### Section 5.0 : Derivation and Correlation of the Viscous Damping Coefficient

In the equations derived in the preceding sections, the only variables that still need to be defined are the viscous damping coefficient  $\lambda$  and c the speed of sound. I tried a number of different expressions for  $\lambda$ , and numerical values for c, in an attempt to correlate the calculations with the test data, repeated from Section 2.0 in Figures 5.1, 5.2, 5.3, and 5.4, for the stuffed test transmission line. I found that the best correlation was achieved by expressing c and  $\lambda$  as functions of the stuffing density. The following relationships between the speed of sound, the damping coefficient, the stuffing density, and frequency are empirical. There was no closed form equation used to determine these characteristics of the stuffed test transmission line.

The values used for the speed of sound are 342 m/sec, 335 m/sec, 325 m/sec, and 320 m/sec for stuffing densities of 0.0 lb/ft<sup>3</sup>, 0.191 lb/ft<sup>3</sup>, 0.382 lb/ft<sup>3</sup>, and 0.573 lb/ft<sup>3</sup> respectively. These values were arrived at after starting with the measured and calculated speeds of sound shown in Table 2.4.

The mathematical expression for the viscous damping coefficient  $\lambda(\omega)$  is shown below.

$$\begin{split} \lambda_{tube} &\coloneqq 10 \cdot \frac{newton \cdot sec}{m^4} \\ \lambda_{fiber} &\coloneqq D \cdot \frac{ft^3}{lb} \cdot 1570 \cdot \frac{newton \cdot sec}{m^4} \\ \text{order} &\coloneqq 2 - \frac{1}{0.2} \cdot \left( D \cdot \frac{ft^3}{lb} - 0.2 \right) \cdot \Phi \left( D \cdot \frac{ft^3}{lb} - 0.2 \right) + \frac{1}{0.2} \cdot \left( D \cdot \frac{ft^3}{lb} - 0.4 \right) \cdot \Phi \left( D \cdot \frac{ft^3}{lb} - 0.4 \right) \\ \lambda_r &\coloneqq \lambda_{tube} + \lambda_{fiber} \cdot \left( \frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \cdot \left[ 1 + \left( \frac{r \cdot d\omega}{50 \cdot Hz} \right)^{order} \right]^{-1} \end{split}$$

The expression for  $\lambda(\omega)$  was arrived at by iterating the calculation with different constant values of viscous damping. The first term  $\lambda_{tube}$  represents a small loss that is applied to the empty tube. This keeps the empty tube calculations from growing to infinity at the quarter wavelength frequencies. The second term  $\lambda_{fiber}$  represents the viscous damping coefficient for the fiber filled tube. Notice that this expression is linear with the stuffing density D. With only these two terms, I achieved good correlation at the frequencies above 200 Hz. At the lower frequencies, the calculated response was over damped so I experimented with different high pass filter functions. The final expression for  $\lambda(\omega)$  is modified by a high pass filter. The high pass filter starts as a second order filter and transitions to a first order filter as the stuffing density increases. Again, the expression for  $\lambda(\omega)$  and the values for c were arrived at empirically to recreate the test data shown in Figures 5.1, 5.2, 5.3, and 5.4.

In Figure 5.5 the expressions for the speed of sound and the damping coefficient have been plotted. The speed of sound plot, at the top of the page, shows that the minimum speed of sound for stuffing densities less than 1.0 lb/ft<sup>3</sup> is approximately 319 m/sec. This corresponds to a process approximately midway between adiabatic and

isothermal. The figure just below shows the frequency dependent damping coefficient curves for 1.00 lb/ft<sup>3</sup>, 0.75 lb/ft<sup>3</sup>, 0.50 lb/ft<sup>3</sup>, 0.25 lb/ft<sup>3</sup>, and 0.00 lb/ft<sup>3</sup> as seen from top to bottom in the plot. From this second plot it can be seen that as the stuffing density and frequency increase, so does the magnitude of the damping coefficient.

