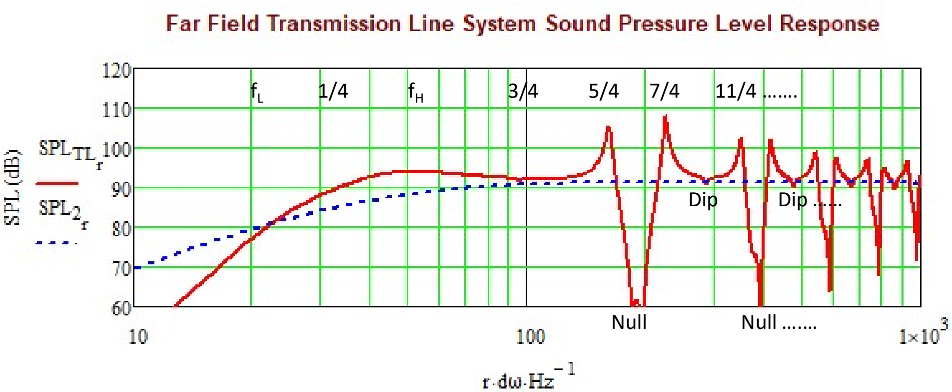
First Dip

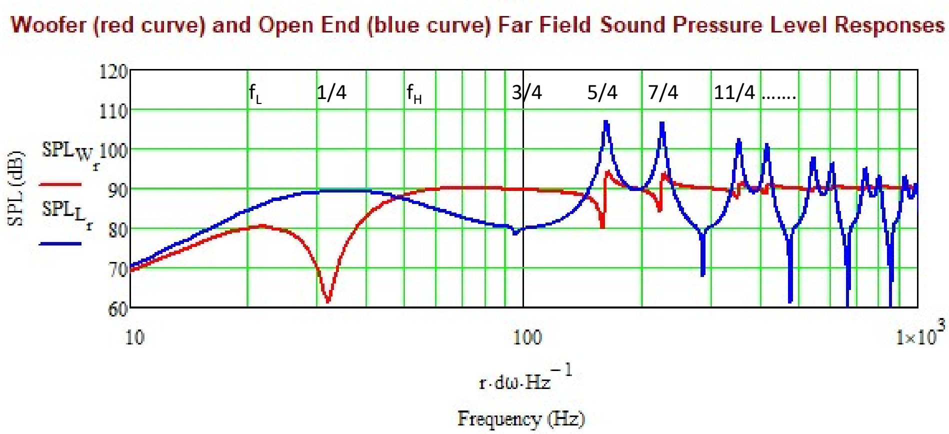


The first dip in the upper plot shown on slide 30 occurs midway between the seven-quarter and eleven-quarter system resonances. The volume velocity and pressure profiles shown on the left indicate a three-quarter wave is produced between the driver and the close-end. The acoustic impedance of the closed end shorts the rest of the TL (p ~ 0 and U is a maximum). Notice that the y-axes are still scaled by a factor of 10, the results are not as responsive as a resonance.

The cosine wave volume velocity and displacement at the driver end and the open-end are equal in magnitude, very small, but out of phase meaning the front of the driver and the open-end are in phase. The tremendous reduction in the volume velocity at the open-end eliminates the +6 dB of additional SPL output seen in the dips for the end loaded TL.

