Section 4.0 : Sound Radiation Pattern from the Mouth of a Horn

In the previous section, the acoustic impedance at the mouth of a horn was calculated. Distributed simple sources were used to model the mouth geometry moving as a piston in an infinite baffle. The output of all the simple sources was integrated to calculate the total acoustic impedance of the horn's mouth. One additional property of the mouth of the horn needs to be derived before the complete acoustic response of a horn can be simulated. At any position in the listening environment, the distances from any two individual simple sources being used to represent the mouth will be slightly different. As the frequency increases, these small differences in path length start to represent larger phase differences leading to cancellation and reinforcement of the summed SPL at the listening position. Expressions for the SPL, as functions of distance and angular location with respect to the horn's axis, will be derived for circular and rectangular mouth geometries.

Circular Horn Mouth :

To derive the SPL radiation pattern of a circular horn mouth, the piston in an infinite baffle analogy is assumed again. Figure 4.1 shows the geometry definition used to derive the far field SPL at an arbitrary position in space for a vibrating circular piston in an infinite baffle. The first step in calculating the radiation pattern is to express r_p in terms of r_o. At distances in the far field, the relationship between these vectors can be stated as follows

$$r_p = r_o - r\cos(\phi)\cos(\alpha)$$

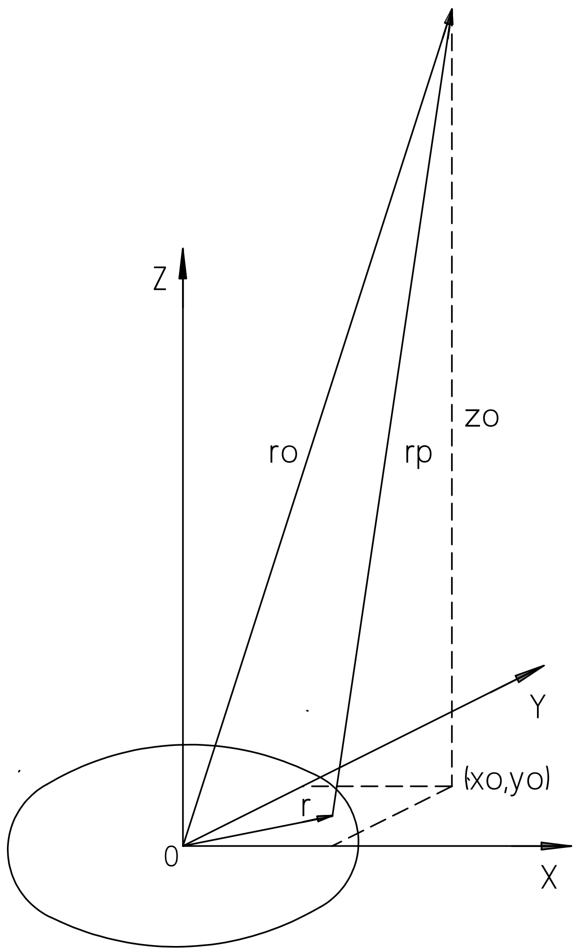
where ϕ and α are the angles that r and r_o form with the X axis. Since the response of the circular piston is axisymmetric, only the solution in the X-Z plane is required to describe the complete 3D radiation pattern. For the simple source at the tip of r, the pressure generated at a point in the far field X-Z plane is shown below.

$$\mathrm{dp}(r_p, t) = \frac{I\sqrt{2}\,\rho\, f\, u_{rms}\, dr\, r\, d\phi\, \mathbf{e}^{(I(\omega t - k r_p))}}{r_p}$$

Substituting the expression for r_p and formulating an integral to sum all of the point sources on the piston's surface yields the following expression.

$$\mathrm{p}(r_o, t) = \frac{I\sqrt{2}\,\rho\, f\, u_{rms}\, \mathbf{e}^{(-I k r_o)}\, \mathbf{e}^{(I\omega t)} \int_0^a \int_0^{2\pi} \mathbf{e}^{(I k r\cos(\phi)\cos(\alpha))}\, r\, d\phi\, dr}{r_o}$$

