Transmission Line and Back Loaded Horn Physics

Introduction

In order to differentiate between a transmission line and a back loaded horn, it is really important to understand the physics that make them work acoustically. If we consider a driver mounted at the closed end of an expanding path with the far end open to the room then the behavior of the air volume will help define if the geometry is behaving as a transmission line or a back loaded horn. The following discussion is derived from my horn and transmission line articles and represents my personal understanding and definitions. Other people may or may not have a common explanation and naming/labeling scheme. But in the end, the air in the volume will behave as governed by the laws of physics and will not be influenced in the least by arguments over different definitions or naming/labeling conventions.

The first step is to define what a horn is in terms of its acoustic and physical properties. Consulting Wikipedia [reference : http://en.wikipedia.org/wiki/Horn_(acoustic)] the following definition is provided.

"A horn is a tapered sound guide designed to provide an acoustic impedance match between a sound source and free air. This has the effect of maximizing the efficiency with which sound waves from the particular source are transferred to the air. Conversely, a horn can be used at the receiving end to optimize the transfer of sound from the air to a receiver."

The easy part of this definition is the "tapered sound guide" geometry, the physical property. Everybody can visualize this type of geometry and in many people's minds this is the sole property that defines a back loaded horn. But the tougher part of the definition is the acoustic property, an "acoustic impedance match" between the source and the air in the room, which is not so easy to visualize. The second part of the definition is determined by the effective size of the horn's mouth, the resulting acoustic impedance, and the frequency range being reproduced.

A one dimensional wave equation, as shown below, can be used to describe the air motion in both transmission line and horn geometries. The second term on the left side of the equals sign accounts for the flare geometry in the equation of motion. If m is equal to zero the geometry is straight, if m is not zero the geometry is contracting or expanding along the length depending on the sign. The third term on the left of the equals sign accounts for distributed fiber damping in the air volume as defined by λ a viscous damping coefficient. This equation of motion is a combined result that can be used to mathematically describe a fiber filled transmission line or an empty horn. Restating the equation by setting the frequency dependent damping term λ to zero leaves the classic exponential horn wave equation found in most acoustics texts.

$$c^2\left(\left(\frac{\partial^2}{\partial x^2}\xi(x,t)\right)+m\left(\frac{\partial}{\partial x}\xi(x,t)\right)\right)-\frac{\lambda\left(\frac{\partial}{\partial t}\xi(x,t)\right)}{\rho}=\frac{\partial^2}{\partial t^2}\xi(x,t)$$

Solving the equation of motion requires the application of two boundary conditions. At the driver end, a unit oscillating velocity is typically assumed as one boundary condition. At the open end, the acoustic impedance of a circular piston mounted on an infinite baffle is usually assumed as the second boundary condition. The open end boundary condition is a source of frequency dependent mass loading (imaginary part) and resistive damping (real part).

How a Transmission Line and an Exponential Horn Work

To understand how a transmission line and an exponential horn work, let's start with a straight transmission line and plot the acoustic impedance at the driver end and the ratio between the applied volume velocity at the driver end and the resulting volume velocity at the open end (remember that volume velocity is air velocity times the cross-section area). By incrementally increasing the cross-sectional area of the transmission line's open end, changes in the plotted results demonstrate the physics involved in the workings of an exponential horn. Table 1 below summarizes the geometries used for this study. In the table, the lower cut-off frequency f_c typically used for describing a horn is calculated based on the open end (the transmission line's terminus or the horn's mouth) cross-sectional area using the following equation.

$$S_L = \frac{\left(\frac{c}{2 f_c}\right)^2}{\pi}$$

Table 1 : Transmission Line and Horn Geometries

Geometry	S_0	S_L / S_0	S_L	L	f_c	Figure
a	0.047	1	0.047	0.820	447.2	1
b	0.047	2	0.094	0.820	316.2	2
c	0.047	3	0.141	0.820	258.2	3
d	0.047	4	0.188	0.820	223.6	4
e	0.047	5	0.235	0.820	200.0	5
f	0.047	10	0.471	0.820	141.4	6
g	0.047	15	0.706	0.820	115.4	7
h	0.047	20	0.942	0.820	100.0	8
Units	m^2	---	m^2	m	Hz	---

Figure 1 shows the results for the straight transmission line. The fundamental resonance in the acoustic impedance plot occurs at 92 Hz which matches within 5% the calculated quarter wavelength prediction for a straight transmission line. In the quarter wavelength calculation below the physical length is increased to include an approximate end correction term.

$f_0 = c / (4 \times L_{effective})$
$f_0 = (344 \text{ m/sec}) / (4 \times (0.820 \text{ m} + 0.6 \times ((0.047 \text{ m}^2) / \pi)^{1/2})) = 96 \text{ Hz}$

The subsequent resonant peaks are at 3, 5, 7, …. multiples of the fundamental 1/4 wavelength frequency. The height and sharpness of the peaks, in the impedance and the volume velocity ratio plots, decrease as frequency increases due to the rising resistive (real) component of the open end's acoustic impedance. The acoustic impedance of the circular piston boundary condition assumed for the open end of a quarter wavelength resonator is plotted in Figure 9 as a function of frequency.

Standing waves result when sound waves traveling the length of the transmission line are reflected back towards the driver end. Reflections occur at low frequencies because the mouth is relatively small and the acoustic impedance acts like a slug of mass (almost like a wall). In Figure 9, the straight transmission line's fundamental standing wave corresponds to an acoustic impedance value with real and imaginary components consistent with approximately 0.41 on the horizontal axis. To control the peaky acoustic output from the open end of the transmission line, fiber stuffing would need to be added to viscously damp the sharp resonances generated by the standing waves.

Figure 2 shows the plots for an expanding transmission line geometry that has twice the open end area as the original straight transmission line plotted in Figure 1. Notice that the fundamental resonance has increased from 92 Hz to 103 Hz. Also, all of the peaks and nulls are a little less pronounced and broader. In Figure 9, this expanding transmission line's fundamental standing wave corresponds to an acoustic impedance value with real and imaginary components consistent with approximately 0.65 on the horizontal axis. For this geometry less fiber stuffing would be needed to control the peaks in the acoustic output from the open end

Figures 3, 4, and 5 continue to increase the open end area. As the heights of the peaks in the acoustic impedance and volume velocity ratio plots decrease and broaden, the valleys between successive peaks start to fill in and the lower bound of the plotted data rises. In each sequential plot, the acoustic damping provided by the open end increases and extends lower in frequency meaning less fiber stuffing would be required. Reviewing the data in Table 1, the lower cut-off frequency f_c is dropping in frequency as the open end's cross-sectional area increases.