Higher Frequency Nulls and Dips

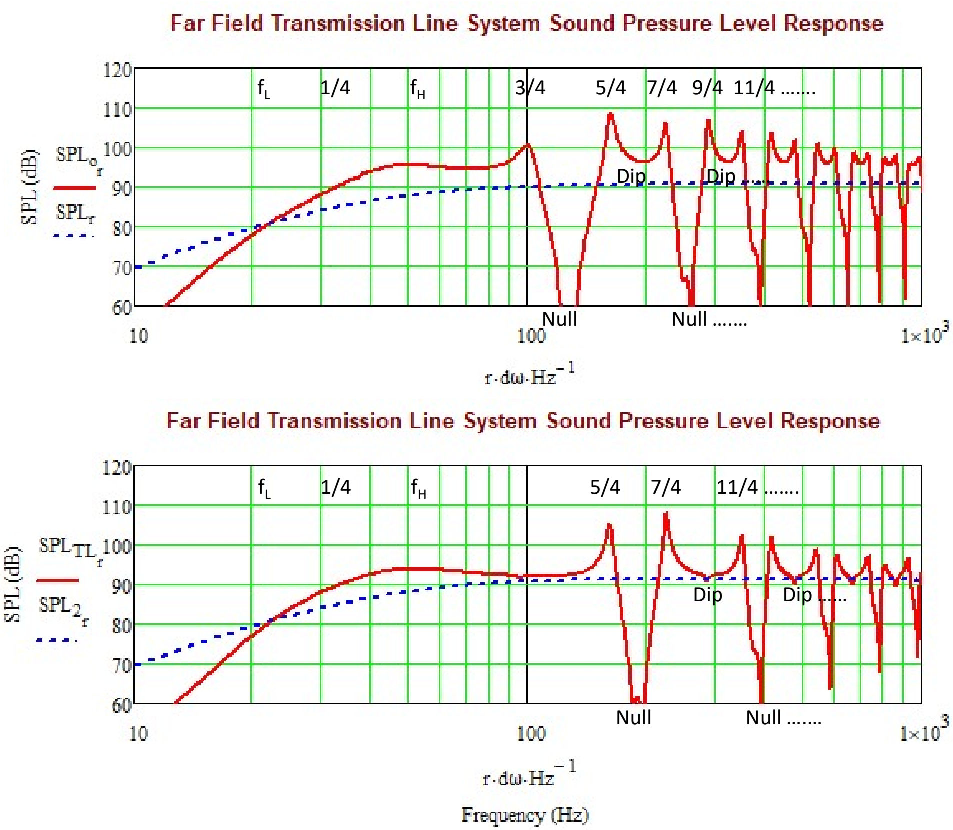




The first null and dip volume velocity and pressure profiles along the TL length were shown in the previous two slides. By offsetting the driver, the TL has essentially been split into a closed-end TL and an open-ended TL. The back of the driver sees two parallel paths, the path with the lower acoustic impedance ($Z_{acoustic}$ = p / U) captures most of the the driver's volume velocity.

Offsetting the driver to approximately one-third of the length is a special case to eliminate the 3/4 resonance standing wave. The plotted results are easily predicted and understandable. If more or less of an offset were used the plotted SPL would get messy, but the same physics apply and there would be nulls and dips interspersed between the quarter-wave peaks.

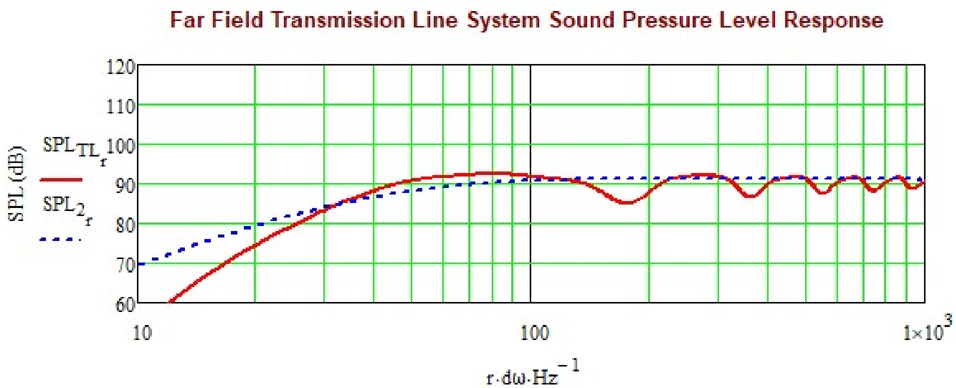
Higher Frequency Nulls and Dips

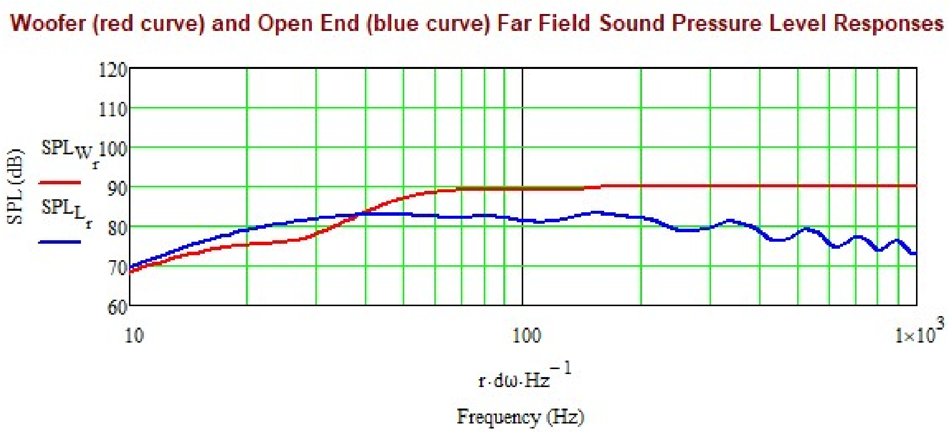


The top plot shows the combined driver and open-end SPL response and the infinite baffle SPL response for the end loaded driver. The bottom plot shows the combined driver and open-end SPL response and the infinite baffle SPL response for the offset driver.

The deep nulls and dips calculated at 1 m are artifacts of the software's assumption that the driver and TL open-end are coincident and mounted on an infinite baffle. For a typical floor standing TL, with the driver near seated ear height and the open-end near the floor, the severe nulls and shallow dips will not occur at 1 m. The combined responses shown here for an end loaded or offset driver TL are not real or accurate. Again, this is a limitation in almost all TL design software.

Offset Driver w/ Stuffing – Calculated Response





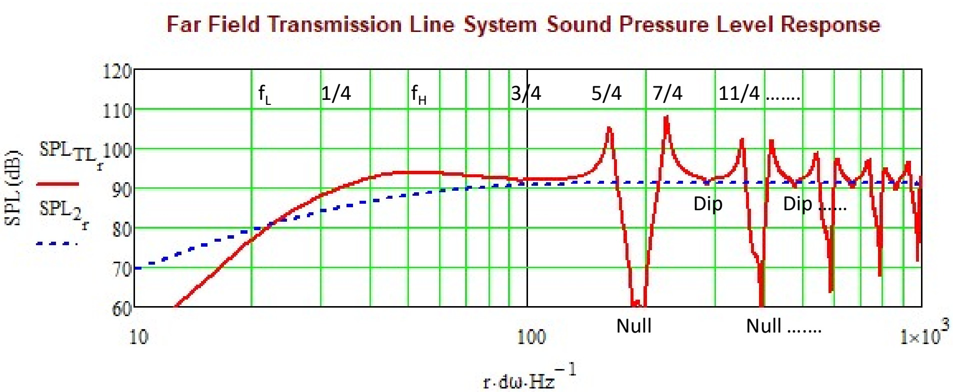
The set of plots on the left add uniform fiber stuffing to the offset driver model. Fiber stuffing with a density of 0.5 lb/ft^3 was applied along the entire length of the TL.