Figure 4.1 : Circular Piston Geometry Definition



Evaluating the integral is fairly straight forward requiring trigonometry and careful substitutions. After evaluating the double integral, and making all of the appropriate substitutions, the following well know expression results.

$$p(r_0, \theta, t) = \frac{I \sqrt{2}\, \rho\, f\, \pi\, a^2\, u_{rms} \left[2 \frac{J_1(k\, a \sin(\theta))}{k\, a \sin(\theta)} \right] \mathbf{e}^{(-I\, k\, r_o)}\, \mathbf{e}^{(I\, \omega\, t)}}{r_o}$$

The angle θ is measured between r_o and the Z axis.

Rectangular Horn Mouth :

To derive the SPL radiation pattern of a rectangular horn mouth, the piston in an infinite baffle analogy is assumed again. Figure 4.2 shows the geometry definition used to derive the far field SPL at an arbitrary position in space for a vibrating rectangular piston in an infinite baffle. The first step in calculating the radiation pattern is to express r_p in terms of r_o. At distances in the far field, the relationship between these vectors can be stated as follows

$$r_p = r_o - (x\cos(\alpha) + y\cos(\beta))$$

where α and β are the angles that r_o forms with the X axis and Y axis respectively. Since the response of the rectangular piston is symmetric in the X-Z and X-Y planes only the patterns in these two planes are required to describe the complete 3D radiation pattern. For the simple source at the tip of r, the pressure generated at a point in the far field is shown below.

$$\mathrm{dp}(r_p, t) = \frac{I\sqrt{2}\,\rho\, f\, u_{rms}\, dx\, dy\, \mathbf{e}^{(I(\omega t - k r_p))}}{r_p}$$

Since $1/r_p \sim 1/r_o$ in the far field.

$$\mathrm{dp}(r_o, t) = \frac{I\sqrt{2}\,\rho\, f\, u_{rms}\, \mathbf{e}^{(I\omega t)}\, \mathbf{e}^{(-I k r_o)}\, \mathbf{e}^{(-I k x\cos(\alpha))}\, \mathbf{e}^{(-I k y\cos(\beta))}\, dx\, dy}{r_o}$$