By Figure 6, the acoustic impedance is settling in and oscillating only slightly about (ρ x c) / S_0 while the ratio of volume velocities is approaching a constant value of three for frequencies above 300 Hz. The phase angles also show none of the large 180 degree phase swings typically associated with standing wave resonances. No fiber stuffing is required for this geometry.

Figures 6, 7, and 8 all appear to show horn-like responses. In each case the damping provided by the open end's acoustic impedance boundary condition is sufficient to suppress the standing wave resonances seen in the previous plots. Fiber stuffing would not be required for any of these geometries. The last geometry defined in Table 1, and shown in Figure 8, is what I define as a consistent horn geometry since the exponential flare and the mouth size are both tuned to a lower cut-off frequency f_c of 100 Hz. All of the other rows in Table 1 are transmission lines geometries transitioning to compromised horn geometries as you move down the rows of the table

To summarize the results shown in Figures 1 through 8, the geometric transition from straight unstuffed transmission line to consistent exponential horn geometry was studied by increasing the open end cross sectional area. The study held the length and the driven end cross sectional area constant while maintaining exponentially flared geometries. The geometries that lie between the transmission line and the final consistent exponential horn are all in a gray area of transmission line or compromised exponential horn designs. The first observation made was that the fundamental quarter wavelength resonant frequency of the transmission line rose as the geometry expanded along the length. This is consistent with expanding TL design results described elsewhere on my site.

At the same time, increasing the open end's area increased the acoustic damping boundary condition. This results in the attenuation and broadening of the resonant peaks, and filling in of the deep nulls that exist between the peaks, typically associated with the higher harmonics of a transmission line's fundamental quarter wavelength resonance. As the expanding transmission line's open end area continues to increase, this effect starts to become more evident at lower harmonics and eventually even at the fundamental resonant frequency. The damped resonant peaks spread and merge, filling in the valleys between them, producing relatively constant acoustic impedance above the lower cut-off frequency f_c. As transmission line geometry transitions to a consistent exponential horn there is no longer evidence of standing waves, at distinct frequencies, producing a series of sharp peaks and nulls in the plotted responses. As the open end area increases, the acoustic boundary condition's damping extracts sound energy that would normally be reflected back into the geometry and broadcasts it out into the room yielding an increasingly efficient sound energy transfer.

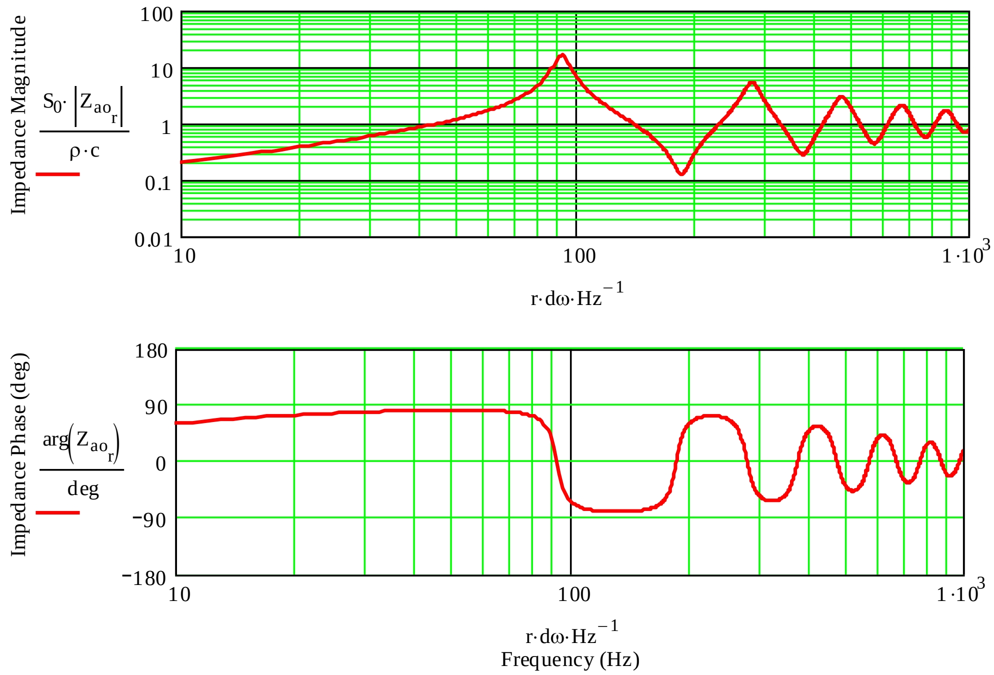
A properly sized and designed exponential horn, a consistent design, is a non-resonant or in other words a highly damped acoustic enclosure. Without the full damping supplied by the mouth, compromised exponential horns exhibit weak quarter wavelength standing waves similar to a transmission line enclosure. This counters one prevailing myth about standing waves in horns; there are no half wavelength standing wave resonances associated with a horn geometry. All longitudinal standing wave resonances in transmission lines and compromised horn geometries exhibit quarter wavelength pressure and velocity distributions.

To understand the increased efficiency attributed to horn loading a driver, the volume velocity ratio Ξ has been plotted in the bottom two plots of Figures 1 through 8. Volume velocity at the open end is important because it can be used directly to calculate the pressure and thus the SPL at some location out in the listening environment. In all of the plots the volume velocity ratio at 10 Hz is equal to unity, what goes in the driver end comes out of the open end. But as we move up above 100 Hz, the transmission line response is seen as a series of tall narrow peaks while the more horn like response becomes an elevated value across all frequencies. The volume velocity ratio Ξ in Figures 1 exhibits discrete peaks the first exceeding a ratio of 5 while the remaining peaks are all below a ratio of 5. Comparing this to Figure 8, the volume velocity ratio above 100 Hz is consistently hovering between 4 and 5. The series of plots 2 through 7 track the changes that take place in the volume velocity ratio curve as the geometry transitions from transmission line to consistent exponential horn geometry.