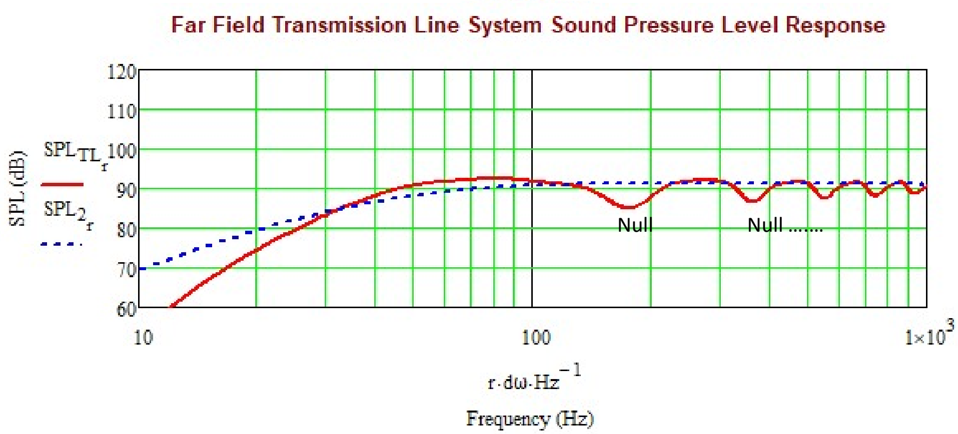
The top plot shows the combined driver and TL open-end SPL response and the infinite baffle SPL response. The lower plot shows the separate driver and open-end SPL responses.

In the lower plot, the fiber stuffing has provided enough damping to control the quarter-wave peaks as evident by the flat smooth driver SPL (red curve) response. The open end (blue curve) response shows a few small wiggles, but the SPL is far enough below the driver results that they do not impact the summed SPL response.

In the upper plot, there are a series of shallow nulls which are still caused by the assumption that the driver and open end are coincident and mounted on an infinite baffle.

Comparison of Offset Driver Responses w/o and w/ Stuffing



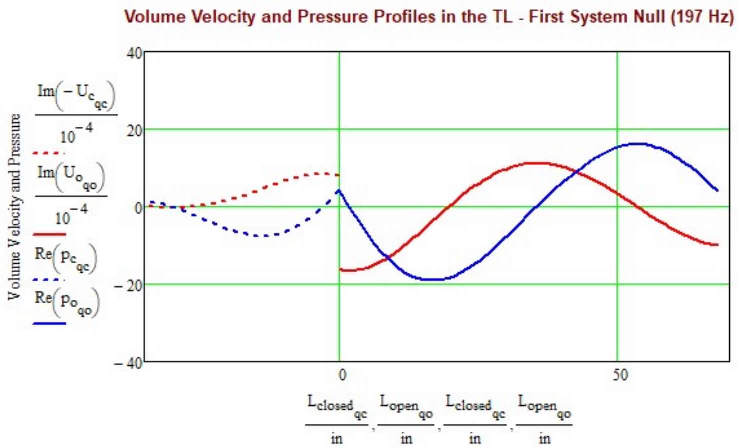


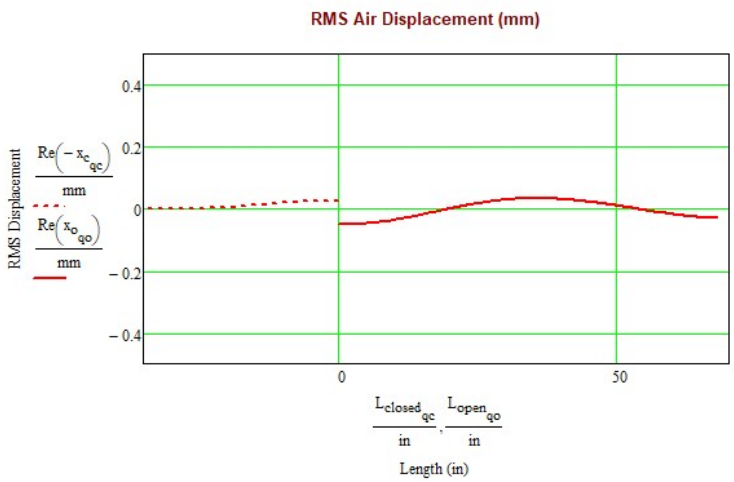
The top plot shows the combined driver and TL open-end SPL response for the empty offset driver TL. The lower plot shows the same SPL response but with uniform fiber stuffing filling the entire length of the TL.

In the bottom plot the peaks associated with the standing wave resonance have been mitigated. The shallow nulls still exist where the deep nulls previously occurred, they are not real and artifacts of the simulation software.

The following slide shows the air vibration and addresses the cause of the first null. The full set of slides showing the air vibration for each identified frequency between f_L and 7/4 are not included since they have been efficiently damped so as not to influence the summed SPL response.

