Section 6.0 : Advanced Transmission Line Modeling Techniques

In Section 4.0, two simple models were derived for a closed end and an open end transmission line. In both of these models the driver is located at one end of the transmission line and the cross-sectional area is monotonically varied based on a taper ratio m and the exponential area equation found in Section 3.0. For a straight-line geometry m = 0 and this is equivalent to the test line geometry shown in Figure 2.1 with an open or a closed end boundary condition.

In most transmission line designs, the driver is not all the way at one end of the line and the cross-sectional area is much more complicated with changes in area occurring at several locations along the line's length. To model these more complicated enclosures, a second set of MathCad worksheets were formulated and are presented in this section.

Offsetting the Driver in a Transmission Line :

To address the offset driver geometry, the MathCad model "TL Open End.MCD" needed to be rewritten to reflect the situation shown in Figure 6.1. When a driver is installed offset from the closed end of the transmission line, the line is split into two separate sections. Above the driver is typically a short transmission line with a closed terminus. Below the driver is a longer transmission line with an open terminus. Notice that these two transmission lines are in parallel as seen from the back of the driver.

The equivalent acoustic circuit and equivalent electrical circuit for the transmission line with an offset driver are shown in Figures 6.2 and 6.3 respectively. Notice that in the equivalent acoustic circuit, the acoustic impedance of the closed line and the acoustic impedance of the open line are in parallel. Also notice the relationship between the volume velocities at the back of the driver.

$$U_d = U_c + U_o$$

In Figure 6.1, recognize that as the driver moves into the cabinet, the air volume displaced moves into the closed ended transmission line and into the open ended transmission line. The volume split will depend on the relative values of the two acoustic impedances Z_{ac} and Z_{ao}. Also notice that the volume velocity of the air at the terminus is no longer a function of the volume velocity of the driver. The volume velocity of the air at the terminus is a function of the volume velocity of the air entering the open ended transmission line.

$$\frac{U_L}{S_L} = \frac{\varepsilon\, U_o}{S_o}$$

For the first quarter wavelength mode, offsetting the driver has no significant impact. The most obvious impact of offsetting the driver in a transmission line is a dip or null in the terminus response. This shows up in both stuffed and unstuffed terminus responses. In the frequencies above the first quarter wavelength resonance, the dramatic attenuation of the terminus output can locally reduce if not eliminate the ripple typically seen in most transmission line system SPL responses.

The reason for the severe dip or null in the terminus response can be found in Figure 4.2 for an unstuffed closed end transmission line. In this figure the first half wavelength mode is seen as a spike at 171 Hz. But at the frequency corresponding to a quarter wavelength standing wave, ~86 Hz, a deep null can be seen in the pressure response. Since the acoustic impedance of a transmission line is the ratio of the pressure to the volume velocity, a deep null must also exist in the acoustic impedance of the closed end line. A deep null in this acoustic impedance will tend to short the parallel impedances in Figure 6.2 and all of the driver's volume velocity will be directed into the closed end. The means that the output from the terminus will be strongly attenuated.

