Section 7.0 : Design of a Back Loaded Exponential Horn

For the past few years, a back loaded horn MathCad worksheets have been available for downloading from my site. There have been quite a few updates to this worksheet which extended the scope and corrected minor bugs. The worksheet was derived to simulate the back loaded horn geometry shown in Figure 7.1. The model solves the equivalent acoustic and electrical circuits shown in Figures 7.2 and 7.3 respectively. By default a unit input of 1 watt, into an assumed 8 ohm voice coil resistance, is applied as a constant voltage of 2.8284 volts RMS.

At this point, a complete analysis of the equivalent circuits could be performed and the derivations would drag on for many pages. But this type of analysis, while providing some very useful sizing relationships, would probably not provide any intuitive feel for the workings of an exponential back loaded horn speaker. So instead of a rigorous mathematical derivation, the understanding gained in the preceding sections will be used to produce a set of simulations intended to characterize how a back loaded exponential horn works and what trade-offs can be made to optimize the final horn system performance. In all of the following simulations, it is assumed that the cross-sectional areas are circular. Square and rectangular cross-sectional areas will be examined in a separate study.

Probably the most important results presented so far are the resistive nature of the acoustic impedance of the horn and the potential for a large increase in the volume velocity ratio E above the lower cut-off frequency f_c. This means that the back of the driver is radiating into a pure acoustic resistance that is a function only of the air density, the speed of sound, and the horn throat area.

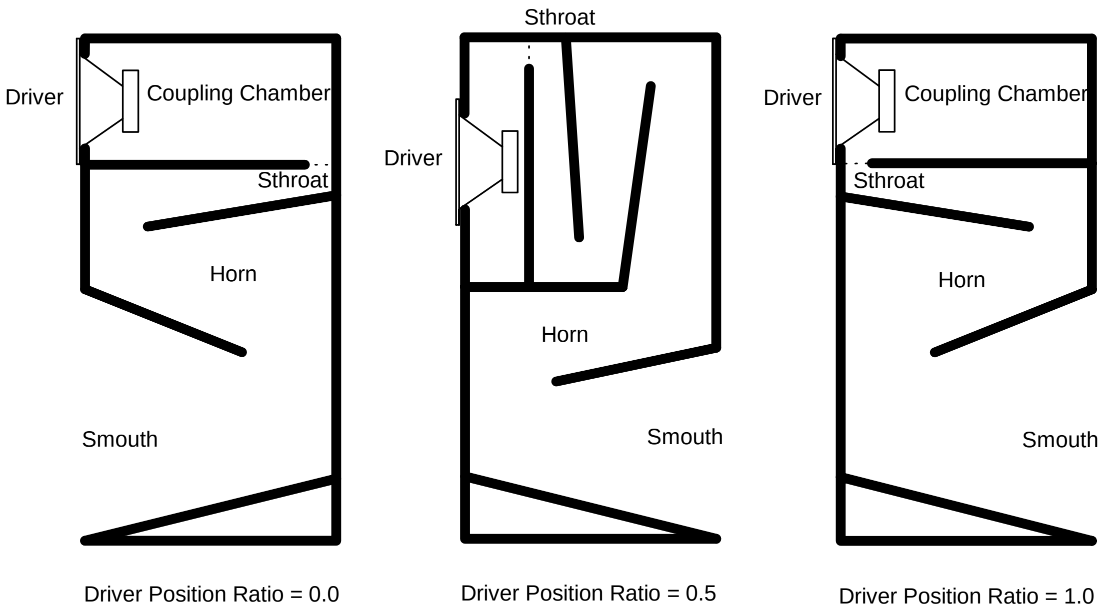
$$Z_{throat} = \frac{\rho\, c}{S_0}$$

Keeping this in mind, while looking again at Figure 7.1, we are left with a driver mounted in an enclosure where the front of the driver is radiating into the room and the back of the driver is radiating into a pure acoustic resistance which also radiates into the room but with a time delay due to the length of the horn.

The back loaded horn speaker system is more complicated due to the two different sources radiating into the room. The problem is to design a system where these two sources work well together producing a desirable SPL response across the entire frequency range. <u>A consistent design mates a driver with an exponential horn geometry when both have the same characteristic frequencies</u>. This last statement is an important definition that will be assumed throughout the remainder of this section.

The resulting motion of the driver’s cone, the driver’s volume velocity, is amplified by the horn to become a greater volume velocity at the horn’s mouth. This result produces an increase in low frequency system efficiency compared to a closed or ported box. Again, looking back at Figures 5.2 through 5.5 there is no evidence of a strong resonance in the horn which would be seen as peaks in the magnitude response and rapid phase shifts. All of this depends on the horn being sized to act as a horn and not a resonant transmission line as demonstrated in Figures 5.8 through 5.15.

Figure 7.1 : Back Loaded Horn Geometry



where :

Horn Geometry is defined by :

$S_0 = S_{throat}$ = throat area

$S_L = S_{mouth}$ = mouth area

L_{horn} = horn length

Coupling Chamber Geometry is defined by :

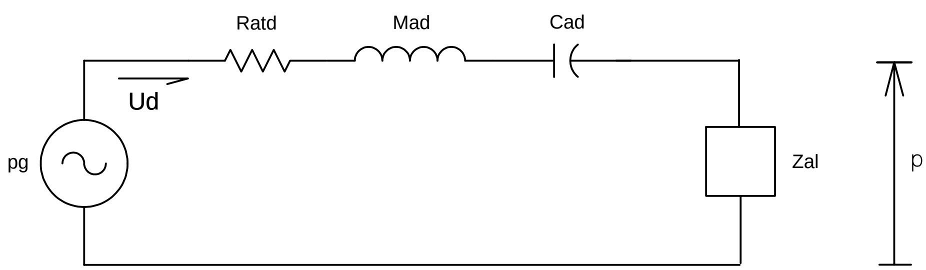
S_{DF} = coupling chamber area at the closed end

S_{LF} = coupling chamber area at the throat end

ξ = driver position ratio ($0 < \xi < 1$)

L_F = coupling chamber length

Figure 7.2 : Acoustic Equivalent Circuit for a Back Loaded Horn Speaker



where :

p_g = pressure source
= $(e_g\ Bl) / (S_d\ R_e)$

R_{ad} = driver acoustic resistance
= $(Bl^2 / S_d^2)\ [Q_{ed} / ((R_g + R_e)\ Q_{md})]$

R_{atd} = total acoustic resistance
= $R_{ad} + (Bl)^2 / [S_d^2\ ((R_g + R_e) + j\omega\ L_{vc})]$

C_{ad} = driver acoustic compliance
= $V_d / (\rho_{air}\ c^2)$

M_{ad} = driver acoustic mass
= $(f_d^2\ C_{ad})^{-1}$

Z_{al} = horn acoustic impedance (including coupling chamber)

U_d = driver volume velocity
= $S_d\ u_d$

u_d = driver cone velocity

then :

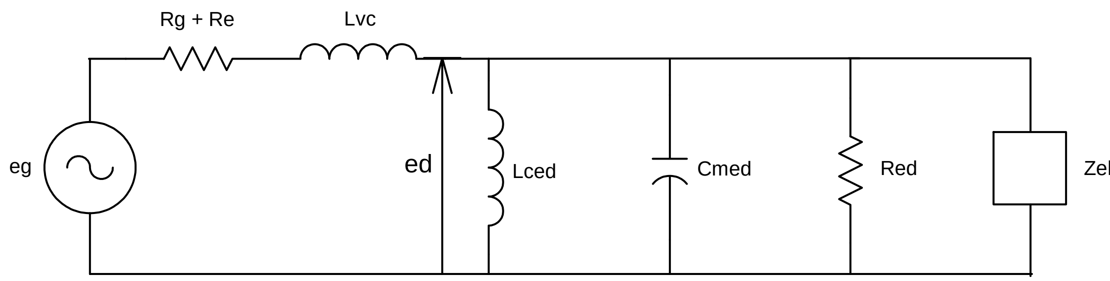
U_L = mouth air volume velocity
= $E\ U_d$

E = U_L / U_d = volume velocity ratio

u_L = mouth air velocity
= $\varepsilon\ u_d$

ε = u_L / u_d = velocity ratio

Figure 7.3 : Electrical Equivalent Circuit for a Back Loaded Horn Speaker



where :

e_g = voltage source
= 2.8284 volt

R_g+R_e = electrical resistance of the amplifier, cables, and voice coil

L_{vc} = voice coil inductance

L_{ced} = inductance due to the driver suspension compliance
= $[C_{ad} (Bl)^2] / S_d^2$

C_{med} = capacitance due to the driver mass
= $(M_{ad} S_d^2) / (Bl)^2$

R_{ed} = resistance due to the driver suspension damping
= $R_e (Q_{md} / Q_{ed})$

Z_{el} = horn equivalent electrical impedance (including coupling chamber)
= $(Bl)^2 / (S_d^2 Z_{al})$

e_d = $Bl\ u_d$

The Generic Driver :

Before any simulations can be run, a generic driver needs to be defined that is easily adjustable to different combinations of Thiele / Small[8,9,10,11] parameters. The following driver parameters have been defined and are intended to represent a typical eight inch diameter full range driver such as those produced by Lowther, Fostex, or AER. All of the results that follow are really intended for full range driver applications but should also be applicable to woofer or mid-bass drivers.