First Null

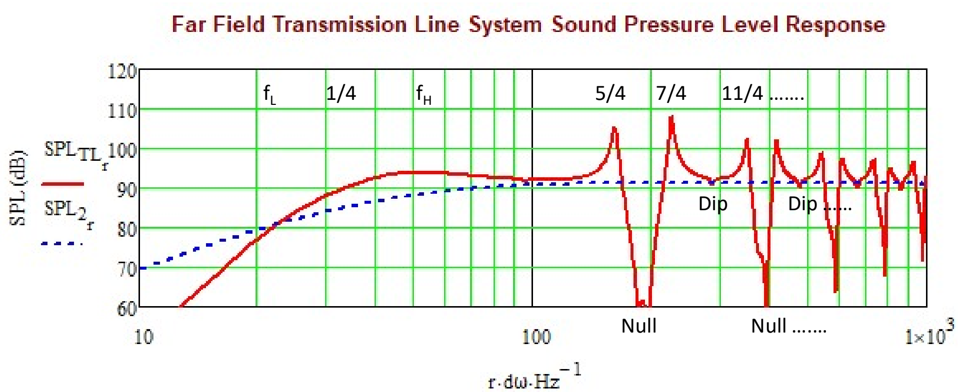


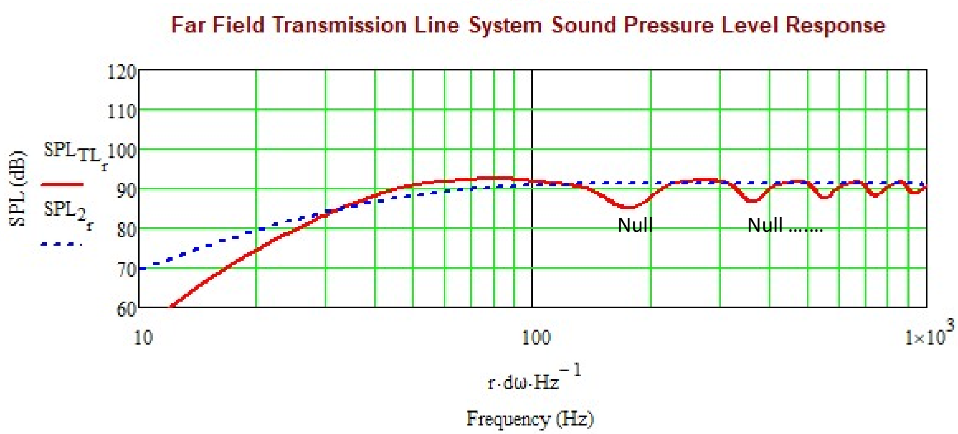


The first null in the lower plot shown on the previous slide still occurs midway between the five-quarter and seven-quarter system resonances. The volume velocity and pressure profiles shown on the left indicate that one complete cycle is produced along the length of the TL between the driver and the open-end. Notice that the y-axes have been scaled up by a factor of 10, the results are not nearly as responsive as a resonance.

The volume velocity and displacement between the driver and the open-end are full cosine waves but no longer equal in magnitude, this means that the front of the driver and the open-end are out of phase leading to the shallow null in the lower plot on the previous slide. Both outputs are producing sound waves, the summation adds but not completely destructive like the empty TL results shown in the upper plot on the previous slide.