Extending these observations about the volume velocity ratio curves, to the acoustic SPL output produced by transmission lines and consistent exponential horns, leads to the following understanding of why the horn speaker is so efficient. The damping provided by the real part of the acoustic impedance at the horn's mouth efficiently transfers sound energy into the listening room environment at all frequencies above the lower cut-off frequency f_c. Without this constant transfer of energy, a significant portion of the sound energy is reflected back into the flared geometry producing standing waves at discrete frequencies related to the geometry's length and flare rate. These standing waves produce narrow bands of higher SPL in the listening room due to the peaking resonance of the volume velocity at the open end. The consistent horn's efficient transfer of sound energy into the room produces a more uniform higher SPL output across the frequency spectrum and removes the potential for peaky acoustic output due to axial standing waves associated with a transmission line or compromised horn designs. To battle the peaky acoustic output of transmission lines, fiber stuffing is typically distributed along the length of the geometry which in turn reduces the SPL output generated at the open end..

Figure 1 : Acoustic Impedance and Volume Velocity Ratio for S_L / S_0 = 1 in Table 1

Acoustic Impedance at the Throat of the Horn



E = (Volume Velocity at the Mouth of the Horn) / (Volume Velocity at the Throat of the Horn)

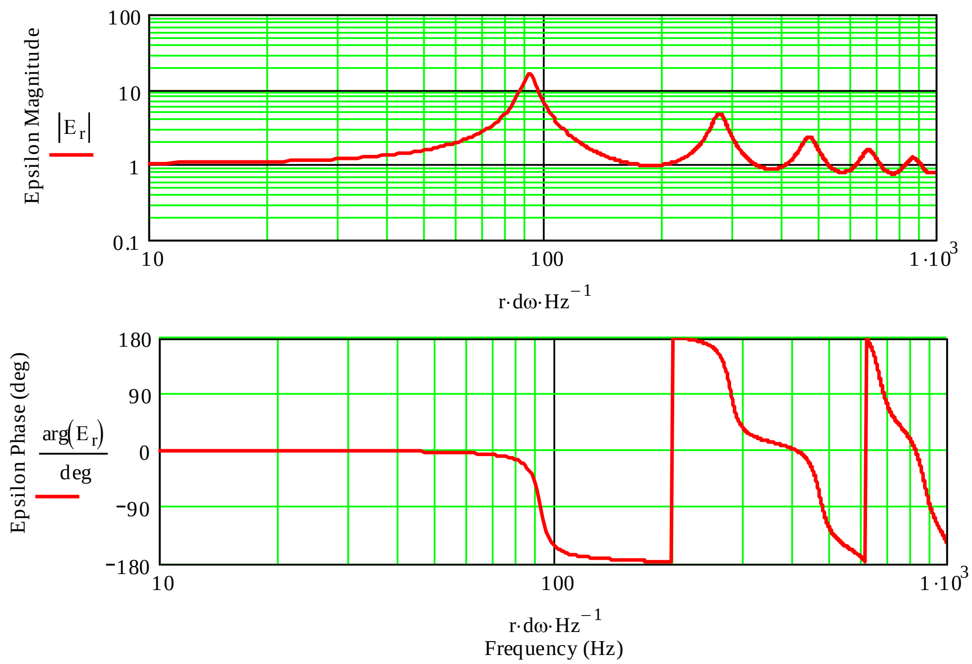
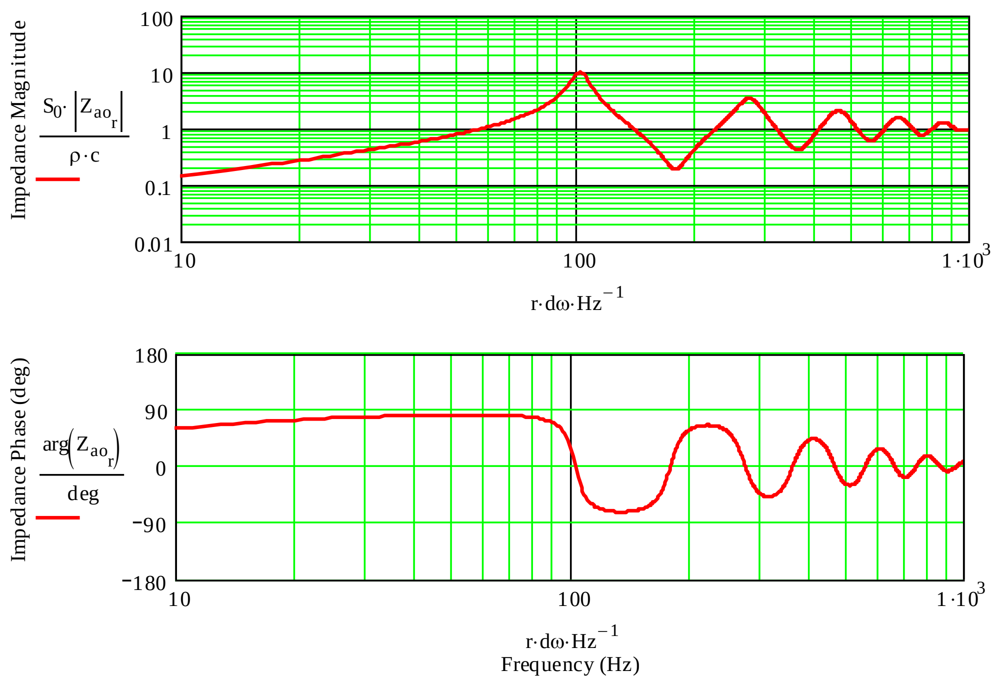


Figure 2 : Acoustic Impedance and Volume Velocity Ratio for S_L / S_0 = 2 in Table 1

Acoustic Impedance at the Throat of the Horn



E = (Volume Velocity at the Mouth of the Horn) / (Volume Velocity at the Throat of the Horn)