The generic full range driver is defined below based on key input properties and some derived properties. When looking at the relationships used to calculate the derived properties, please keep in mind that MathCad internally automatically converts frequency in Hertz to frequency in rad/sec. This property of MathCad leads to equations that may not look exactly like those familiar to the DIY speaker designer. In equations containing a frequency term, a 2π multiplier may be missing or added depending on the desired units of the result.

Driver Thiele / Small Parameters : Generic Driver Derivation

$f_d := 50 \cdot Hz$ $\qquad Q_{md} := 4$

$R_e := 8 \cdot ohm$ $\qquad Q_{td} := 0.2$

$L_{vc} := 0 \cdot mH$ $\qquad M_{md} := 14 \cdot gm$

$S_d := 205 \cdot cm^2$

Derived Thiele / Small Parameters

$$Q_{ed} := \left(\frac{1}{Q_{td}} - \frac{1}{Q_{md}}\right)^{-1} \qquad Q_{ed} = 0.2$$

$$C_{md} := \left(M_{md} \cdot f_d^2\right)^{-1} \qquad C_{md} = 7.2 \times 10^{-4} \frac{m}{newton}$$

$$V_{ad} := C_{md} \cdot \left(\rho \cdot c^2 \cdot S_d^2\right) \qquad V_{ad} = 43.0 liter$$

$$\eta_o := V_{ad} \cdot \left(2 \cdot \pi \cdot c^3 \cdot Q_{ed} \cdot f_d^{-3}\right)^{-1} \qquad \eta_o = 2.5\%$$

$$SPL := 112 + 10 \cdot \log(\eta_o) \qquad SPL = 96.0 \text{ dB}$$

$$Bl := \left(\frac{f_d \cdot R_e \cdot M_{md}}{Q_{ed}}\right)^{0.5} \qquad Bl = 12.9 \frac{newton}{amp}$$

Now that a generic driver has been defined, baseline horn geometry can be formulated and a simulation run to calculate the on-axis anechoic SPL response, the electrical impedance, and the impulse pressure response. Modifications can be made to the generic driver, and the exponential horn geometry, to study the changes that occur in the calculated responses.

Baseline Exponential Horn Design :

The first simulation to be run, and results presented, will be referred to as the baseline design. The lower cut-off frequency f_c is specified as 50 Hz to match the driver's f_d. A coupling chamber will not be included in the baseline horn geometry. The throat area is set equal to the driver's S_d so that a length can be calculated. This is a consistent back loaded horn design.

To start the design process, the area of the horn mouth is calculated using Equation (5.3).

$$S_{mouth} = (1 / \pi) \times (c / (2 \times f_c))^2$$

$$S_{mouth} = (1 / \pi) \times (342 \text{ m/sec} / (2 \times 50 \text{ Hz}))^2$$

$$S_{mouth} = 3.723 \text{ m}^2 = 5771 \text{ in}^2$$

$$S_{mouth} = 181.6 \times S_d$$

Using Equation (5.2), the flare constant is calculated next.

$$m = (4 \pi f_c) / c$$

$$m = (4 \pi 50 \text{ Hz}) / 342 \text{ m/sec}$$

$$m = 1.837 \text{ m}^{-1}$$

And finally, the horn's length is calculated using Equation (5.1) after setting the throat area equal to S_d.

$$L_{horn} = \ln(S_{mouth} / S_{throat}) / m$$

$$L_{horn} = \ln(181.6 / 1) / 1.837 \text{ m}^{-1}$$

$$L_{horn} = 2.831 \text{ m} = 111.5 \text{ in}$$

The horn's geometry is now completely defined. Looking at the dimensions shown above, not many DIYer's would build a horn with a mouth this big. But if we follow this design through, a number of results will be presented that provide some insights leading to a reduction of the horn's size towards something more manageable. Substituting the dimensions and areas into one of the back loaded horn MathCad worksheets the acoustic impedance, the volume velocity ratio, the SPL, the electrical impedance, the driver displacement, and the impulse response are calculated.

Figure 7.4 shows the calculated acoustic impedance at the horn's throat and the volume velocity ratio between the throat and the horn mouth. As was observed in the previous sections, the acoustic impedance is purely resistive above the lower cut-off frequency f_c. Also, notice that the volume velocity ratio exhibits only a phase shift associated with distance traveled. In other words, the volume velocity ratio's phase increases linearly with frequency.

Figure 7.5 presents the back loaded horn system SPL response (solid red curve) along with the driver in an infinite baffle response (dashed blue curve) as a reference in the top plot. In the bottom plot, the driver (solid red curve) and horn mouth (dashed blue curve) contributions to the back loaded horn system's SPL response are shown. A zero dB reference is used in all of the back loaded horn SPL plots corresponding to the basic driver SPL at 1 m for 1 watt of input. For the baseline generic driver this value is 96 dB. Obviously the plotted response is nowhere near the desired smooth and flat SPL response. But looking at the different peaks and nulls, a few key observations can be made that will increase the understanding of the back loaded horn's performance.

First, in the bottom plot of Figure 7.5 a series of nulls are evident in the mouth SPL response starting at 700 Hz. These are the same nulls seen in the front loaded horn plots due to the large size of the mouth and the distance at which the response is calculated. At the 1 m calculation distance, as frequency increases and the wavelength decreases, the sound radiated from different points on the mouth cancel. This is due to the difference in the path lengths to the reference position corresponding to 180 degree phase differences in the pressure at specific frequencies. As the reference position changes, the frequency at which the nulls start to appear also changes. For example, at a 10 m distance the first null appears at approximately 6000 Hz. Looking at the location of the deep nulls in the bottom plot, and transferring the frequency locations to the top plot, some of the anomalies can easily be explained in the system SPL response.

A second set of smaller peaks and nulls can also be seen in the system SPL response shown in the top plot of Figure 7.5. These are not evident in the driver and mouth SPL response shown in the bottom plot. These peaks and nulls start just above 100 Hz, are spaced closer together in the frequency domain, and increase in depth as frequency increases. A simple explanation is also possible for the cause of these peaks and nulls. Going back to Figure 7.4, remember the acoustic impedance acting on the back of the driver is purely resistive and the ratio of the volume velocities exhibits a phase shift that is only a function of distance. The sound radiated from the back of the driver is 180 degrees out of phase with the front of the driver. The sound produced at the mouth is also 180 degrees out of phase but with the addition of a phase shift associated with traveling the length of the horn. Above the lower cut-off frequency f_c, the back loaded horn can be visualized as two sources, 180 degrees out of phase, separated by a distance equal to the length of the horn. An estimate can now be made of the frequencies at which the driver and mouth add to form a peak or cancel to form a null and the results are shown at the top of the following page. Remember that MathCad automatically internally converts frequency in Hertz to frequency in rad/sec and this property leads to equations that may not look exactly like those familiar to the DIY speaker designer. In equations containing a frequency term, a 2π multiplier may be missing or added depending on the desired units of the result.

Cancellation and Reinforcement Frequencies

$L_{horn} = 2.831\text{m}$

If the two sources (driver and mouth) were of equal strength :

$$f_{null} := \frac{2\cdot\pi c}{L_{horn}} \qquad f_{peak} := \frac{2\cdot\pi\cdot c}{2\cdot L_{horn}}$$

$1\cdot f_{null} = 120.787\text{Hz}$	$1\cdot f_{peak} = 60.393\text{Hz}$
$2\cdot f_{null} = 241.574\text{Hz}$	$3\cdot f_{peak} = 181.180\text{Hz}$
$3\cdot f_{null} = 362.360\text{Hz}$	$5\cdot f_{peak} = 301.967\text{Hz}$
$4\cdot f_{null} = 483.147\text{Hz}$	$7\cdot f_{peak} = 422.754\text{Hz}$
$5\cdot f_{null} = 603.934\text{Hz}$	$9\cdot f_{peak} = 543.541\text{Hz}$
$6\cdot f_{null} = 724.721\text{Hz}$	$11\cdot f_{peak} = 664.327\text{Hz}$
$7\cdot f_{null} = 845.507\text{Hz}$	$13\cdot f_{peak} = 785.114\text{Hz}$

The frequencies shown above correlate reasonably well with the peaks and nulls in the system SPL response plotted in Figure 7.5. As the relative magnitudes of the driver and the mouth SPL change, so do the depths of the nulls at the frequencies where they are 180 degrees out of phase. There are just a couple of more results to be presented to complete the characterization of the baseline back loaded horn calculated response.

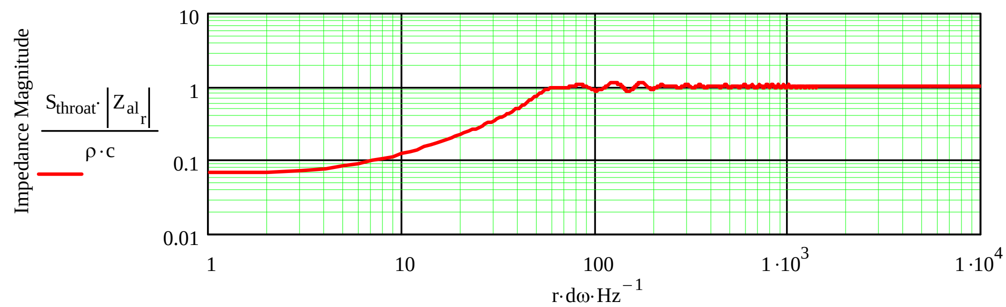
Figure 7.6 presents the electrical impedance magnitude and phase of the driver in the baseline back loaded horn (solid red curve) and in an infinite baffle (dashed blue curve). Since the back loaded horn represents a pure acoustic resistance, the back loaded horn system impedance curve exhibits almost the same characteristics as a driver mounted in an infinite baffle but with the driver's resonant frequency reduced by the additional mass of the air in the horn.