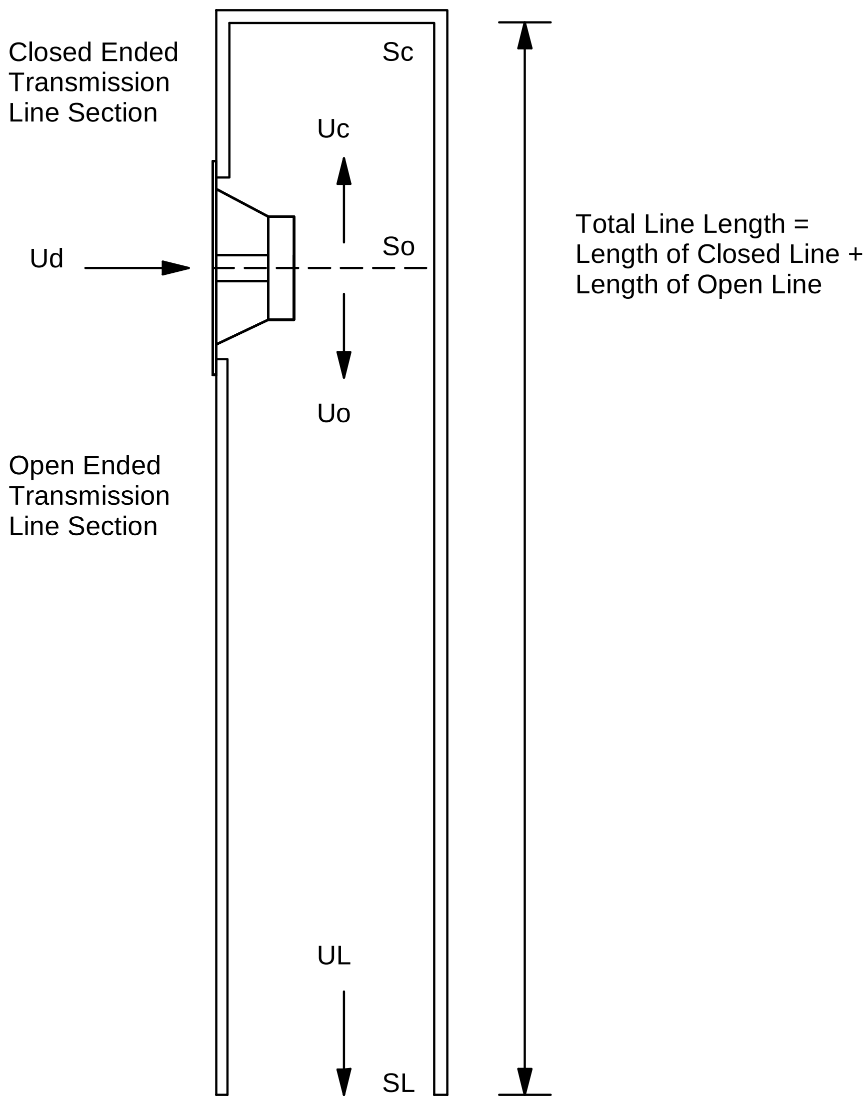
For a stuffed transmission line, the same phenomenon can be observed. The damping from the fibrous tangle will tend to reduce the sharpness of all the resonant peaks and at the same time reduce the depth of the first null resulting from the closed end portion of the transmission line. By reducing the sharpness of the peaks and nulls, the frequency content is also locally broadening for the volume velocities into the open and closed ends of the transmission line. The resulting shallower dip will not attenuate the terminus response as much as the unstuffed line but it will still have a significantly wider effect on the total SPL system response.

In summary, offsetting the driver from the end of a transmission line will dramatically reduce the terminus SPL response at the quarter wavelength frequencies of the closed end section of the transmission line. By shifting the position of the driver, the position of the dips in the terminus response can be moved higher or lower in frequency. This geometry can be used to reduce the midrange ripple inherent in a transmission line design with a driver mounted at one end. It should also be recognized that shifting the driver does not change the basic quarter wavelength frequencies that would be calculated for the entire transmission line length. Only the amount of excitation applied to each quarter wavelength mode is changed.

If the driver is offset to a location at approximately one third the length of a straight transmission line, the 3/4 wavelength standing wave can be totally suppressed. In fact, the impact from every other quarter wavelength mode (3/4, 7/4, 11/4, …) will be essentially removed from the system SPL response.

The MathCad worksheet "TL Open End.MCD" was reconfigured to model the equivalent circuits in Figures 6.2 and 6.3. The third worksheet is called "TL Offset Driver.MCD". This MathCad computer model can be used to simulate an offset driver, in a tapered, straight, or expanding fiber damped transmission line. This model will handle a wide variety of geometries with Dacon Hollofil II stuffing densities between 0.0 lb/ft^3 and 1.0 lb/ft^3. The only restriction on the geometry is that the closed end cross-sectional area S_c must be greater than zero. If the user is trying to simulate a pointed closed end, a very small numerical value for the area can be specified for S_c.