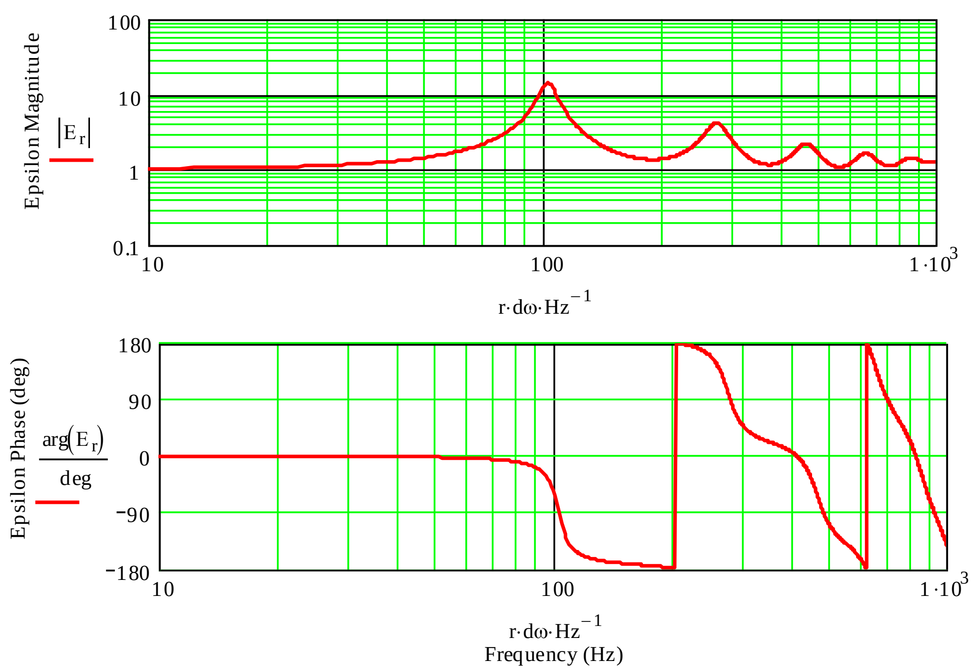
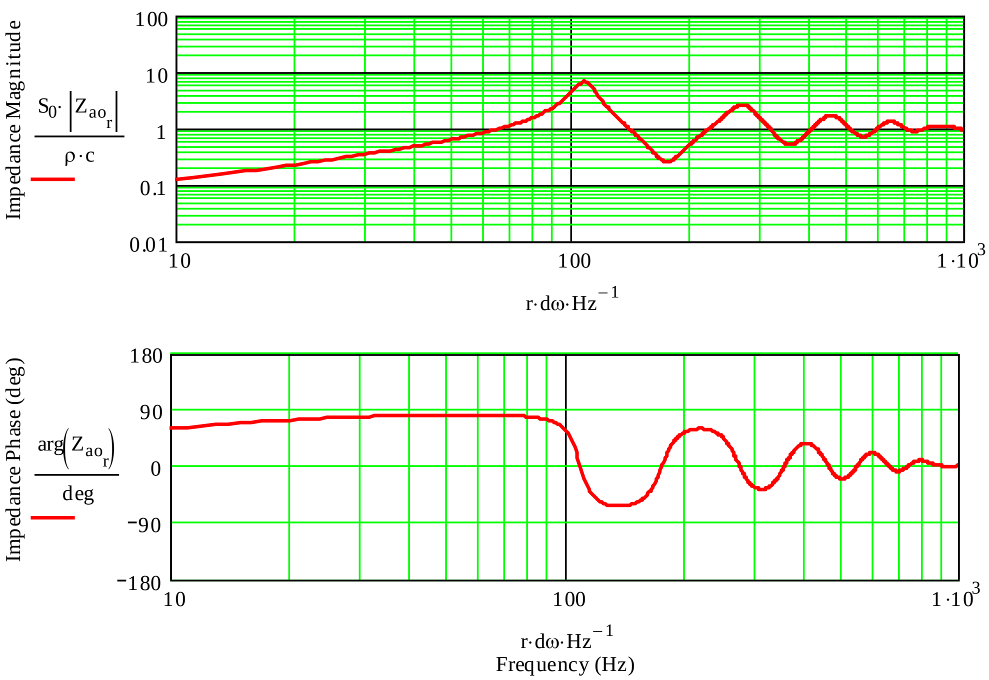


Figure 3 : Acoustic Impedance and Volume Velocity Ratio for S_L / S_0 = 3 in Table 1

Acoustic Impedance at the Throat of the Horn



E = (Volume Velocity at the Mouth of the Horn) / (Volume Velocity at the Throat of the Horn)

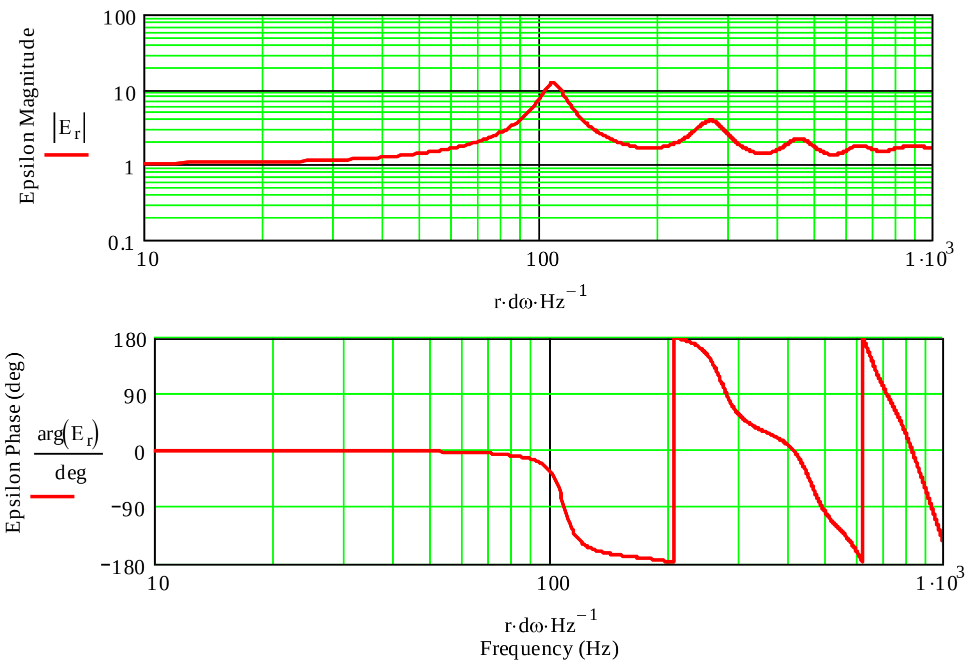
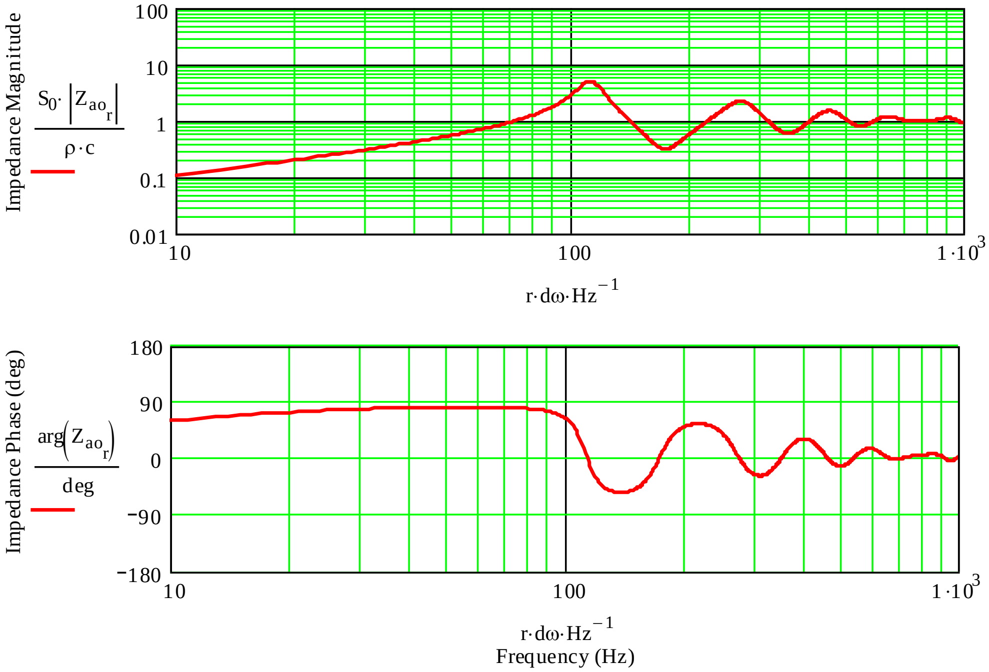