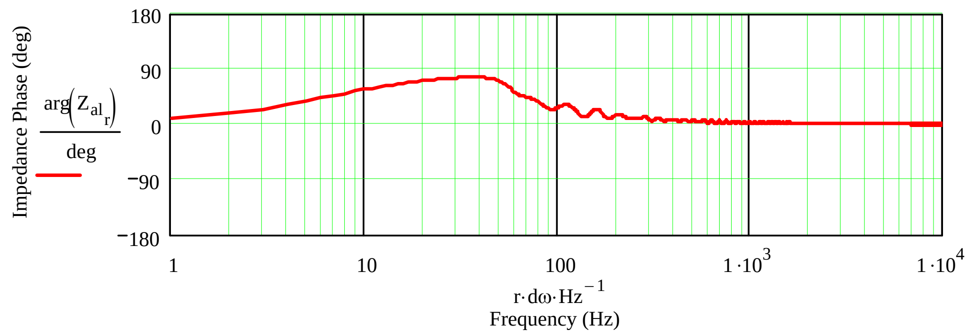
Figure 7.7 presents the driver displacement for the driver in the baseline back loaded horn (solid red curve) and in an infinite baffle (dashed blue curve). The curves are almost identical. Unlike the bass reflex and closed box enclosures, there is no reduction in the driver displacement curve. The consistent back loaded horn design does not provide any advantage over an infinite baffle with respect to driver displacement.

Figure 7.8 shows the impulse response of the back loaded horn system. The driver pulse arrives first followed by an inverted pulse that is produced by the horn mouth. The sharpness of the initial pulse is significantly better then the other more common styles of speaker enclosures.

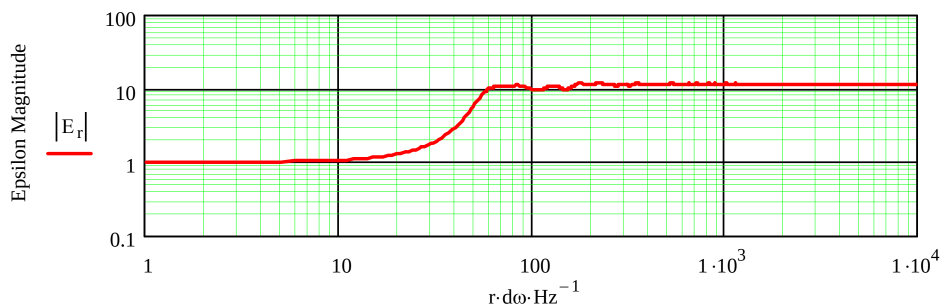
Figure 7.4 : Back Loaded Exponential Horn Response - Baseline Configuration

Resulting Acoustic Impedance for the Back Loaded Horn





Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m ^{3}/sec Driver Excitation



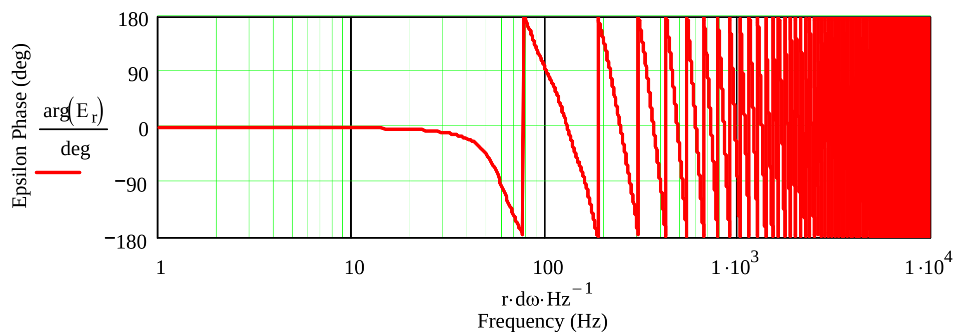
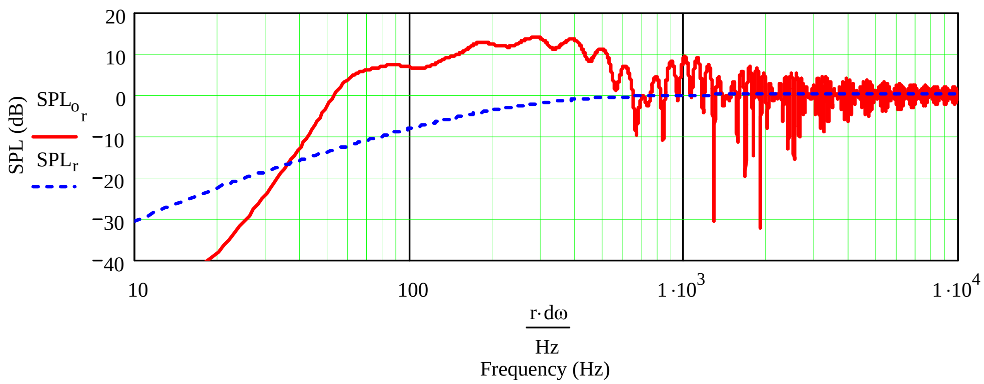