Figure 6.1 : Offset Driver Geometry



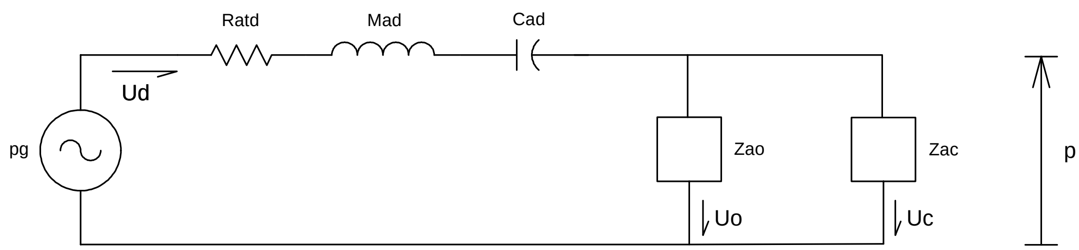
where

S_o = cross-sectional area behind the driver
U_o = volume velocity into the open ended line

S_c = cross-sectional area at the closed end
U_c = volume velocity into the closed ended line

S_L = cross-sectional area at the open terminus end
U_L = volume velocity exiting the terminus of the line

Figure 6.2 : Acoustic Equivalent Circuit for a Transmission Line with an Offset Driver



Where :

p_g = pressure source
= $(e_g\, Bl) / (S_d\, R_e)$

R_{ad} = driver acoustic resistance
= $(Bl^2 / S_d^2)\, [Q_{ed} / ((R_g + R_e)\, Q_{md})]$

R_{atd} = total acoustic resistance
= $R_{ad} + (Bl)^2 / [S_d^2\, ((R_g + R_e) + j\omega\, L_{vc})]$

C_{ad} = driver acoustic compliance
= $V_d / (\rho_{air}\, c^2)$

M_{ad} = driver acoustic mass
= $(f_d^2\, C_{ad})^{-1}$

Z_{ao} = open ended transmission line acoustic impedance

Z_{ac} = closed ended transmission line acoustic impedance

U_d = driver volume velocity
= $S_d\, u_d$

u_d = driver cone velocity

U_o = open ended volume velocity

U_c = closed ended volume velocity

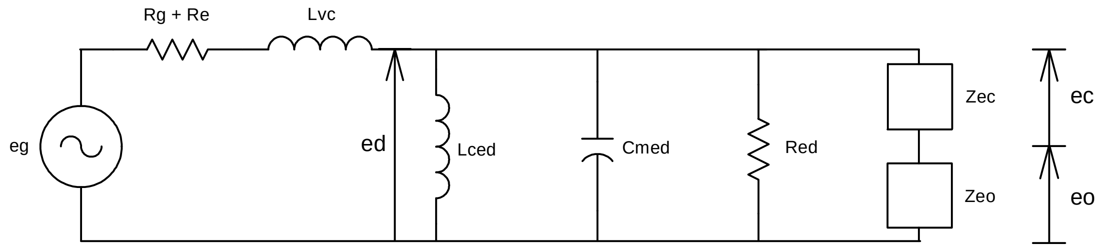
U_d = $U_o + U_c$

then :

u_L = terminus air volume velocity
= $\varepsilon\, u_o$

ε = u_L / u_o

Figure 6.3 : Electrical Equivalent Circuit for a Transmission Line with an Offset Driver



Where :

e_g = voltage source
= 2.8284 volt

R_g+R_e = electrical resistance of the amplifier, cables, and voice coil

L_{vc} = voice coil inductance

L_{ced} = inductance due to the driver suspension compliance
= $[C_{ad}\,(Bl)^2] / S_d^2$

C_{med} = capacitance due to the driver mass
= $(M_{ad}\,S_d^2) / (Bl)^2$

R_{ed} = resistance due to the driver suspension damping
= $R_e\,(Q_{md} / Q_{ed})$

Z_{eo} = open ended transmission line equivalent electrical impedance
= $(Bl)^2 / (S_d^2\,Z_{ao})$

Z_{ec} = closed ended transmission line equivalent electrical impedance
= $(Bl)^2 / (S_d^2\,Z_{ac})$