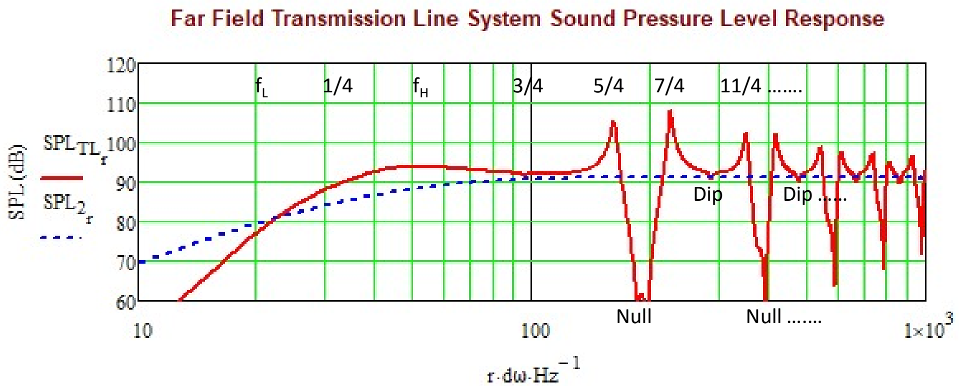
Summary

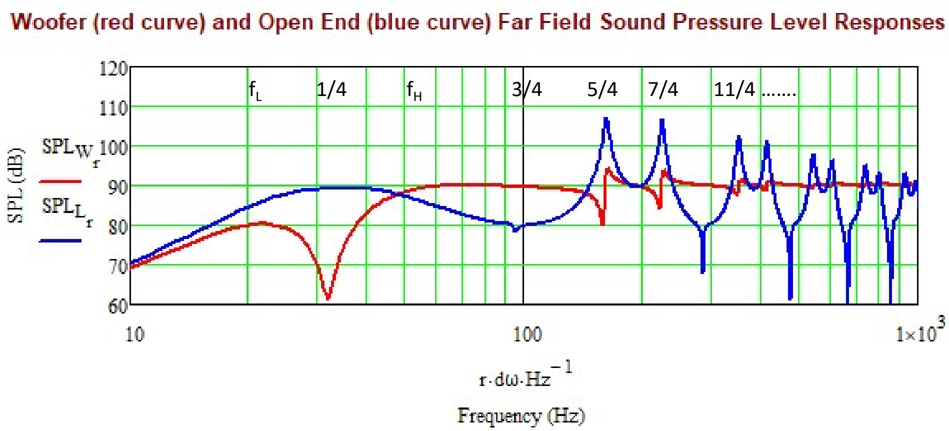




The progression from end loaded driver TL, to offset driver TL, to damped offset driver TL follow the path typically taken by a DIY TL designer. The last two steps in the path are shown in the curves on the left. The software programs used for TL designs will all produce similar results, but the interpretation and understanding of these results is not always correct.

Almost all free TL design software calculate the SPL response at 1 m assuming that the driver and open-end are coincident and mounted on an infinite baffle. This assumption leads to nulls in the system response that do not really exist at 1 m. Often higher densities of fiber damping are specified to try and mitigate these false nulls at the expense of low frequency extension and SPL output.





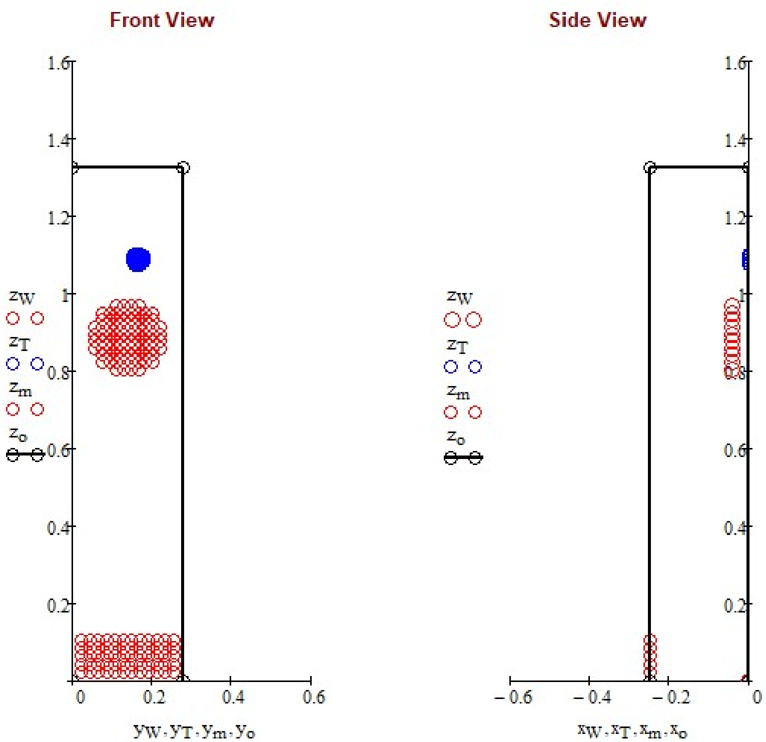
This is a well-behaved sample problem, the one third driver offset places the nulls and dips between the remaining quarter wavelength standing waves and eliminates one series of resonances (3/4, 9/4, 15/4, … standing waves).

A different driver offset, changing geometry along the length, or only partially stuffed would not produce as clean a set of SPL curves. The response would not be as regular or well separated; it would becoming much more ragged. Model a similar sample problem to see these changes in results for yourself.

When assessing SPL response (top plot) consider the individual SPL responses (bottom plot) and try to identify causes of any sharp nulls or dips with respect to the infinite baffle SPL response. There should be an obvious reason, or you need to start looking for an artifact produced by the simulation software.