Making two more substitutions

$$k_x = k\cos(\alpha)$$

$$k_x = k\sin(\theta_x)$$

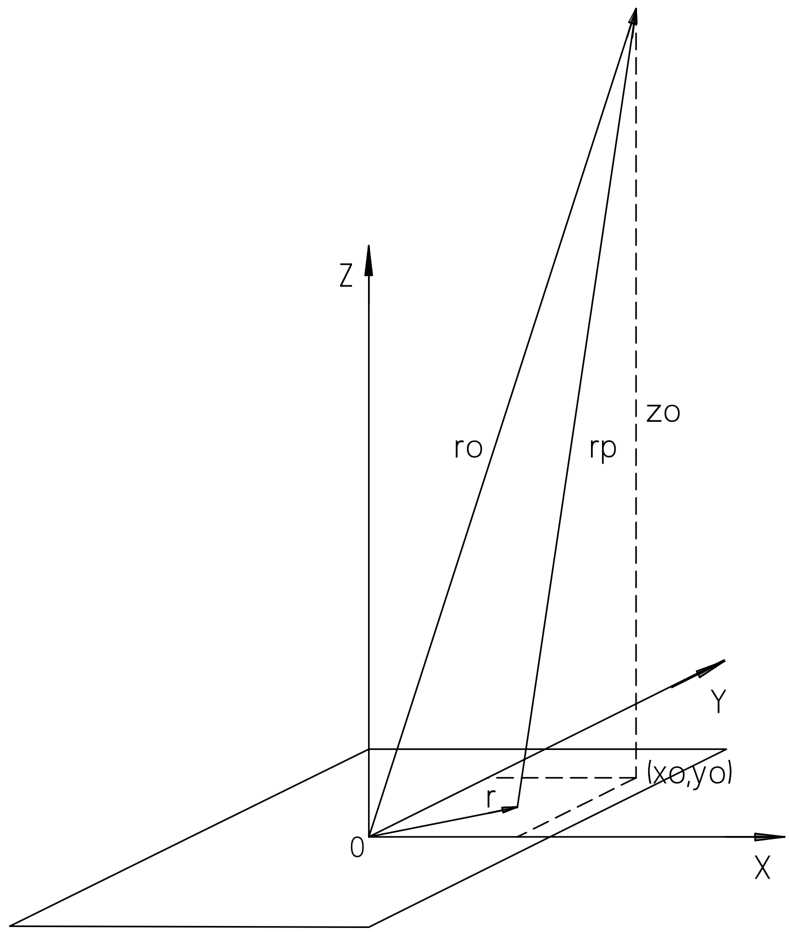
and

$$k_y = k\cos(\beta)$$

$$k_y = k\sin(\theta_y)$$

the differential pressure can be simplified to the following expression. The angles θ_x and θ_y are measured from the Z axis.

Figure 4.2 : Rectangular Piston Geometry Definition



$$\mathrm{dp}(r_o, t) = \frac{I\sqrt{2}\,\rho\, f\, u_{rms}\, \mathbf{e}^{(I\,\omega\, t)}\, \mathbf{e}^{(-I\, k\, r_o)}\, \mathbf{e}^{(-I\, k_x\, x)}\, \mathbf{e}^{(-I\, k_y\, y)}\, dx\, dy}{r_o}$$

Summing the pressure produced by all of the simple sources, making up the rectangular surface, leads to the following integral.

$$\mathrm{p}(r_o, \theta_x, \theta_y, t) = \frac{I\sqrt{2}\,\rho\, f\, u_{rms}\, \mathbf{e}^{(I\,\omega\, t)}\, \mathbf{e}^{(-I\, k\, r_o)} \int_{-a}^{a} \mathbf{e}^{(-I\, k_x\, x)}\, dx \int_{-b}^{b} \mathbf{e}^{(-I\, k_y\, y)}\, dy}{r_o}$$

This integral is really very straight forward to evaluate and the following result is obtained.

$$p(r_o, \theta_x, \theta_y, t) = \frac{I\sqrt{2}\,\rho\, f\, u_{rms}\, \mathbf{e}^{(I\,\omega\, t)}\, \mathbf{e}^{(-I\, k\, r_o)}\, 2\, a \frac{\sin(k_x\, a)}{k_x\, a}\, 2\, b\frac{\sin(k_y\, b)}{k_y\, b}}{r_o}$$

The pressure distribution forms two different patterns in the X-Z and Y-Z planes as determined by the aspect ratio between the rectangle's dimensions.

Summary of Sample Radiation Patterns :

To get a feel for the radiation pattern of the sound pressure emanated by the mouth of a horn, the sample problem from the previous section was used again. Assuming a constant mouth area equal to a 32 inch by 32 inch square, the equivalent radius a_L for a circular mouth of equal area was calculated. The radius of this equivalent area circular piston is 18.054 inches. The following plots show the radiation patterns of a simple source, a circular horn mouth, a square horn mouth, and a rectangular horn mouth all calculated in the far field and at a frequency of 200 Hz. The results have all been normalized to the value produced by the simple source.

Figure 4.3 presents the results for the circular mouth and the square mouth. Notice that at this relatively low frequency the two different shaped mouths produce equivalent pressure radiation patterns. Also notice that compared to the simple source the sound field is starting to become directional.

Figures 4.4 and 4.5 present the results for the circular mouth and two different rectangular mouths. The aspect ratios, width divided by height, are 0.2 and 0.1 respectively. In the horizontal plane, the narrow width allows the rectangular mouth's radiation pattern to approach the simple source result while the circular mouth is becoming directional. In the vertical plane, the two rectangular mouths have become very directional.