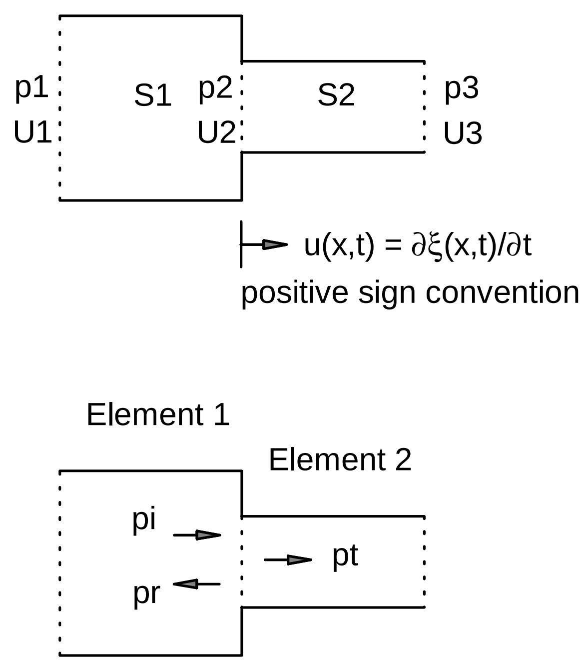
e_d = $Bl\,u_d$

Modeling a Transmission Line with a Change in Cross-Sectional Area :

To address changes in physical geometry, a method was needed to account for the changes in the line's cross-sectional area. In Beranek's[6] text, section 11 of chapter 5, he describes what happens at the junction of two pipes with different cross-sectional areas. The following sketch shows the geometry and the relationship between the pressures and the volume velocities.

Figure 6.4 : Change in a Line's Cross-Sectional Area

At the Junction :
- Pressure is continuous
- Volume velocity is continuous



pi = incident pressure wave
pr = reflected pressure wave
pt = transmitted pressure wave

S1 = area of Element 1
S2 = area of Element 2

At the junction, the pressure and the volume velocity must be the same in both pipes. As a pressure wave travels along pipe 1 and arrives at the junction, a part of the wave will be transmitted into pipe 2 while a second part of the wave will be reflected back into pipe 1. Transmission and reflection of a wave will occur at any discontinuity in the acoustic impedance. A discontinuity can be a change in the cross-sectional area, a sharp change in the taper rate of the cross-sectional area, or a sudden change in the fiber stuffing density.

To apply Beranek's methods to a transmission line using the one-dimensional acoustic element shown in Figure 4.1, and the transfer matrix of Equation (4.4), divide the line into sections based on changes in the cross-sectional area and changes in the stuffing density. For the two-element pipe, shown in Figure 6.4, the following transfer matrices can be written.

$$\begin{bmatrix} U_1 \\ p_1 \end{bmatrix} = \begin{bmatrix} -\dfrac{Z_{22}}{Z_{21}} & \dfrac{1}{Z_{21}} \\ -\dfrac{Z_{11}\,Z_{22}}{Z_{21}} + Z_{12} & \dfrac{Z_{11}}{Z_{21}} \end{bmatrix} \begin{bmatrix} U_2 \\ p_2 \end{bmatrix}$$

$$\begin{bmatrix} U_2 \\ p_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{Z_{33}}{Z_{32}} & \dfrac{1}{Z_{32}} \\ -\dfrac{Z_{22}\,Z_{33}}{Z_{32}} + Z_{23} & \dfrac{Z_{22}}{Z_{32}} \end{bmatrix} \begin{bmatrix} U_3 \\ p_3 \end{bmatrix}$$

Combining these two equations.

$$\begin{bmatrix} U_1 \\ p_1 \end{bmatrix} = \begin{bmatrix} -\dfrac{Z_{22}}{Z_{21}} & \dfrac{1}{Z_{21}} \\ -\dfrac{Z_{11}\,Z_{22}}{Z_{21}} + Z_{12} & \dfrac{Z_{11}}{Z_{21}} \end{bmatrix} \begin{bmatrix} -\dfrac{Z_{33}}{Z_{32}} & \dfrac{1}{Z_{32}} \\ -\dfrac{Z_{22}\,Z_{33}}{Z_{32}} + Z_{23} & \dfrac{Z_{22}}{Z_{32}} \end{bmatrix} \begin{bmatrix} U_3 \\ p_3 \end{bmatrix}$$