Figure 7.5 : Back Loaded Exponential Horn Response - Baseline Configuration

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

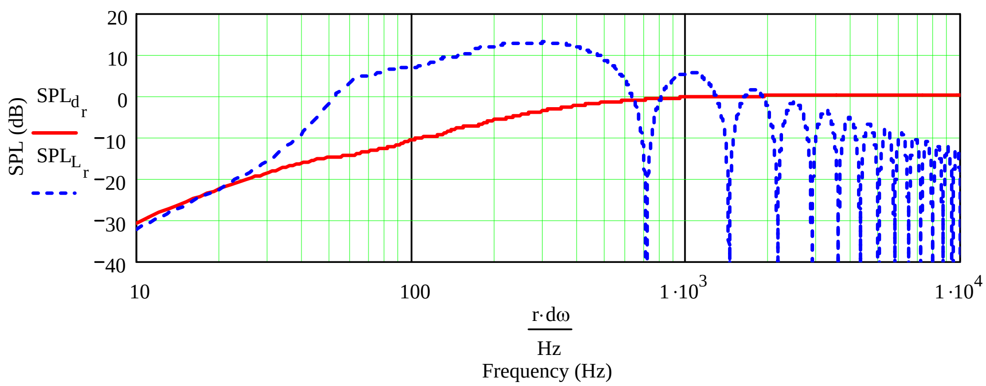


Figure 7.6 : Back Loaded Exponential Horn Response - Baseline Configuration

Back Loaded Horn System and Infinite Baffle Impedance

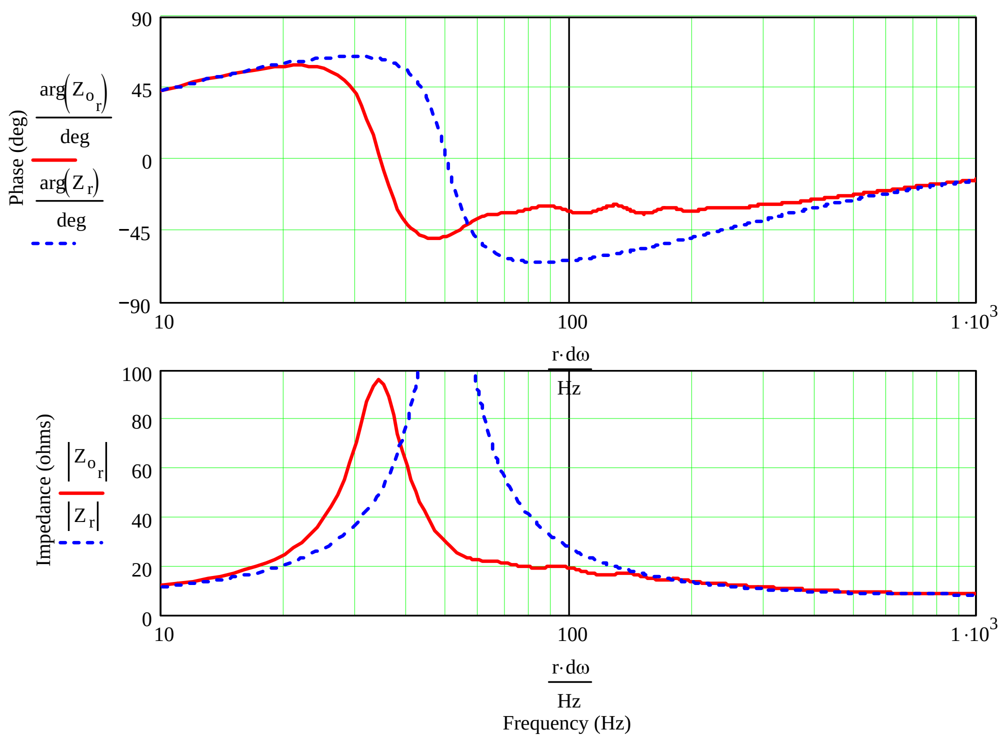


Figure 7.7 : Back Loaded Exponential Horn Response - Baseline Configuration

Woofer Displacement

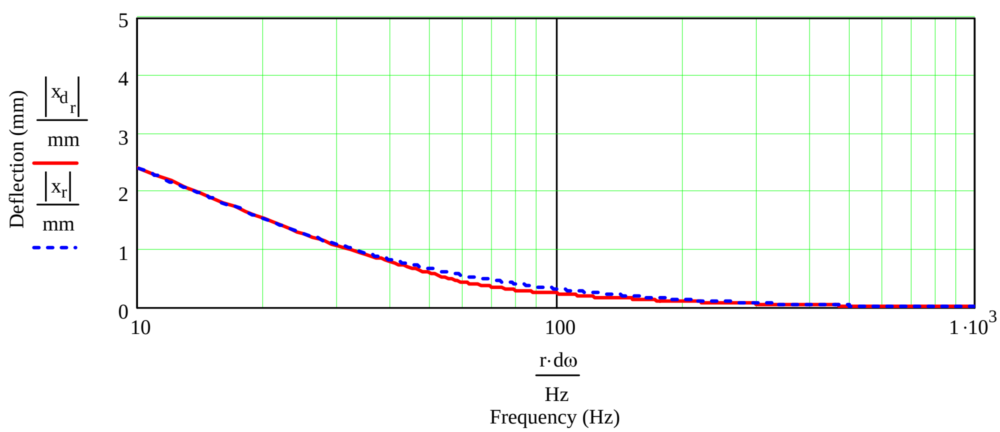
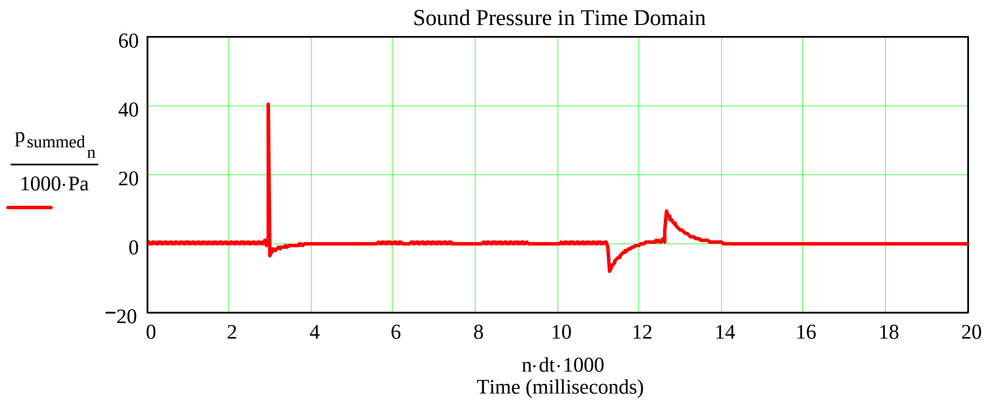


Figure 7.8 : Back Loaded Exponential Horn Response - Baseline Configuration

System Time Response for an Impulse Input



Adding a Coupling Chamber to the Baseline Exponential Horn Design :

Based on the plots shown in Figure 7.5, there is a real advantage to adding a coupling chamber to the baseline design. The ultimate goal of a back loaded horn design is to augment the driver's rolling off low frequency response by horn loading the output from the back side of the driver. The increase in efficiency generated by horn loading will provide significant bass reinforcement which in theory can also negate the baffle step loss associated with most other enclosure styles. Adding 3 to 6 dB of additional bass SPL response, over the frequency range below the baffle step frequency, will yield a flat frequency response when the speaker is located in a real listening room. But at some frequency, the output from the horn needs to be rolled off so that the system efficiency is equal to the driver's reference SPL value.

The simulations presented in this section will include a coupling chamber that rolls off the horn response at a higher cut-off frequency f_h of 100 Hz. The chamber volume V is calculated, in the same manner as demonstrated in Section 5, using Equation (5.4).

$$V = (342 \text{ m/sec} \times 0.021 \text{ m}^2) / (2\,\pi\,100 \text{ Hz}) \times (1000 \text{ liters} / \text{m}^3) = 11.158 \text{ liters}$$

Placing this coupling volume, modeled as a lumped compliance, in series with the baseline horn geometry and rerunning the MathCad simulation produces the results plotted in the figures that follow.

Figure 7.9 shows the calculated acoustic impedance seen by the back of the driver and the volume velocity ratio between the driver and the horn mouth. As was observed in the previous section, the acoustic impedance is purely resistive above the lower cut-off frequency f_c. Adding a coupling volume rolls off both the acoustic impedance and the volume velocity ratio.

In Figure 7.10, the back loaded horn system SPL response (solid red curve) along with the driver in an infinite baffle response (dashed blue curve), as a reference, are shown in the top plot and the driver (solid red curve) and horn mouth (dashed blue curve) contributions to the back loaded horn's system SPL response are shown in the bottom plot. Clearly the coupling volume has significantly improved the system SPL response.

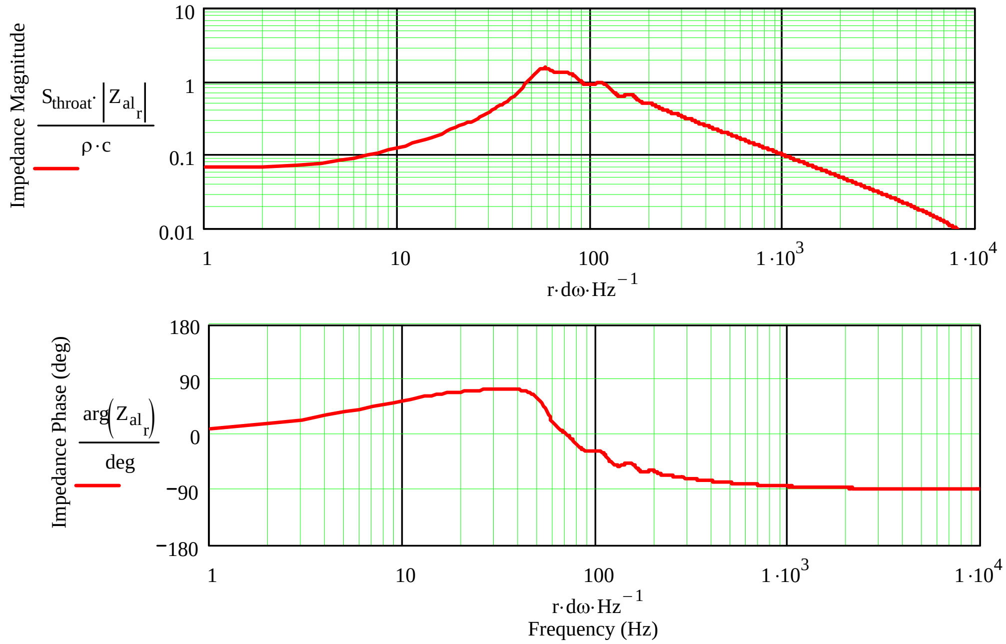
Figure 7.11 presents the electrical impedance magnitude and phase of the driver in the baseline back loaded horn (solid red curve) and in an infinite baffle (dashed blue curve). The impact of the coupling chamber is seen as a second broad and rounded impedance hump centered at approximately 125 Hz.

Figure 7.12 shows the driver displacement for the driver in the baseline back loaded horn (solid red curve) and in an infinite baffle (dashed blue curve). Again the curves are almost identical. The back loaded horn, with a coupling chamber, appears to provide a small advantage over an infinite baffle with respect to driver displacement. It should also be noted that in the listening room this displacement curve will generate the driver's full rated SPL at 1 m for 1 watt input all the way down to the horn's lower cut-off frequency. The bass output from the back loaded horn system is more efficient then the driver's rated SPL output and fills in for the rolling off response of the driver.

Figure 7.13 displays the impulse response of the back loaded horn system. The driver pulse arrives first followed by an inverted pulse that is produced by the horn mouth. The sharpness of the initial pulse is still significantly better then other common types of enclosures. Notice that the secondary pulse from the horn mouth is almost flat due to the filtering out of the higher frequencies by the coupling chamber.