Figure 4 : Acoustic Impedance and Volume Velocity Ratio for S_L / S_0 = 4 in Table 1

Acoustic Impedance at the Throat of the Horn



E = (Volume Velocity at the Mouth of the Horn) / (Volume Velocity at the Throat of the Horn)

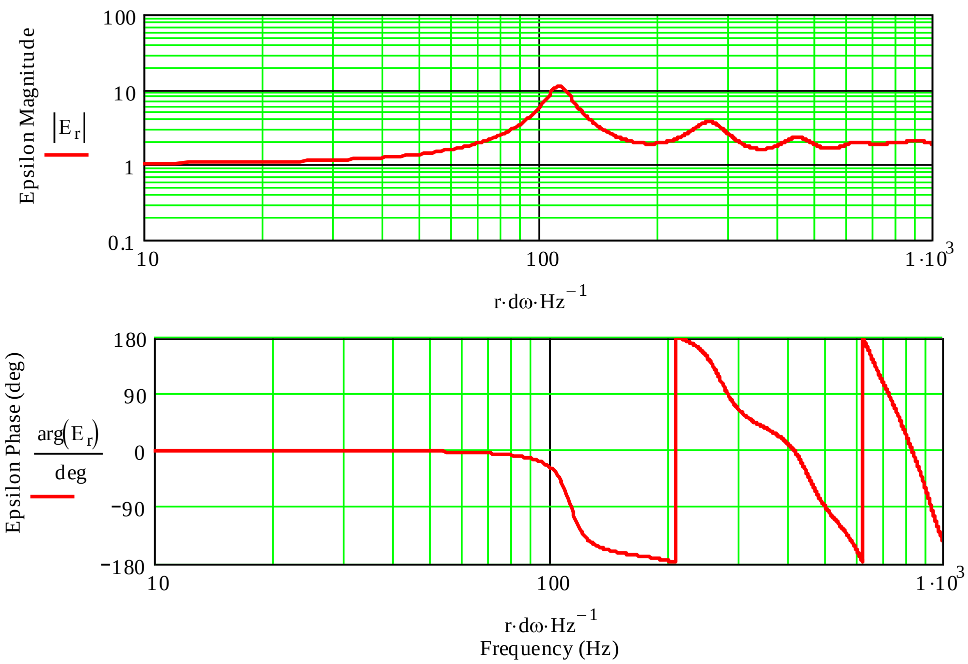
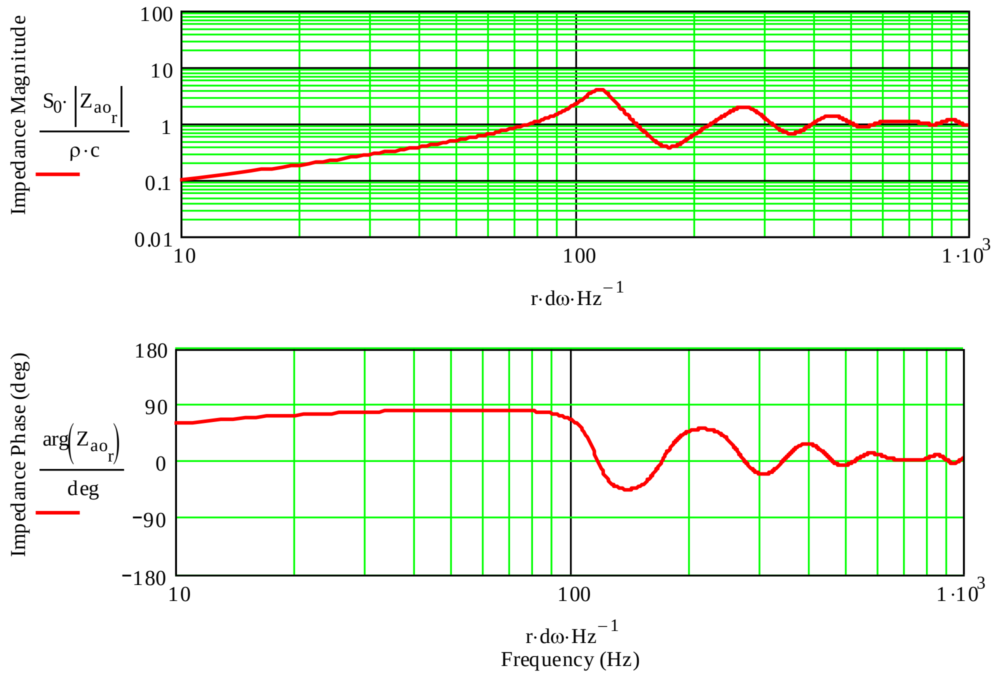


Figure 5 : Acoustic Impedance and Volume Velocity Ratio for S_L / S_0 = 5 in Table 1

Acoustic Impedance at the Throat of the Horn



E = (Volume Velocity at the Mouth of the Horn) / (Volume Velocity at the Throat of the Horn)