So the pressure and volume velocity at one end of a complicated geometry can now be expressed in terms of the pressure and volume velocity at the far end. Simple multiplication of 2 x 2 matrices can be performed to quickly work through 1, 5, 10, or even 100 one-dimensional acoustic elements. After compiling all of the elements, the pressure and volume velocity at one end of a transmission line is a related to the pressure and volume velocity at the far end of the line by a single 2 x 2 transfer matrix. The boundary conditions described in Section 4.0 can now be applied to calculate the acoustic impedance and if applicable the terminus volume velocity.

The MathCad model "TL Sections.MCD is based on the transfer matrix method described above and simulates an offset driver in a fiber filled transmission line. The number of acoustic elements can be increased in both the closed and open ended sections of the transmission line so that very complex geometry changes and stuffing density schemes can be modeled.

Behavior of Tapered, Straight, and Expanding Transmission Lines :

After making the corrections and improvements to the MathCad worksheets over the past two years, my understanding of tapered and expanding transmission lines improved considerably. I began to see some real benefits for both tapered and expanding transmission lines. One thing I studied was the changes that occur, in the straight transmission line's quarter wavelength resonant frequencies, when a tapered or expanding geometry is introduced.

The following example illustrates the differences in the quarter wavelength resonant frequencies for a tapered, a straight, and an expanding transmission line. Assume that the three transmission lines all have the same length, the same internal volume, and are modeled without any internal stuffing. The basic geometry is defined below for the straight transmission line designed for a 40 Hz quarter wavelength resonant frequency.

Area = (10 in) (10 in) = 100 in^2
= 0.065 m^2

Length = c / (4 x f)
= (342 m/sec) / (4 x 40 Hz)
= 84.15 in
= 2.14 m

Volume = Area x Length
= 8415.4 in^3
= 137.9 liters

Table 6.1 shows the area at the driven end of the transmission line S_0 and the area at the open end (terminus) of the transmission line S_L for the three different assumed geometries. Again, all three transmission lines have the same length and internal volume.

Table 6.1 : Cross-Sectional Area Definitions

Transmission Line Configuration	S_0 (in^2) at x = 0	S_L (in^2) at x = L
Tapered Line	150	50
Straight Line	100	100
Expanding Line	50	150

Figure 6.5 shows the magnitude of the air velocity at the terminus end of the transmission line assuming a 1 m/sec velocity at the driven end. This applied 1 m/sec air velocity is assumed to be uniform over the entire area S_0. As the frequency of the driven end increases from 1 Hz to 1000 Hz, thirteen separate resonant frequencies are excited. The sharp peaks in the plots in Figure 6.5 define the frequency of each resonance.

Looking in Figure 6.5 at the magnitude of ε for the frequencies below 10 Hz you can see that the value is different for each of the three transmission line geometries. The magnitude is equal to the ratio of the driven area S_0 over the terminus area S_L. This indicates that at very low frequencies, the volume of air moving into the line at x = 0 is equal to the volume of air moving out of the line at x = L. This relationship is derived as follows for frequencies below 10 Hz.

$$\varepsilon = u(L,t) / u(0,t)$$
$$= u(L,t) / (1 \text{ m/sec})$$

$$S_0 \times u(0,t) = S_L \times u(L,t)$$
$$S_0 \times (1 \text{ m/sec}) = S_L \times u(L,t)$$
$$S_0 / S_L = u(L,t) / (1 \text{ m/sec})$$

$$\varepsilon = S_0 / S_L$$

Also notice in Figure 6.5 that the resonant peaks above 100 Hz appear to occur at approximately the same frequencies. However it can also be seen in Figure 6.5, that the first resonance for each of the transmission lines occurs at a different frequency.