Moving Outside the Box

Circular Driver and TL Terminus Simple Source Pattern with Baffle Edge Outline

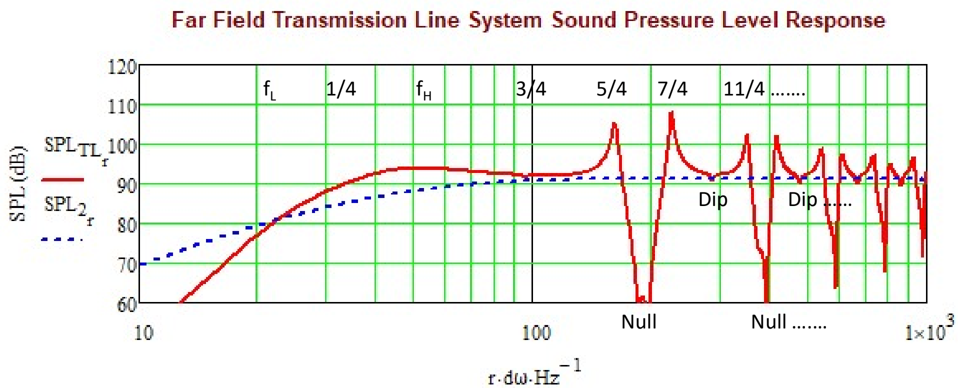


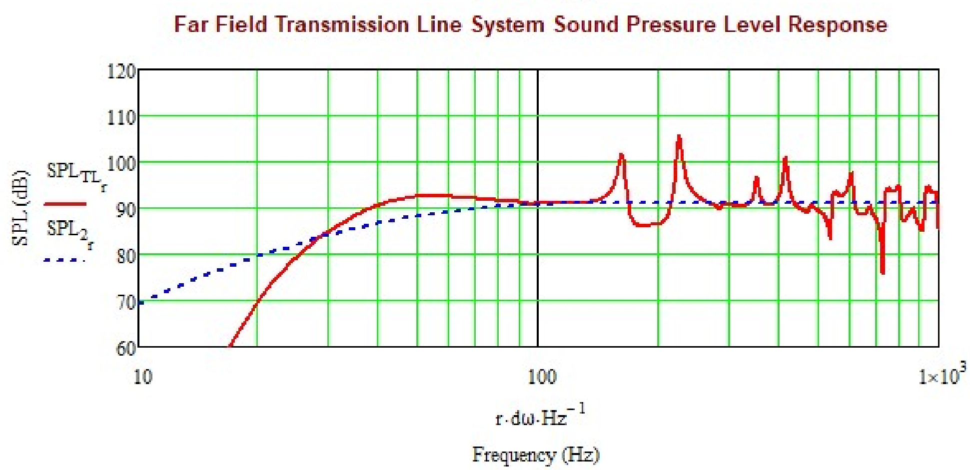
Red sources represent the woofer and terminus.
Blue sources represent the tweeter.
Black outline represents the baffle edge.
Origin is at the bottom front left corner of the enclosure.

The floor standing TL that is equivalent to the sample problem is shown on the left. The driver is offset and closer to the top of the enclosure. The open-end is located at the bottom of the enclosure, close to the floor, and can be placed on the front or rear baffle (pictured) by reconfiguring the internal dividers.

This is the geometry used in the following simulation results. First only the vertical gap between the driver and open-end will be simulated, both still assumed to be on an infinite baffle. Then the actual baffle dimensions and rear open-end location will be introduced into the calculations. Next, fiber stuffing will be added for the full path length and finally advanced corner modeling will be applied.

Non-Coincident Sources but Still Infinite Baffle



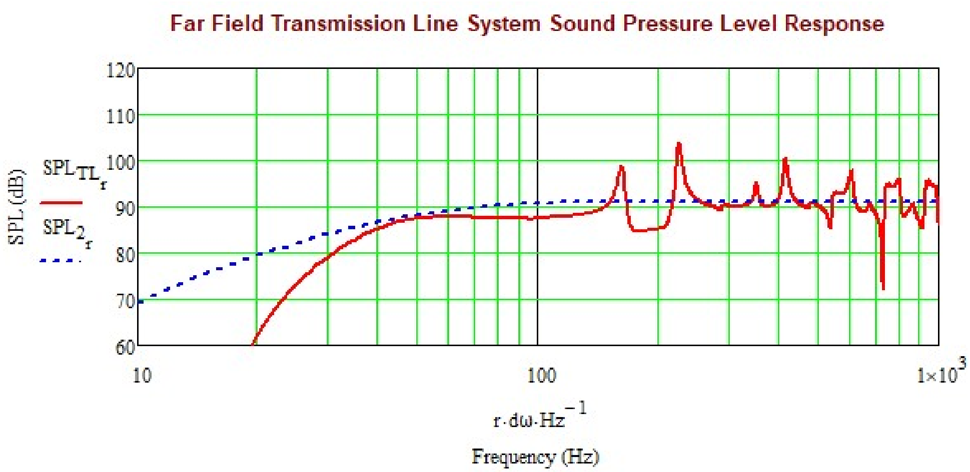


The top plot is reproduced from slide 30; it assumes an empty TL with an offset driver. The driver and open-end are assumed coincident and mounted on an infinite baffle. The SPL response calculated at 1 m contains peaks due to the standing quarter wavelength waves and deep nulls driven by the assumption of coincident sources.

The lower plot vertically separates the driver and open-end but still assumes an infinite baffle. There are still peaks due to the standing quarter wavelength waves, but the deep nulls have been eliminated.

To the best of my knowledge, the only available freeware TL design program that allows the driver and open end to be separated by a prescribed distance is Hornresp. Unfortunately, most of the Hornresp simulations I see presented on the Internet neglect this important input variable.

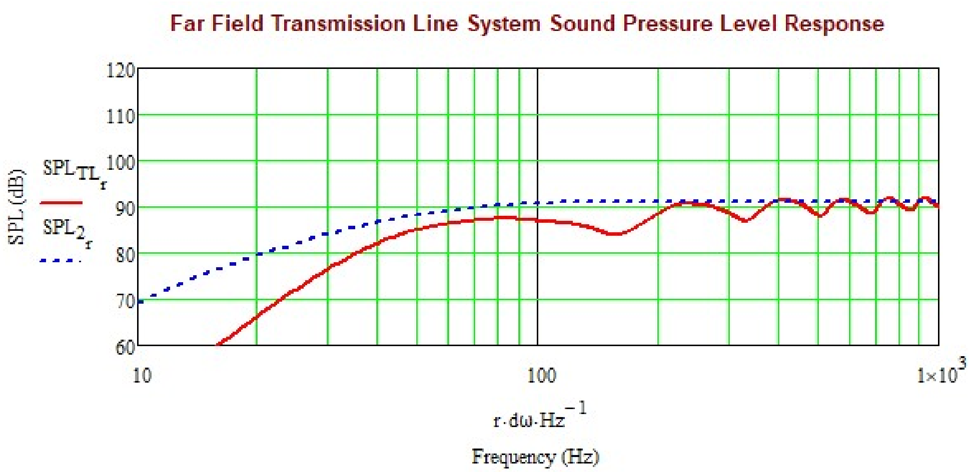
Non-Coincident Sources with Baffle Model



Modeling the finite sized front baffle and the edge sources adds the baffle step into the SPL response. While the SPL is the same at 1000 Hz it is lowered at frequencies below about 300 Hz. This is the transition from radiating into 4π space at low frequencies to radiating into 2π space at mid and higher frequencies.