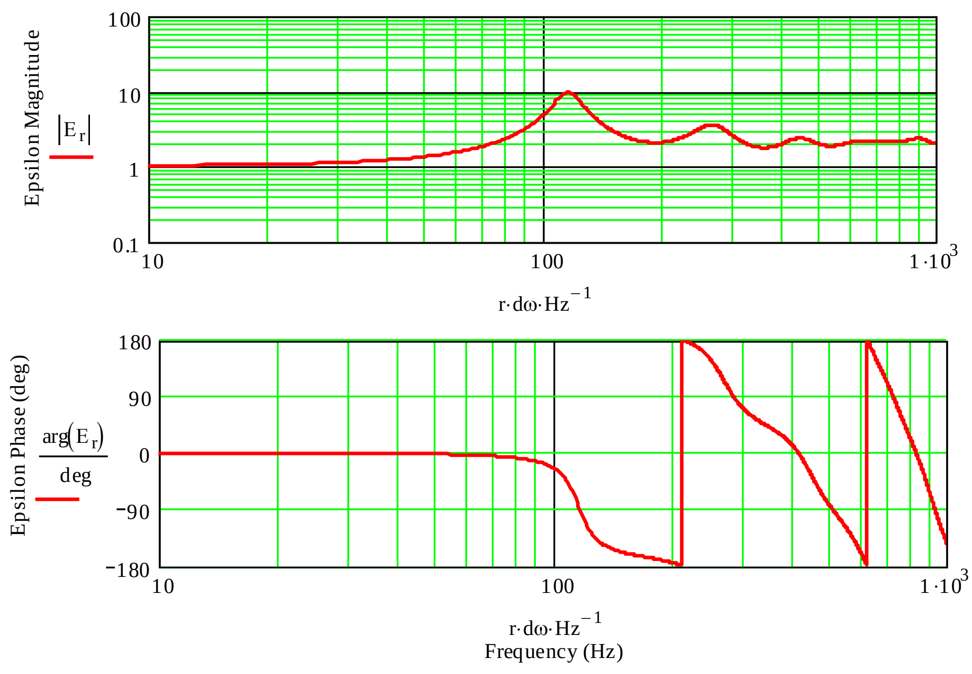
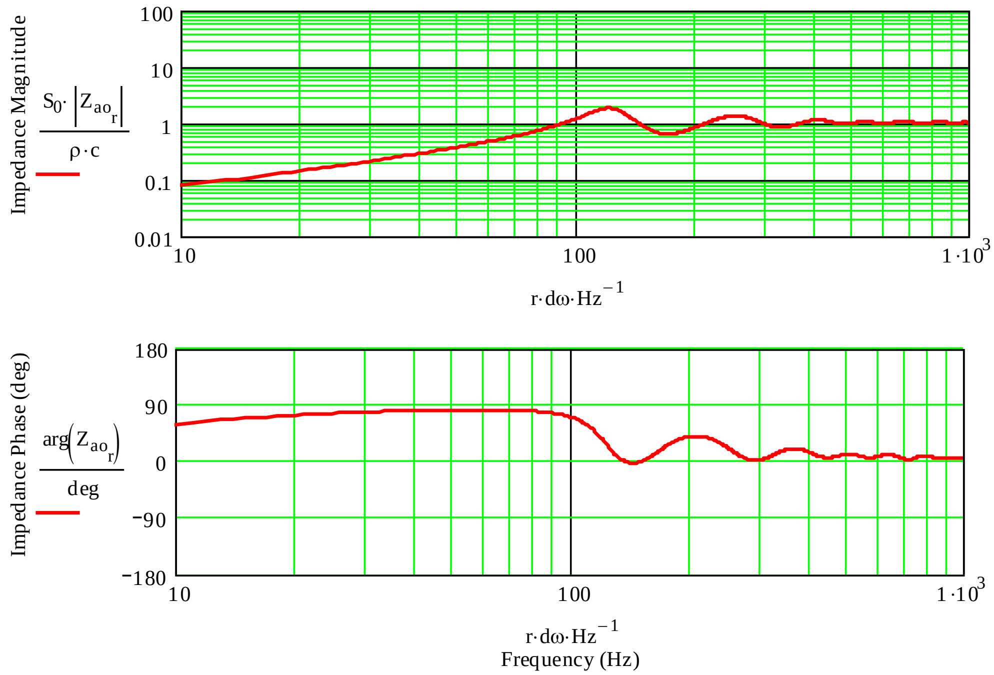


Figure 6 : Acoustic Impedance and Volume Velocity Ratio for S_L / S_0 = 10 in Table 1

Acoustic Impedance at the Throat of the Horn



E = (Volume Velocity at the Mouth of the Horn) / (Volume Velocity at the Throat of the Horn)