The MathCad computer program was used to perform all of the calculations using the "TL Open End.MCD" worksheet. When you look at MathCad worksheets keep in mind that all of the calculations are performed on column vectors with each position in the vector corresponding to a specific frequency. Also recognize that MathCad accepts whatever units are entered and converts them to a consistent set of default units for all of the calculations. Therefore, I tend to work with the length L expressed in inches and the Thiele / Small parameters expressed in metric units. Plots showing the calculated impedance and the SPL magnitude and phase response, for the woofer and the terminus, are shown in Figures 5.6, 5.7, 5.8, and 5.9 for the empty test line and the test line with 100 gm, 200 gm, and 300 gm of Dacron Hollofil II stuffing. These plots also contain the measured impedance and SPL overlaid as dashed lines to show the accuracy of the computer model relative to the test data.

Again the terminus SPL phase plots were used to identify the calculated resonant frequency for each mode. At the quarter wavelength frequencies, the phase angle passes through +90 degrees or -90 degrees. The impedance curves were used to identify the calculated shifted driver resonant frequency. Table 5.1 shows these results.

Mode	Unstuffed	100 gm of	200 gm of	300 gm of
	Line (Hz)	Hollofil	Hollofil	Hollofil
		Stuffing (Hz)	Stuffing (Hz)	Stuffing (Hz)
Driver	22	22	24	26
1/4 Wavelength	93	96	96	95
3/4 Wavelength	212	210	205	200
5/4 Wavelength	340	332	320	315
7/4 Wavelength	471	457	440	432
9/4 Wavelength	603	583	561	551
11/4 Wavelength	735	710	683	670

Table 5.1 : Calculated Resonant Frequencies for the Unstuffed and Stuffed Test Line

After comparing the measured and calculated results shown in Figures 5.6 through 5.9, and the measured and calculated resonant frequencies presented in Tables 2.3 and 5.1, I concluded that the MathCad computer model was in excellent agreement with the test data. Getting test data and calculated results to agree to this degree indicated that the computer model was technically sound and could be used as a design tool for transmission line enclosures.

The same sets of measurements were also performed using long fiber wool. The wool was a much courser fibrous tangle with a larger fiber diameter than the Dacron Hollofil II. The number of wool fibers in a given volume, for the same packing density, was probably significantly less than the number of Dacron fibers. The measured results were similar in appearance to those shown in Figures 5.2, 5.3, and 5.4. Based on this second set of plots, it appeared that the wool might provide a little less viscous damping for the same packing density. If there are fewer wool fibers per unit volume, then it makes sense that the amount of viscous damping is lower. I concluded that there is no magic associated with a wool stuffed transmission line. From these results, and the

inherent problems of smell and insects associated with wool, I decided that the Dacron Hollofil II was the best choice for all of my transmission line designs.

### Summary :

The speed of sound and the damping coefficient for a fiber filled transmission line have been derived empirically from the measured test line data. These relationships have been included in the "TL Open End.MCD" MathCad worksheets and the test line modeled with the Focal 8V 4412 driver in one end. The calculated results and the measured data have been plotted in the same graphs and show excellent correlation.

## Figure 5.1 : Unstuffed Test Line



## Figure 5.2 : Test Line Stuffed With 100 gm of Dacron Hollofil II



# Figure 5.3 : Test Line Stuffed With 200 gm of Dacron Hollofil II

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# Figure 5.4 : Test Line Stuffed With 300 gm of Dacron Hollofil II

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# Figure 5.6a : Calculated Results for an Unstuffed Test Line



Impedance Calculation (Calculated = solid line, Measured = dashed line)

### Figure 5.6b : Calculated Results for an Unstuffed Test Line





Frequency (Hz) Terminus Calculated and Measured Near Field Sound Pressure Level Response



Figure 5.7a : Calculated Results for the Test Line Stuffed With 100 gm of Dacron Hollofil II



Impedance Calculation (Calculated = solid line, Measured = dashed line)

Figure 5.7b : Calculated Results for the Test Line Stuffed With 100 gm of Dacron Hollofil II









Figure 5.8a : Calculated Results for the Test Line Stuffed With 200 gm of Dacron Hollofil II



Figure 5.8b : Calculated Results for the Test Line Stuffed With 200 gm of Dacron Hollofil II









Figure 5.9a : Calculated Results for the Test Line Stuffed With 300 gm of Dacron Hollofil II



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Figure 5.9b : Calculated Results for the Test Line Stuffed With 300 gm of Dacron Hollofil II