At low frequencies, below 100 Hz, these horn mouths are non directional and can be modeled as a simple source. Above 100 Hz, the shape of the horn mouth starts to generate a radiation pattern caused by the cancellation of pressure due to differing path lengths (really phase differences) between the distributed simple sources used to model the mouth geometry.

Some final comments on the data presented in Figures 4.3, 4.4, and 4.5 are worth restating. The on-axis response of any horn mouth geometry will be equal to the equivalent simple source response. The off-axis response can become a complicated 3D radiation pattern generated by a distribution of simple sources used to represent the mouth geometry. Using a low aspect ratio horn mouth, the radiation pattern in one plane can be made to approach the simple source result while in the perpendicular plane it becomes highly directional at a very low frequency.

In this section, and the proceeding one, there are some interesting acoustic results presented related to the horn mouth which give rise to a few options to ponder when designing a horn. The horn mouth is going to be a key variable in the design of successful and consistent horn geometries. Understanding the acoustic impedance and the sound pressure radiation pattern, for a given mouth geometry, is critical.

Figure 4.3 : Pressure Radiation Patterns From a Simple Source, a Circular Horn Mouth, and a Square Horn Mouth (Aspect Ratio = width / height = 1.0) Calculated in the Far Field at a Frequency of 200 Hz.

black curve = simple source
red curve = circular horn mouth
blue curve = square horn mouth (horizontal plane)
green curve = square horn mouth (vertical plane)