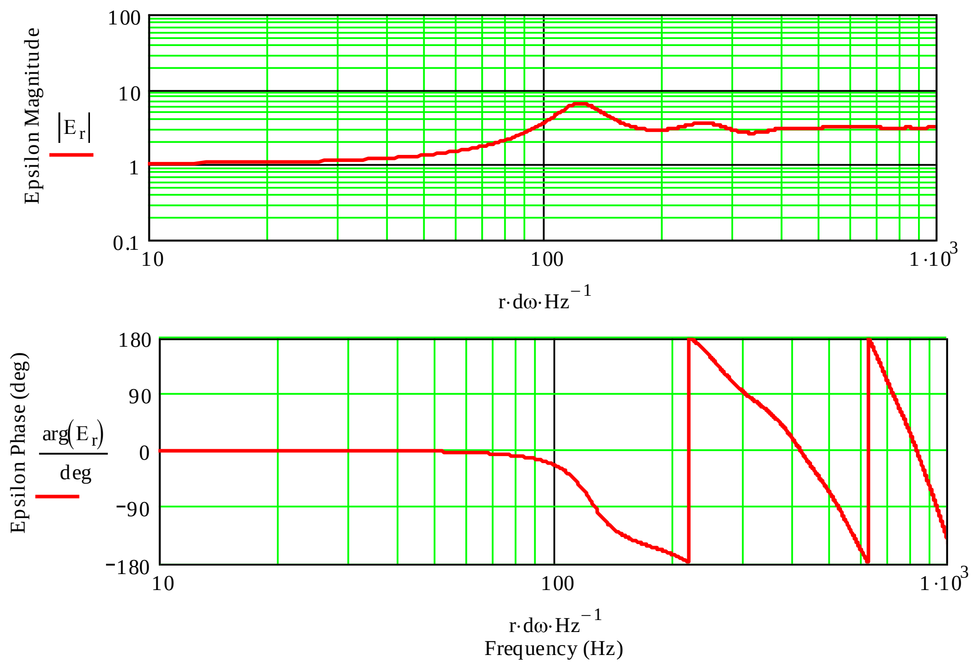
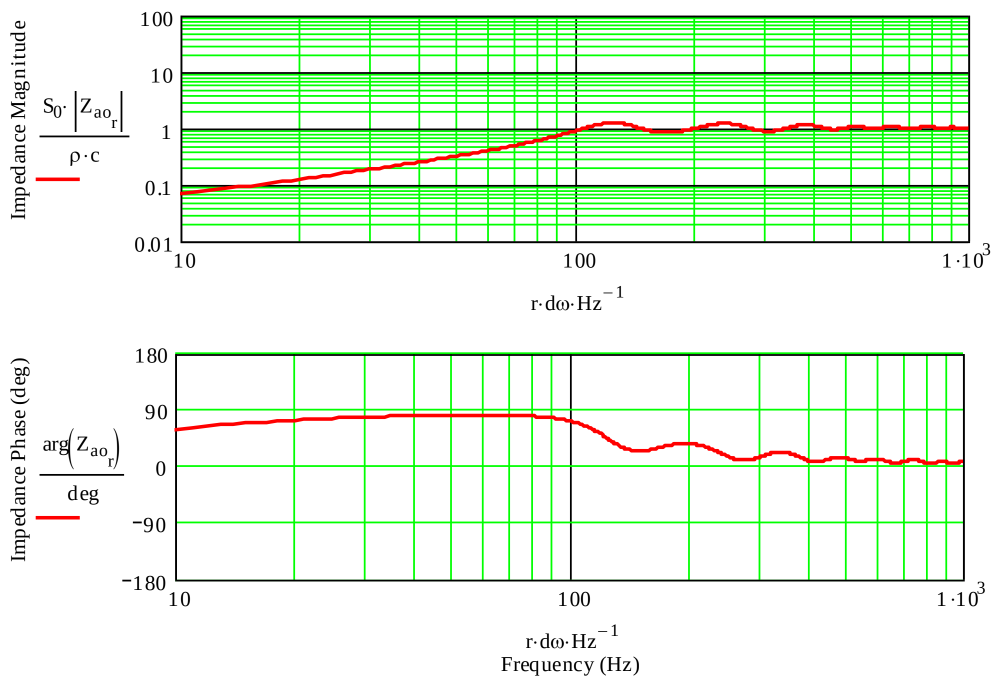


Figure 7 : Acoustic Impedance and Volume Velocity Ratio for S_L / S_0 = 15 in Table 1

Acoustic Impedance at the Throat of the Horn



E = (Volume Velocity at the Mouth of the Horn) / (Volume Velocity at the Throat of the Horn)

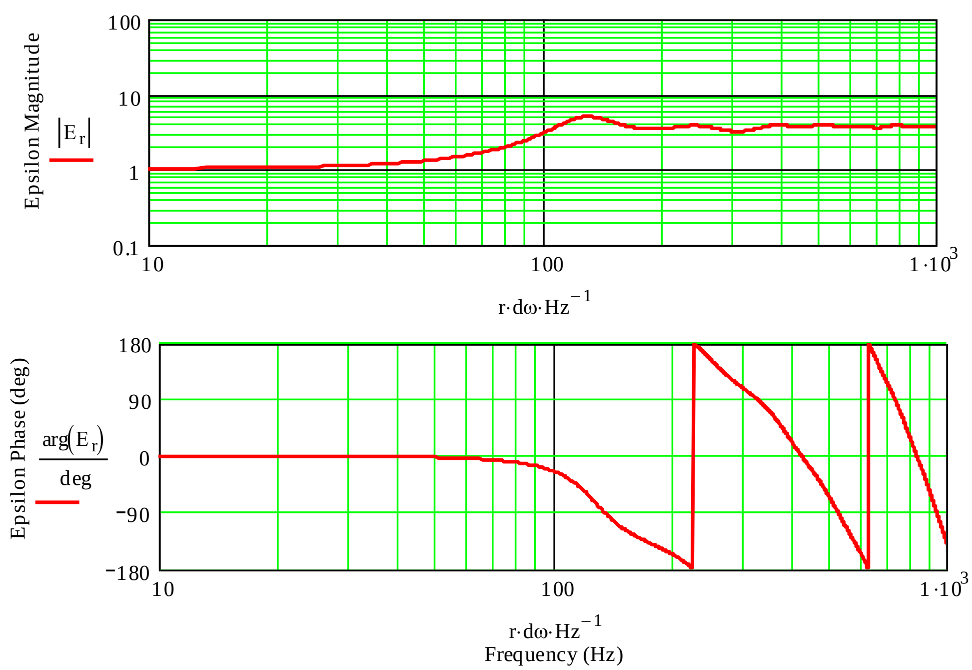
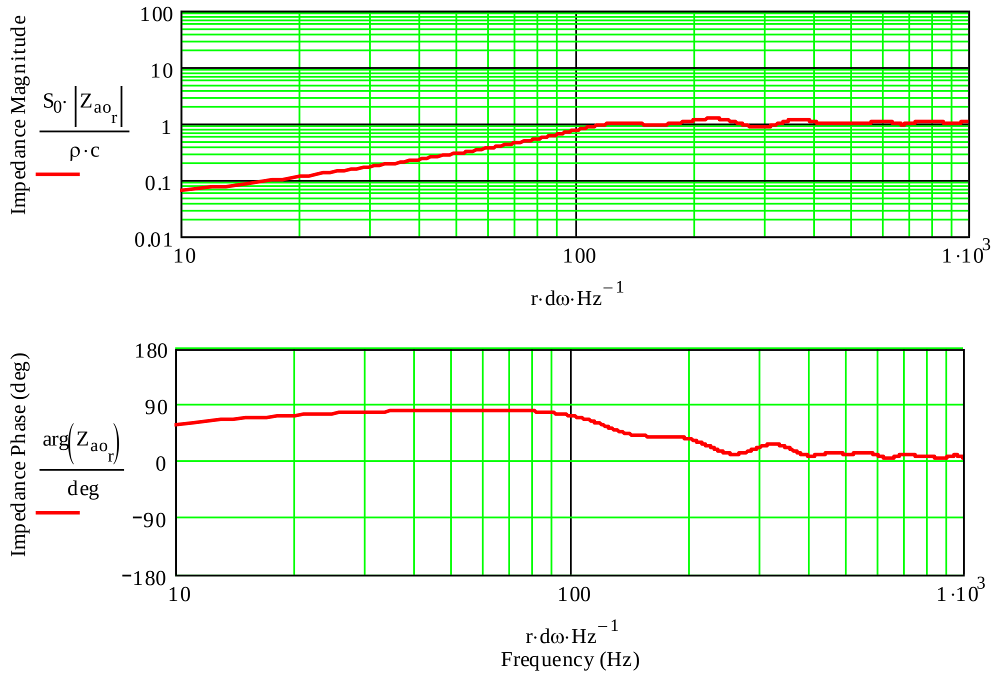