One other thing to notice in this plot, the previous plot, and following plots is the steeper roll-off below the tuning frequency. This is caused by the 1 m distance assumed to the mic on the driver's axis, the open-end contribution is reduced in magnitude and shifted in phase by the extra vertical distance. Pull the mic position out to 3 m and the expected low frequency roll-off is restored along with deeper nulls since the distances from the driver and open-end to the mic position are closer to equal.

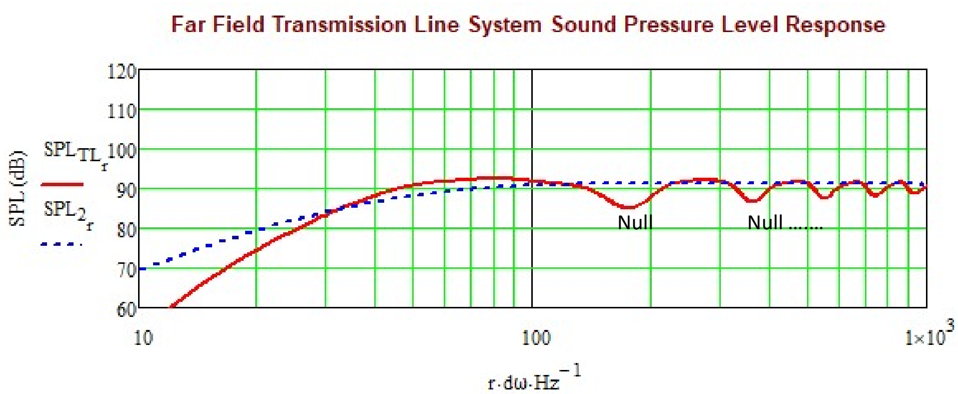
Non-Coincident Sources with Baffle and Stuffing

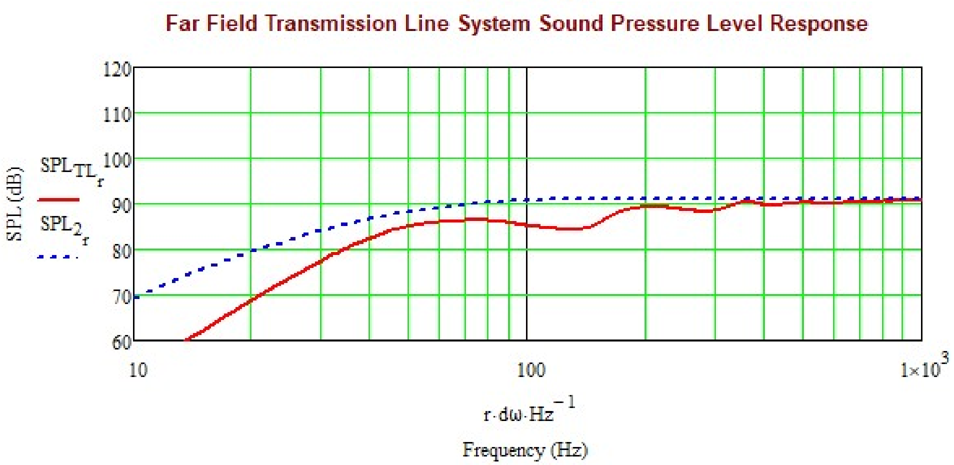


Adding stuffing tames the standing quarter wavelength waves at the enclosure's resonant frequencies. As in the previous calculations, a fiber stuffing density of 0.5 lb/ft^3 was applied to the entire length of the TL. What remains is an undulating SPL response shown on the left.

For a TL speaker system, there will always be SPL output from the driver and the open-end as functions of frequency. The lower plots in slides 20 and 30 show the driver and open-end SPL curves without fiber damping and the lower plot in slide 42 with fiber damping all at a 1 m distance on the driver's axis. Combining these two outputs in some ways is like crossover design when merging two drivers, except in this case geometry is the only method of tuning. Any point in space where the distances to the driver and the open-end are almost equal will result in nulls caused by cancellation at the TL's half wavelength frequencies. Putting the open-end on the rear baffle will help prevent this from happening in front of the speaker around the critical listening position.