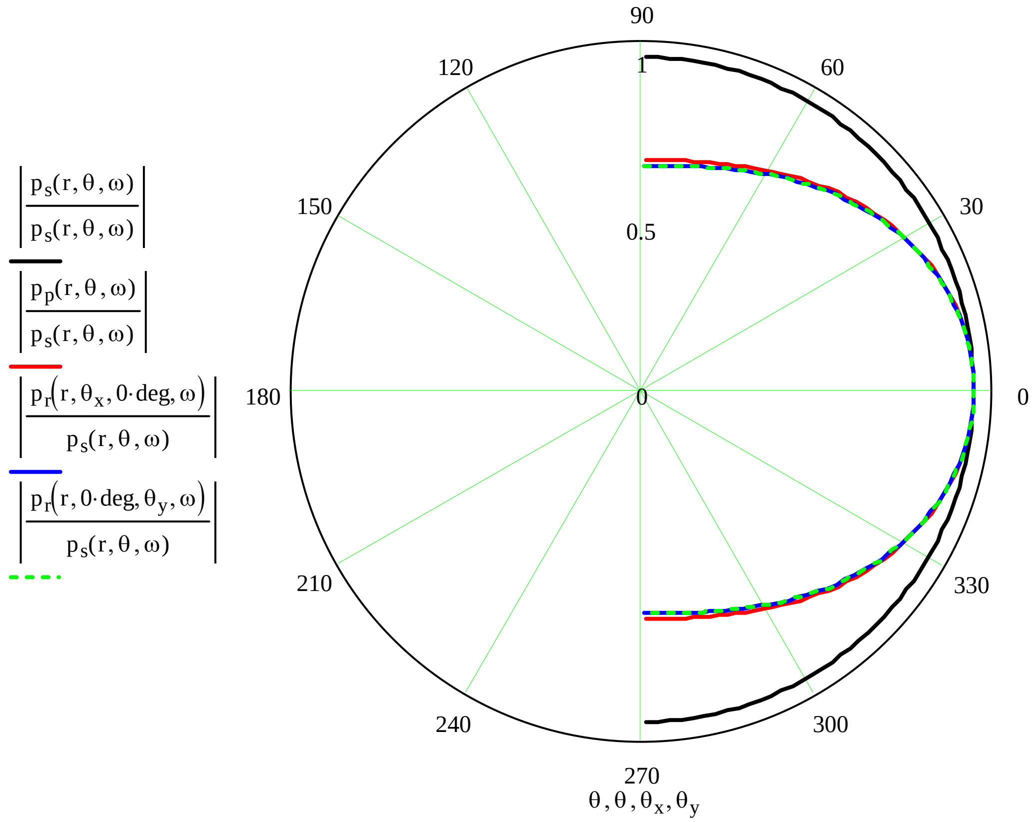


Figure 4.4 : Pressure Radiation Patterns From a Simple Source, a Circular Horn Mouth, and a Rectangular Horn Mouth (Aspect Ratio = width / height = 0.2) Calculated in the Far Field at a Frequency of 200 Hz.

black curve = simple source
red curve = circular horn mouth
blue curve = square horn mouth (horizontal plane)
green curve = square horn mouth (vertical plane)

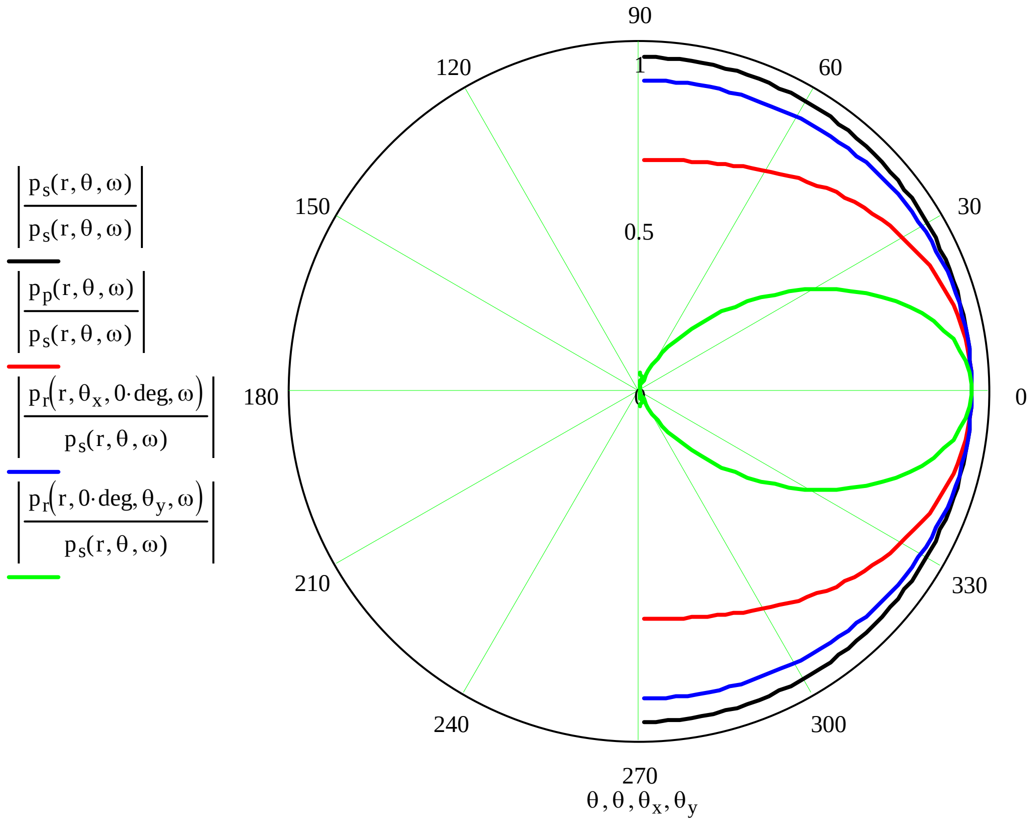


Figure 4.5 : Pressure Radiation Patterns From a Simple Source, a Circular Horn Mouth, and a Rectangular Horn Mouth (Aspect Ratio = width / height = 0.1) Calculated in the Far Field at a Frequency of 200 Hz.

black curve = simple source
red curve = circular horn mouth
blue curve = square horn mouth (horizontal plane)
green curve = square horn mouth (vertical plane)