Figure 9 : Acoustic Impedance and Volume Velocity Ratio for S_L / S_0 = 20 in Table 1

Acoustic Impedance at the Throat of the Horn



E = (Volume Velocity at the Mouth of the Horn) / (Volume Velocity at the Throat of the Horn)

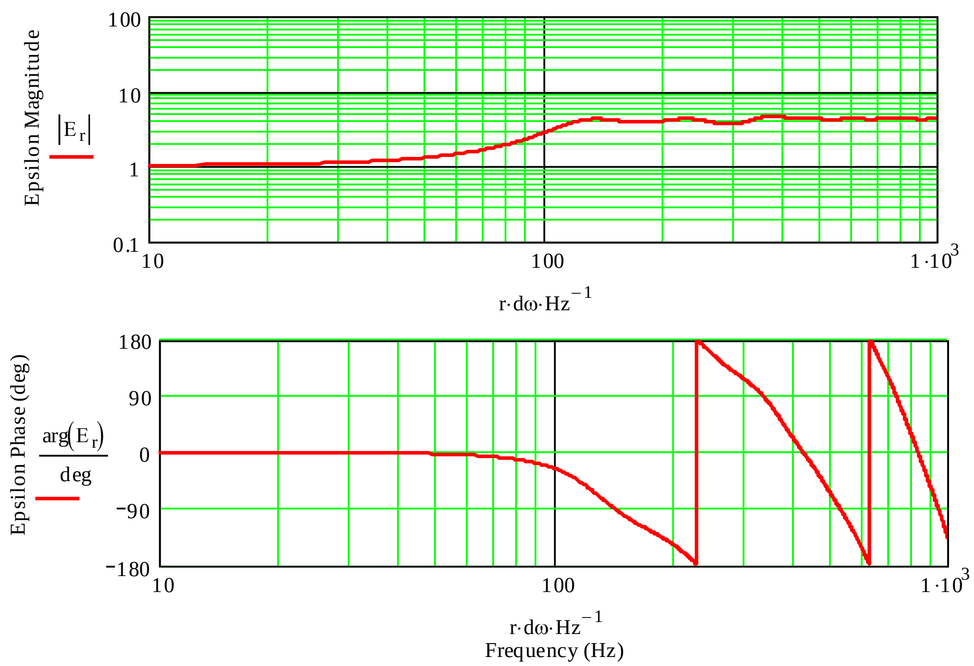
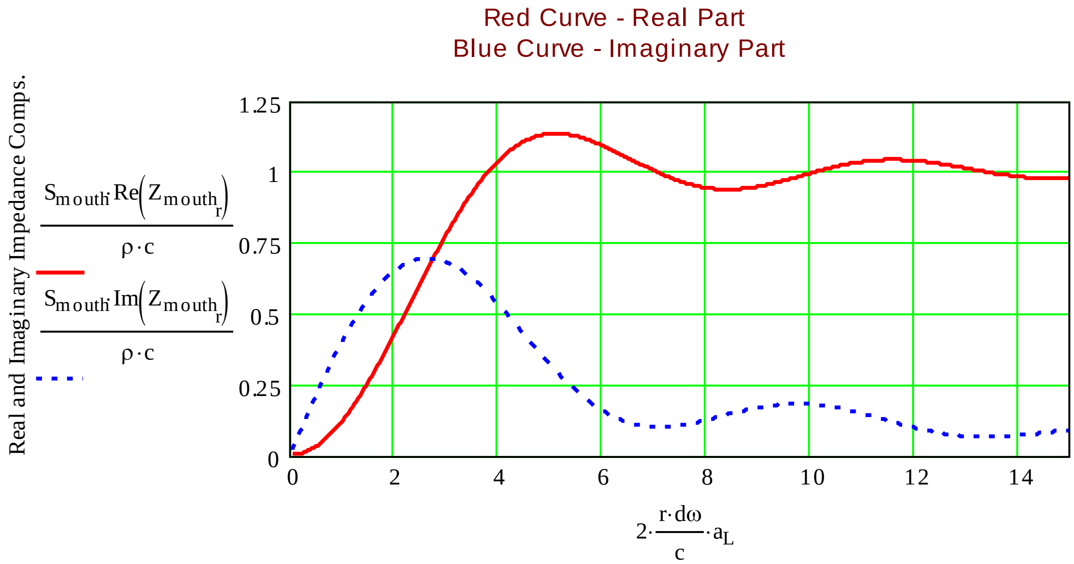


Figure 9 : Circular Horn Mouth Acoustic Impedance



Conclusions

Using my definitions, the key parameter that separates transmission line behavior from horn behavior is the area of the terminus or mouth. At low frequencies, the area of the mouth of a horn needs to be huge to provide the acoustic damping required to control standing wave resonances.

If the terminus or mouth is too small, standing wave resonances will be generated at low frequencies yielding transmission line behavior. However, as frequency increases the acoustic impedance boundary condition at the open end will transition from mass loading to resistive loading and the performance will also shift acoustically from a transmission line response to a horn response.

In Figure 9, the boundary between transmission lines and horns is typically defined at a value of 2 on the horizontal axis. Transmission line behavior would be expected for open end geometries that fall between 0 and 2 on the horizontal axis. As one approaches 2 and extends slightly above 2 there will be a transition region from transmission line to horn behavior. Above 2 the damping provided by the open end is sufficient to control standing wave resonances, efficiently broadcast sound out into the listening room, and horn behavior is exhibited.

Almost all back loaded horn designs found on the Internet are really acting as transmission lines at low frequencies and never approach horn behavior because the mouth is too small. While there are general rules of thumb equations for setting a horn's geometry and the coupling volume between the driver and the horn throat, these equations have all been derived assuming that the mouth size places the design above 2 on the horizontal axis in Figure 9. Blindly applying these relationships to back loaded "horns" with small mouth areas is a hit or miss proposition with respect to the expected system performance. Incorrectly applying horn equations is no better than placing a woofer in a random ported box hoping the two work together producing a smooth and extended low frequency response. Sometimes it works.......