Non-Coincident Sources with Baffle, Stuffing, and Advanced Corner Model

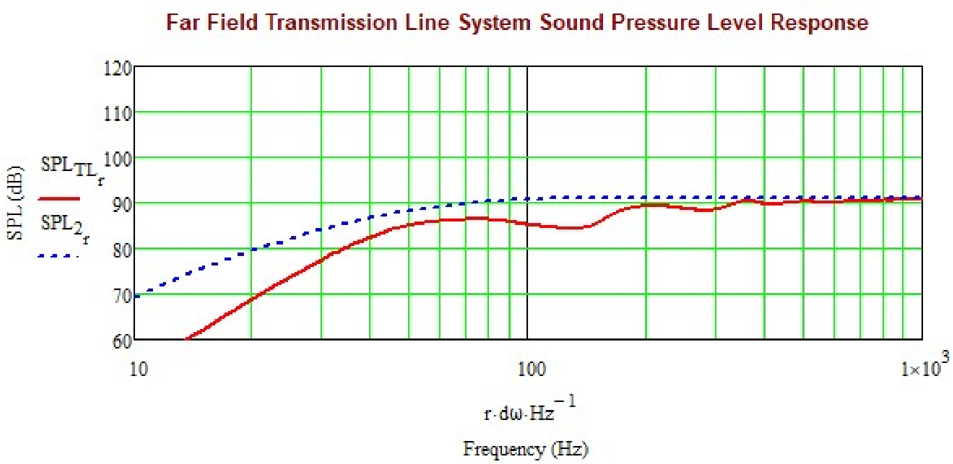




The upper plot is copied from slide 42, it assumes that the driver and open end are coincident, mounted on an infinite baffle, and 0.5 lb/ft3 of fiber stuffing is applied to the entire length of the TL.

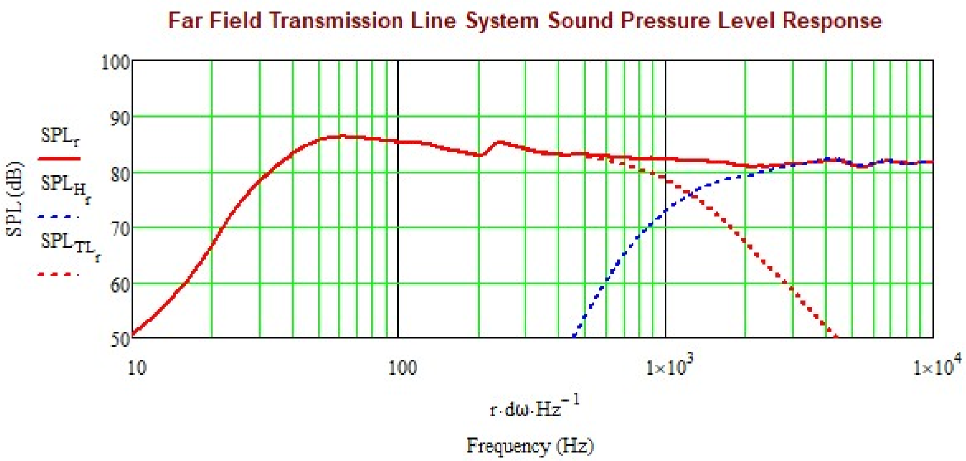
The lower plot puts together all my latest MathCad worksheet upgrades. The driver and open-end are positioned consistent with the speaker shown on slide 47. The baffle step is included in the response. Fiber stuffing with a density of 0.5 lb/ft3 is applied to the entire length of the TL. And finally, advanced corner modeling is used to fold the TL path which raises the tuning slightly while also introducing a low pass filtering effect on the open-end SPL output.

The lower plot is what I would consider the correct simulation result, as of today, for this driver and TL geometry.



The final plotted result on the left can look kind of underwhelming, a lot of work for a somewhat anemic low frequency result.

The addition of the baffle geometry and the relative positions of the driver and open-end is something not calculated in other TL design programs (or most closed or ported box programs, BASTA is an exception). The coincident driver and open-end sources mounted in an infinite baffle model will always look much better at low frequencies. However, measurements will correlate closer with the simulated SPL plot on the left compared to other freeware simulation results.



A straight constant cross-sectional area TL is a poor geometric choice. Tapering the TL, while maintaining the same internal volume and tuning, will shorten the length and push the higher order standing waves (3/4, 5.4, 7/4, …) up in frequency. Fiber or foam damping is more effective as frequency rises so less can be used, only the first ~2/3 of the line typically needs to be damped. Low frequency extension and output is increased; this style of TL enclosure design is a better optimization.

The last step in the TL speaker design is the crossover. A passive or active crossover can address the baffle step losses and smoothly merge the woofer to the midrange or tweeter. Also, room lift at the low frequencies can help even out the final low frequency SPL response curve.

The plot on the left is a simulation of a 5:1 tapered TL, offset driver, open end on the bottom oo the rear baffle, 0.25 lb/ft3 of stuffing in the first 2/3 of the line, single fold, and a passive crossover to the tweeter. A much better result.

Key Take Aways from Part 5

- The freeware TL simulation programs can accurately calculate the resonant frequencies of the speaker system and the individual SPL outputs of the driver and open-end. Most do a good job of modeling the fiber stuffing and in some cases foam lining.
- Accurate simulation results can be produced and plotted for the acoustic impedance, the electrical impedance, the driver SPL response, the driver deflection, and the open-end SPL response.
- Unfortunately, most freeware TL simulation programs treat the driver and open-end as coincident and mounted on an infinite baffle when calculating the combined TL system SPL response at a 1 m distance. This leads to dips and nulls that are artifacts of these assumptions and will not be reproduced in measurements of a finished speaker. To combat this calculated ragged system SPL response, more damping material than necessary is often used which in turn rolls off the low frequency SPL response. The TL designer may be giving away bass output to solve a simulation problem that does not really exist.
- When using one of the freeware TL simulation programs, recognize the assumptions made and the strengths and weaknesses of the tool, factor them into any prediction of the final measurable TL system SPL response. The same can be said for any speaker simulation tool (sealed or bass reflex included).