Table 6.2 summarizes the resonant frequencies of the first five modes for each of the three transmission lines. Also shown, in the second column of Table 6.2, are the ideal quarter wavelength frequencies that would be calculated based on a classical solution of the one-dimensional wave equation. The classic solution of the one-dimensional wave equation can be found in most undergraduate physics or acoustics textbooks. The problem being solved in these textbooks is typically a constant cross-section pipe with a boundary condition specified at each end. The boundary conditions used to solve the wave equation for an open ended transmission line are a sinusoidal velocity applied at the driven end S_0 and a zero pressure (or velocity maximum) defined at the open terminus end S_L.

Table 6.2 : Frequencies of the Quarter Wavelength Standing Waves

Mode Number (n)	Calculated n x c / (4 x L)	Tapered Line	Straight Line	Expanding Line
1	40	30	38	47
3	120	113	114	116
5	200	191	190	190
7	280	269	267	266
9	360	347	343	340
Units	(Hz)	(Hz)	(Hz)	(Hz)

Comparing the second and the fourth columns of Table 6.2, the MathCad model consistently calculates lower resonant frequencies then the classic textbook solution. This is due to the frequency dependent acoustic impedance, see Figure 4.6, specified at the terminus in the MathCad model instead of the zero pressure boundary condition assumed in the textbook solution. The acoustic impedance boundary condition makes the pipe appear to be slightly longer which leads to lower resonant frequencies. A similar situation occurs when sizing the length of a port in a bass reflex enclosure. The effective length of the port, used in most design calculations, is typically longer then the actual physical length of the port.

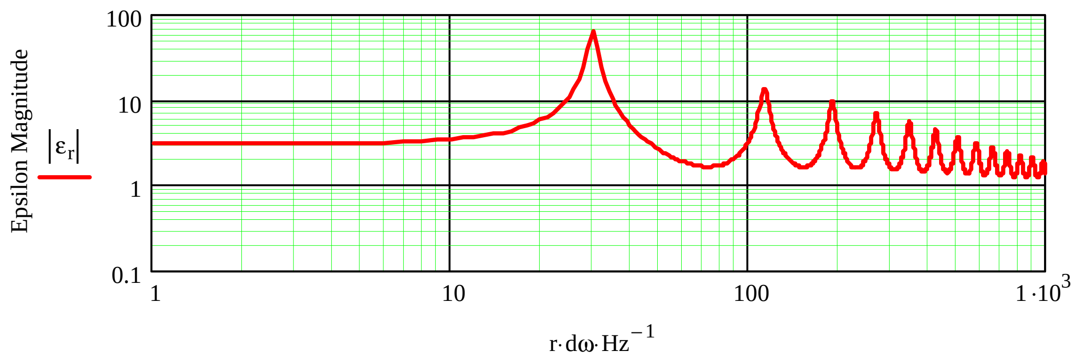
Also notice in Table 6.2, that the resonant frequencies above 100 Hz are approximately the same. As stated previously, when discussing Figure 6.5, the first mode occurs at different frequencies for each of the three transmission line geometries. For the tapered transmission line, the lowering of the first resonant frequency would lead to a shorter line for a 40 Hz design goal. For the expanding transmission line, the increase in the first resonant frequency would have the opposite effect of requiring a longer line for a 40 Hz design goal.

For most transmission lines, the cross-sectional area is usually held constant or tapered. Looking back at the work done by Bailey(7) and then by Bradbury(8), the sketches in Bailey's article would indicate that they were working with test results obtained from tapered transmission lines. Suppose that the expected quarter wavelength resonant frequencies were calculated in the same manner as those shown in the second column of Table 6.2 using the classic equation f = c / (4 x L). Now recognize that the measured results were probably more typical of the resonant frequencies listed in the third column of Table 6.2. The tapered transmission line would have exhibited a lower resonant frequency for the first mode, when compared to a straight transmission line, but correlated closely with the higher frequency modes of the straight transmission line. Bradbury's theory contends that only the low frequency sound waves couple with the fibers, through a viscous damping coefficient, resulting in motion of the fibers and a reduction in the speed of sound due to the added moving mass of the fibers. This postulated reduction in the speed of sound would result in a lower resonant frequency for the first quarter wavelength mode as observed in the test data. At higher frequencies, the fibers are not coupled to the sound waves and do not move, so the speed of sound is unchanged and the resonant frequencies are closer to the expected values.