Figure 7.9 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m ^{3}/sec Driver Excitation

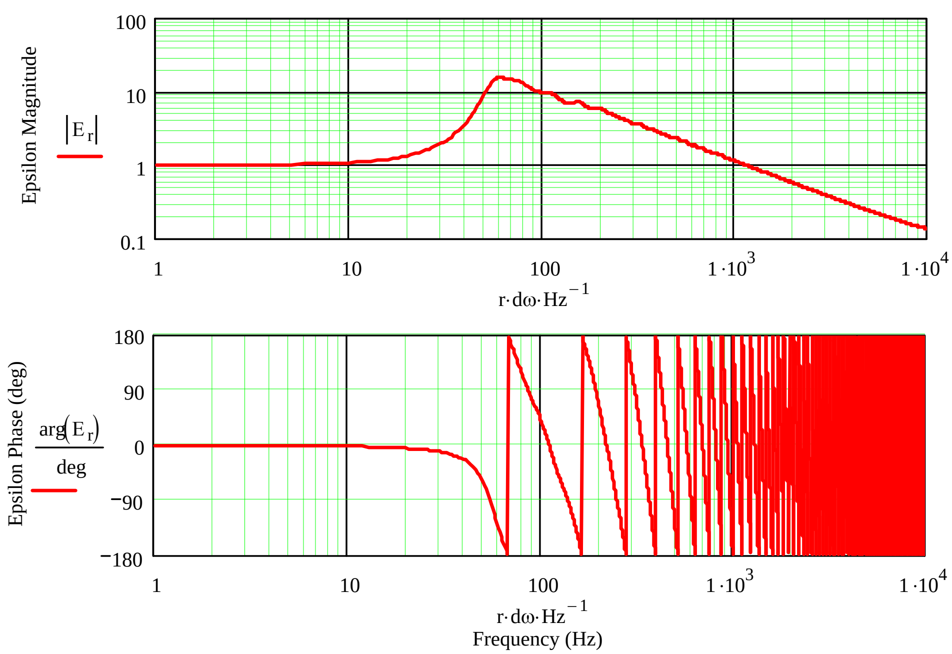
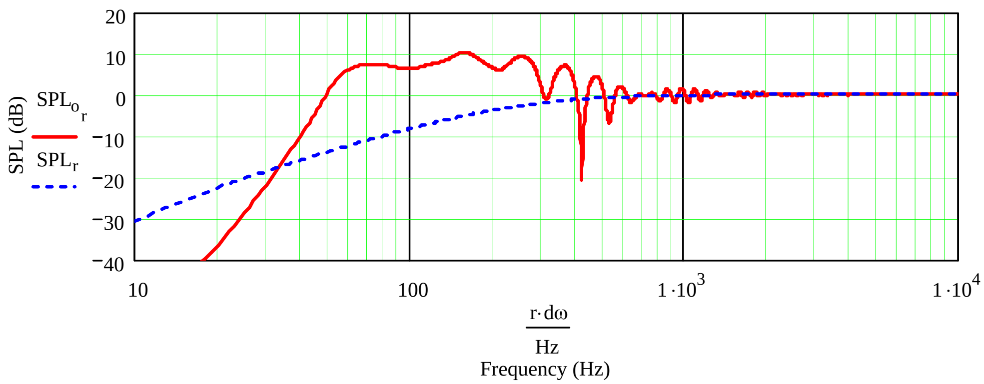


Figure 7.10 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

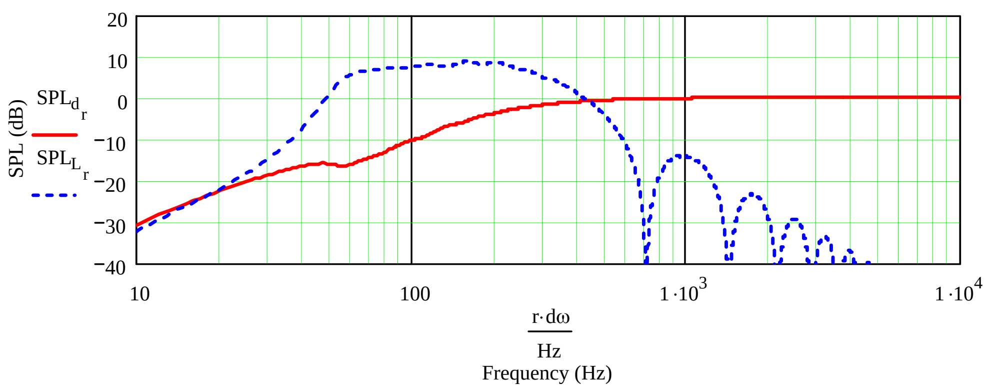


Figure 7.11 : Back Loaded Exponential Horn Response - Baseline Configuration with a Coupling Chamber

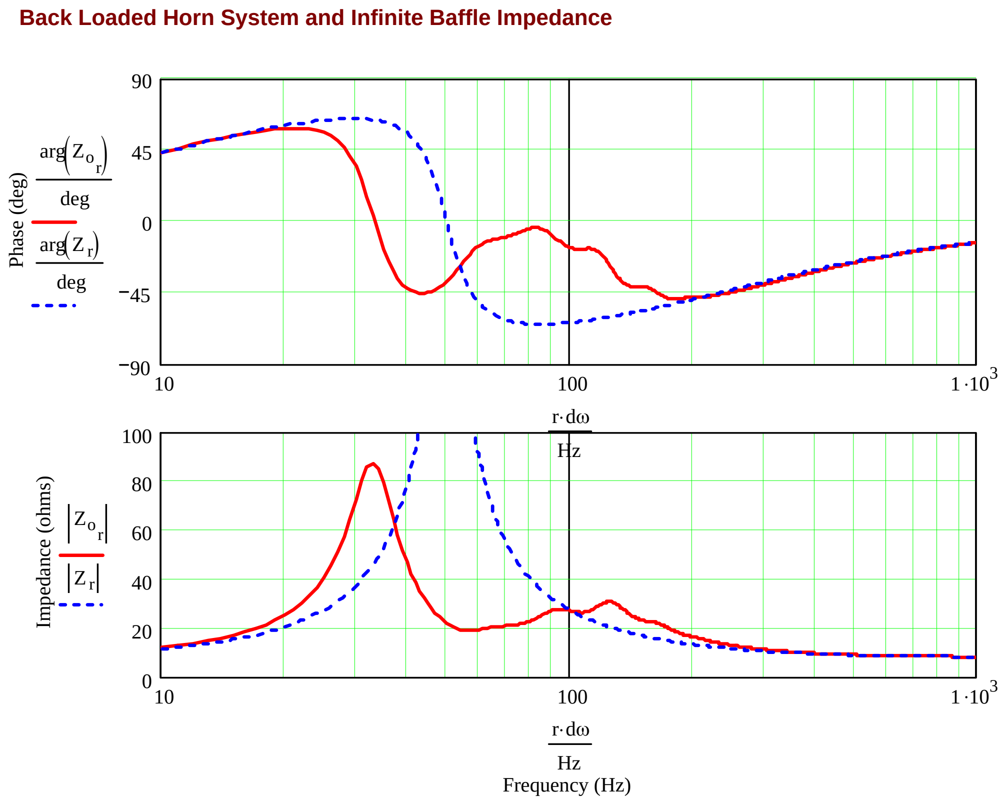


Figure 7.12 : Back Loaded Exponential Horn Response - Baseline Configuration with a Coupling Chamber

Woofer Displacement

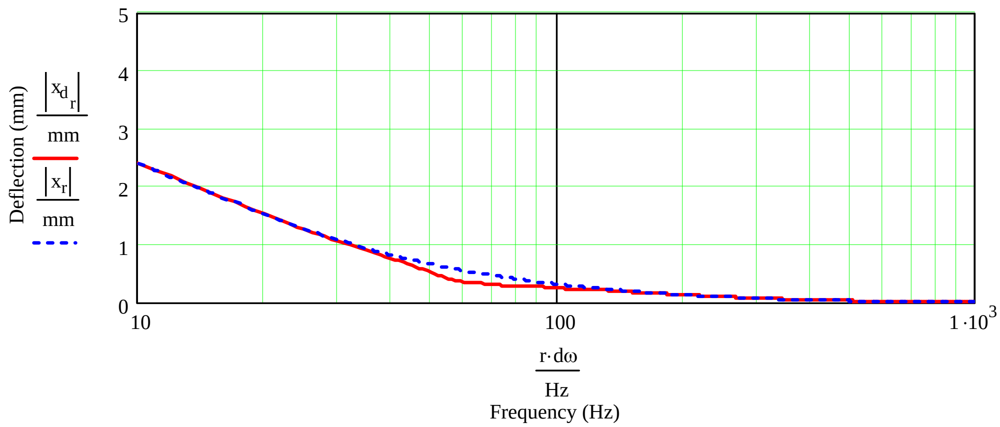
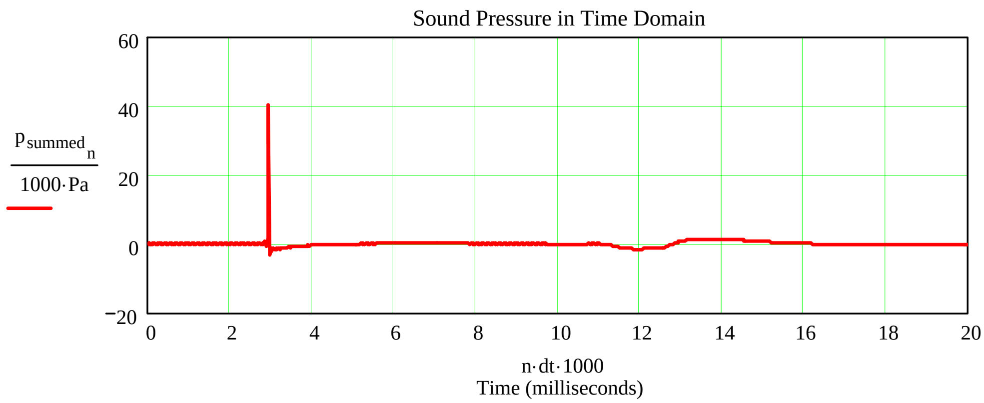


Figure 7.13 : Back Loaded Exponential Horn Response - Baseline Configuration with a Coupling Chamber

System Time Response for an Impulse Input



Unfortunately, the preceding simulation turns out to be a little too optimistic. The coupling chamber was modeled as a lumped compliance when in reality it is a volume as shown in the three back loaded horn configurations in Figure 7.1. The volume of the coupling chamber supports standing waves which will in turn impact the driver and horn mouth output. Consider the coupling volume as a transmission line, with an offset driver, closed at one end and exhausting into an acoustic resistive load, the horn's throat, at the other end. At the horn's throat, a pressure maximum will exist with almost zero velocity which is essentially a closed end boundary condition. Half wavelength standing waves are going to be excited in the coupling chamber.