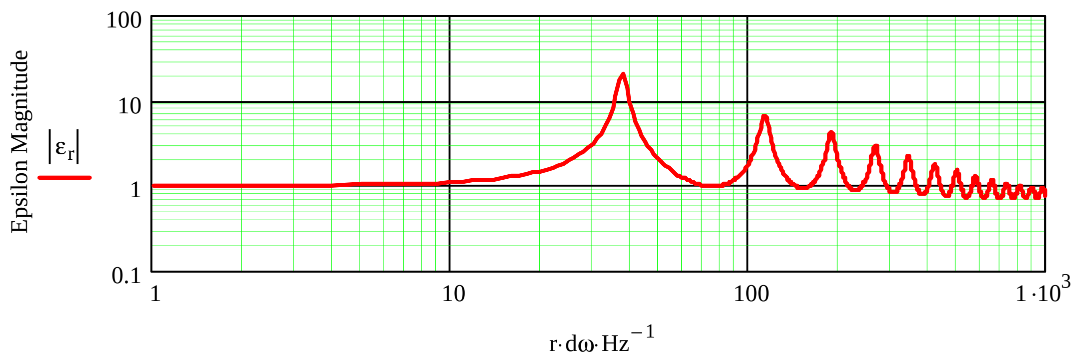
If Bradbury did not include the impact of the tapered geometry on the anticipated quarter wavelength frequencies in his analysis of Bailey's test data, then I have to wonder if this oversight was the impetus for the moving fiber equations. Remember, my conclusion in Section 2.0 was that the fibers are not moving during normal operation of a transmission line. This conclusion was verified in Section 5.0 when the damping coefficient and speed of sound were empirically determined as functions of stuffing density from the test data.

Figure 6.5 : Magnitude of the Terminus Velocity for a 1 m/sec Driven Excitation

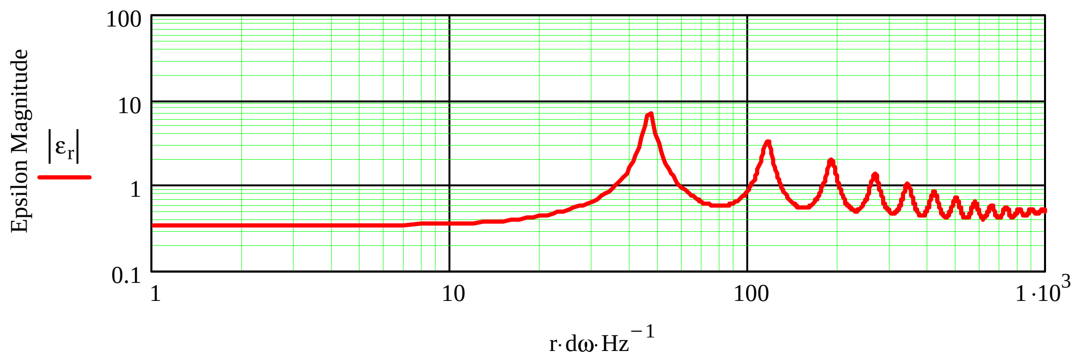
Terminus Velocity for a Tapered Transmission Line



Terminus Velocity for a Straight Transmission Line



Terminus Velocity for an Expanding Transmission Line



Summary :

Two more new MathCad models have been described, “TL Offset Diver.MCD” and “TL Sections.MCD”, in this section. Advanced modeling techniques that can be addressed using these models are offsetting the driver along the length of a transmission line and a completely general geometry and stuffing scheme.

A shift of the first resonant frequency of a straight transmission line as taper or expansion is introduced, has been demonstrated for a constant volume and constant length geometry. Understanding the relationship between the classically calculated first quarter wavelength frequency, $f = c / (4 \times L)$, and the inherent assumption of a straight transmission line geometry is critical. The impact of introducing taper or expansion on this first resonant frequency is one of the keys to designing an optimized transmission line enclosure.