Define the coupling chamber as having a cross-sectional area of twice the throat area, which in this case is also twice the driver's cone area S_d. The length of the coupling chamber is calculated, from the 11.158 liter volume, to be 10.715 inches (or 0.272 m). Three simulations are presented in the following plots corresponding to the three driver positions shown in Figure 7.1. The acoustic impedance, volume velocity ratio, and the SPL responses for the driver ratio ξ = 0.0 are shown in Figures 7.13 and 7.16 respectively. The acoustic impedance, volume velocity ratio, and the SPL responses for the driver ratio ξ = 0.5 are shown in Figures 7.14 and 7.17 respectively. The acoustic impedance, volume velocity ratio, and the SPL responses for the driver ratio ξ = 1.0 are shown in Figures 7.16 and 7.18 respectively.

Assuming that the standing waves of interest are quarter or half wavelengths, corresponding to the length of the coupling chamber, the significant frequencies can be calculated as shown below.

$$f_{1/4} = c / (4 \text{ x } L)$$

$$f_{1/4} = (342 \text{ m/sec}) / (4 \text{ x } 0.272 \text{ m}) = 314 \text{ Hz}$$

$$f_{1/2} = c / (2 \text{ x } L)$$

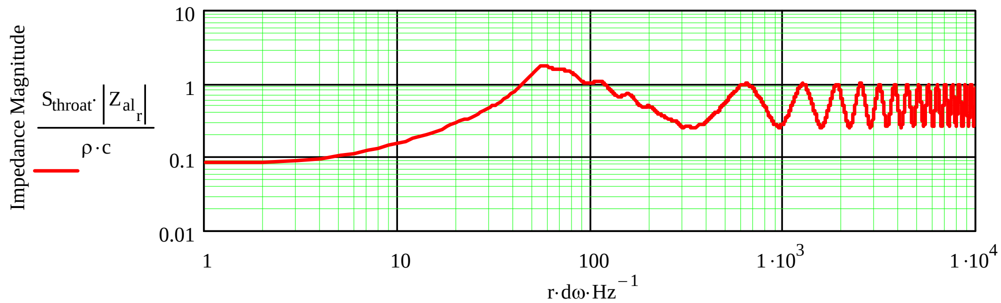
$$f_{1/2} = (342 \text{ m/sec}) / (2 \text{ x } 0.272 \text{ m}) = 628 \text{ Hz}$$

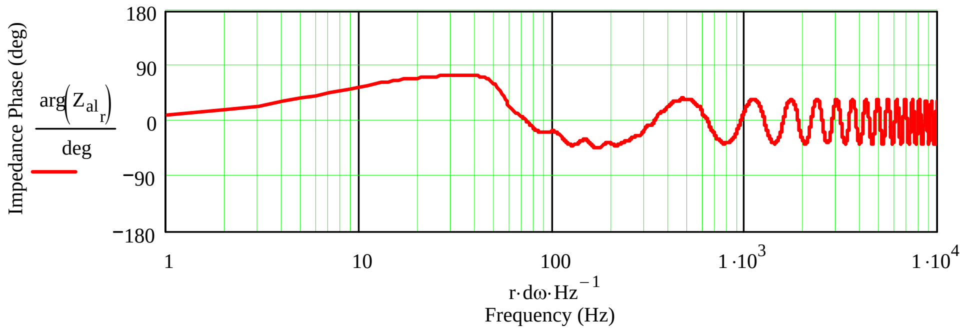
For ξ = 0.0 and ξ = 1.0, the quarter wavelength frequency corresponds to the first null in the acoustic impedance curve and the half wavelength frequency corresponds to the first peak. The first strong standing wave resonance is a half wavelength at 628 Hz.

When ξ = 0.5, the frequency of the first null doubles because the distance from the driver to the closed or throat end of the transmission line, formed by the coupling chamber, has been cut in half. The first resonant frequency for ξ = 0.5 is 628 Hz but the driver is at a pressure minimum as reflected in the acoustic impedance plot by the very sharp and deep null. The first strong standing wave to exert pressure on the driver is the full wavelength mode at 1256 Hz as seen in Figure 7.14.

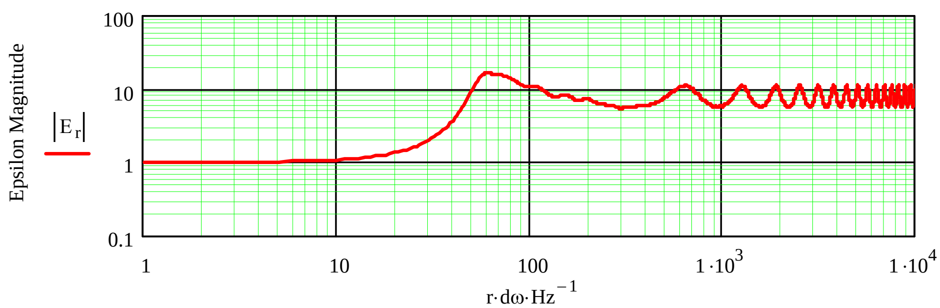
Figure 7.13 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber and a Driver Position Ratio ξ = 0.0

Resulting Acoustic Impedance for the Back Loaded Horn





Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m^3/sec Driver Excitation



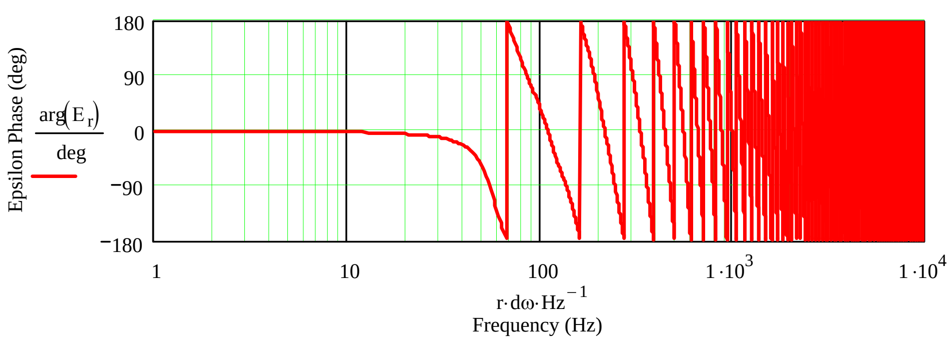
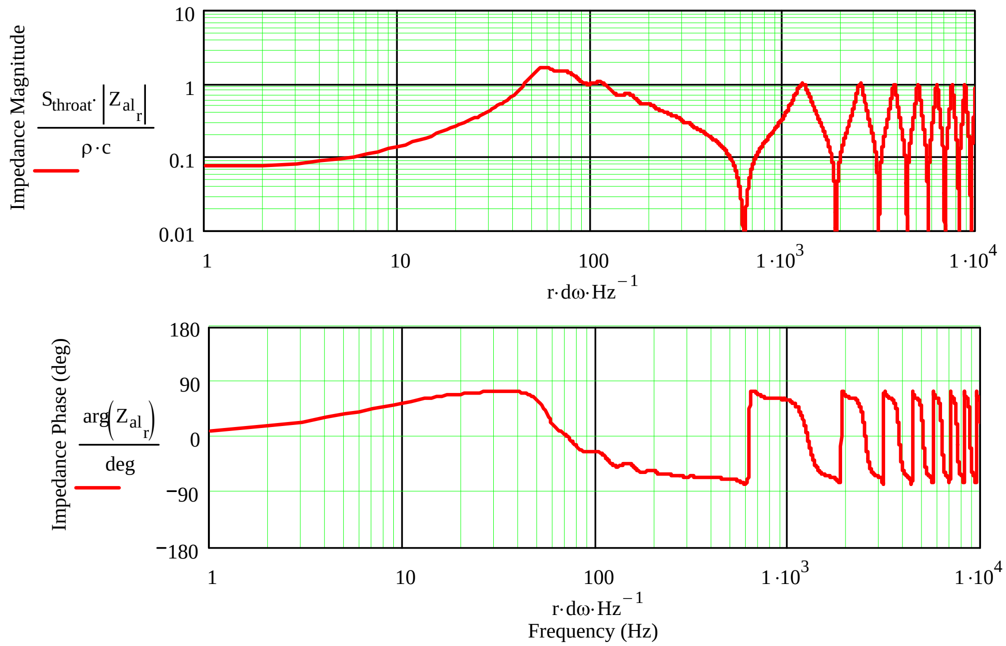


Figure 7.14 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber and a Driver Position Ratio ξ = 0.5

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m^3/sec Driver Excitation

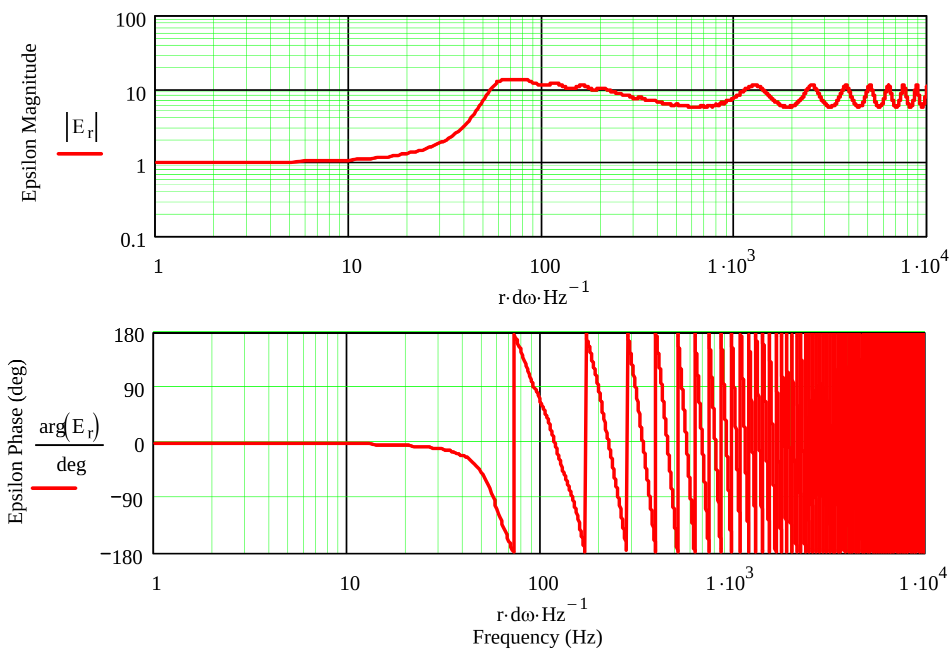
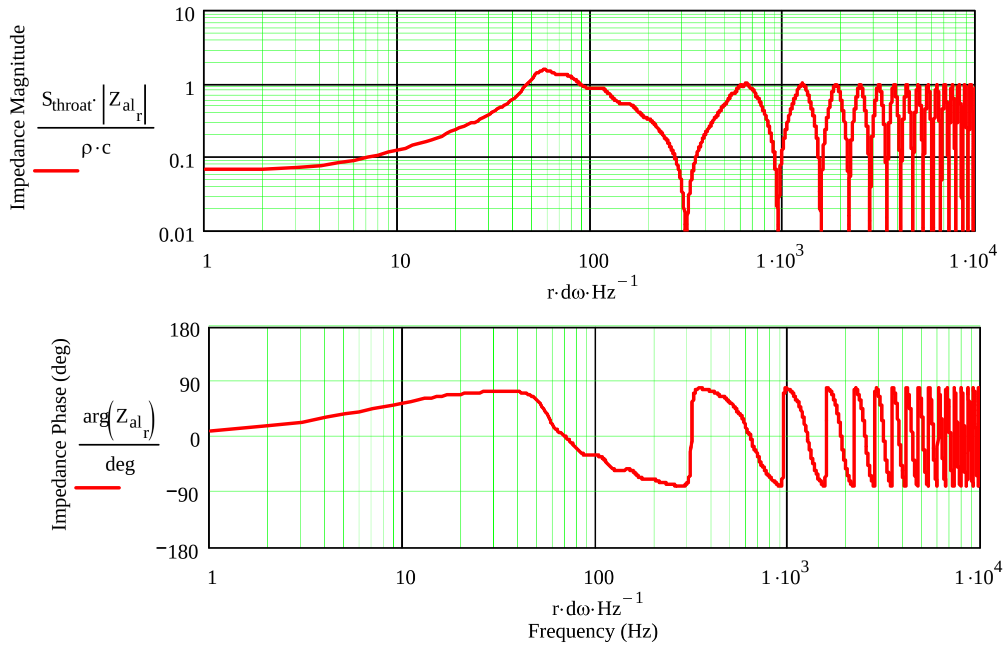


Figure 7.15 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber and a Driver Position Ratio ξ = 1.0

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m^3/sec Driver Excitation

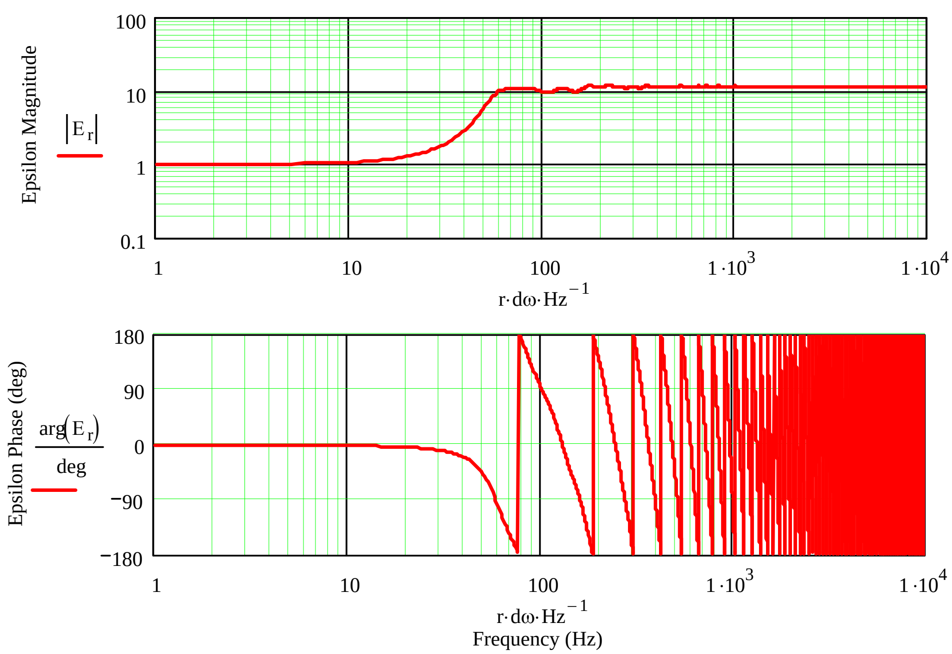
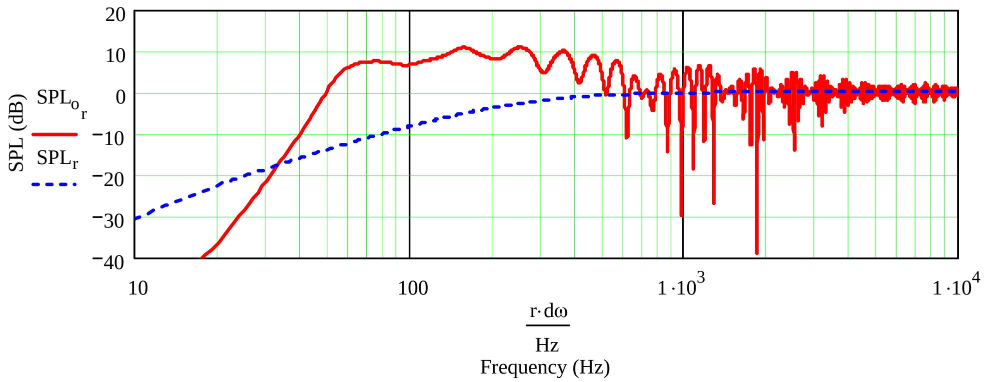


Figure 7.16 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber and a Driver Position Ratio ξ = 0.0

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

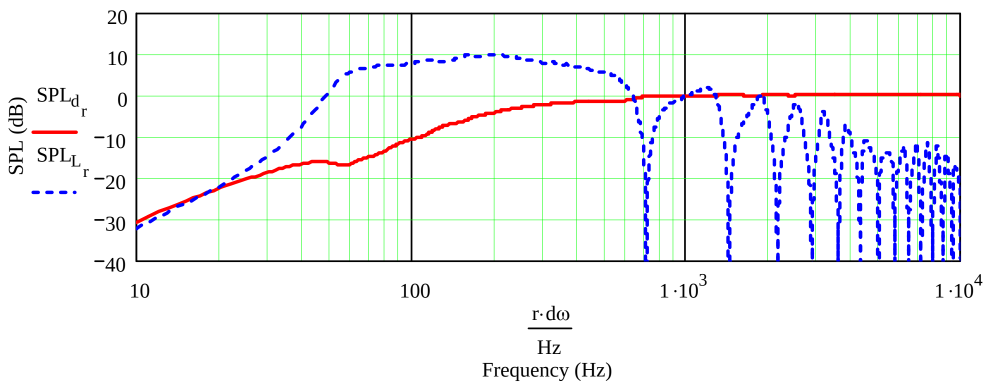
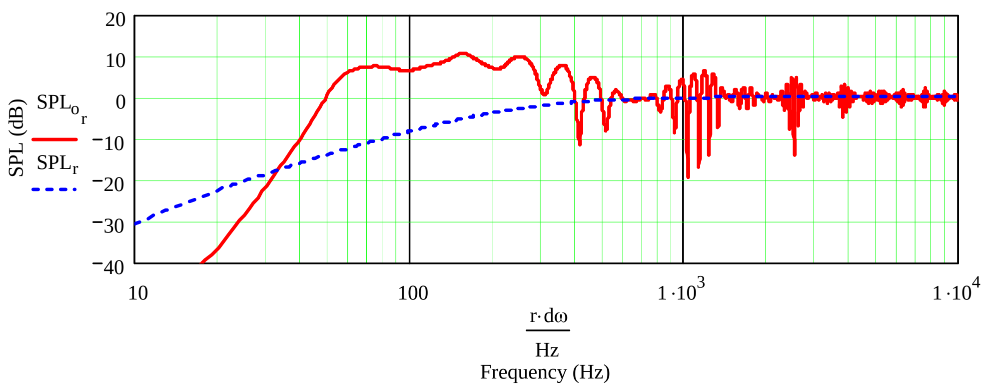


Figure 7.17 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber and a Driver Position Ratio ξ = 0.5

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

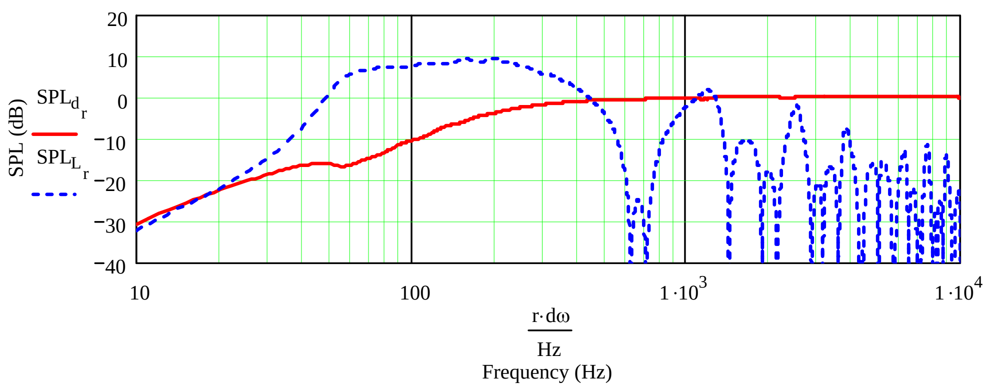
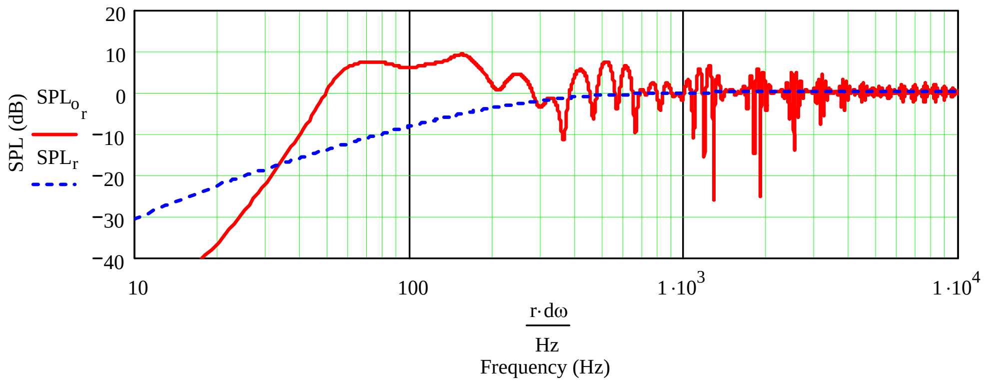
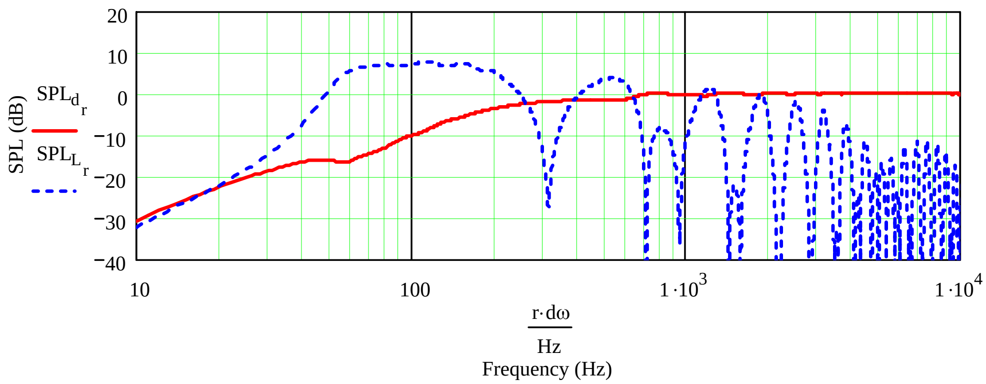


Figure 7.18 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber and a Driver Position Ratio ξ = 1.0

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

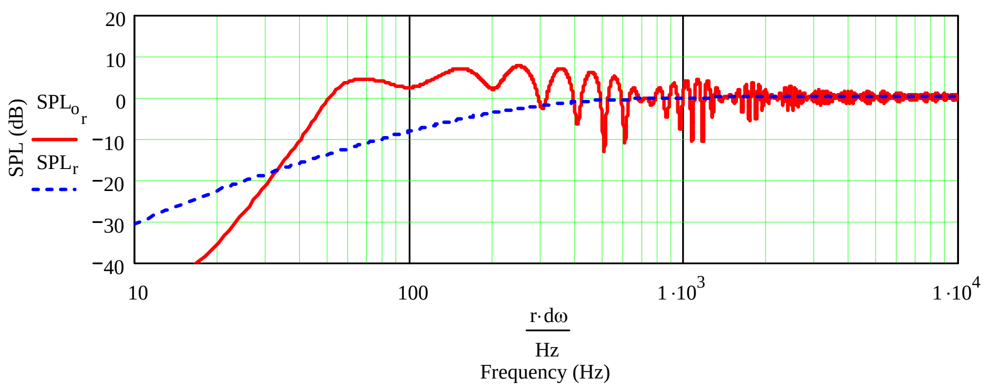


One final set of simulation results will be presented to finish this discussion of the coupling chamber. By adding 0.5 lb/ft^3 of fiber stuffing to the coupling volume, and 0.125 lb/ft^3 of fiber stuffing to the first one third of the horn's length, many of the peaks and nulls can be reduced. Figures 7.19, 7.20, and 7.21 show the SPL traces with the fiber damping included in the simulation for the driver position ratios ξ = 0.0, ξ = 0.5, and ξ = 1.0 respectively.

In each of the figures, a 3 to 4 dB boost of the lower bass response has been achieved. The nulls still exist but they have been reduced significantly. At this point, this particular analysis will stop in favor of looking at some other back loaded horn geometries and eventually pushing towards smaller horn designs. The basic consistent design sizing and trade-offs have been demonstrated.

Figure 7.19 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber, Driver Position Ratio ξ = 0.0, and Fiber Damping

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

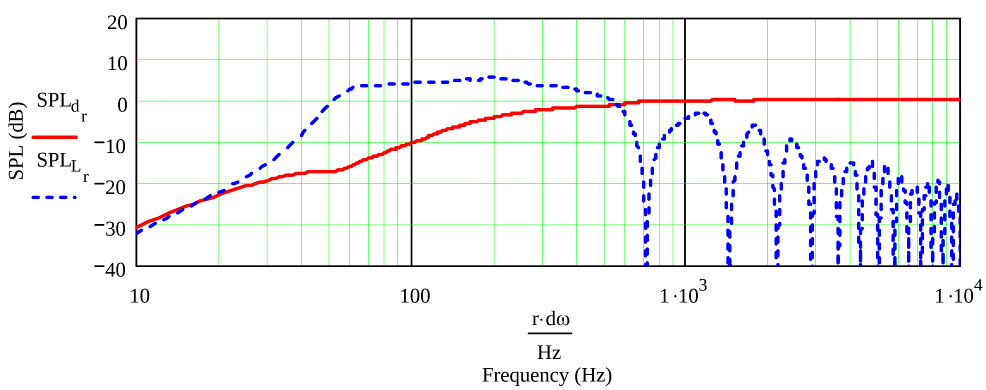
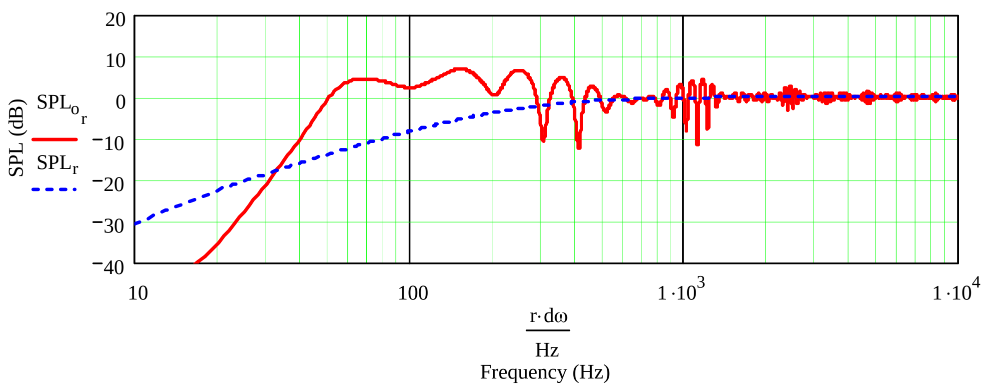


Figure 7.20 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber, Driver Position Ratio ξ = 0.5, and Fiber Damping

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

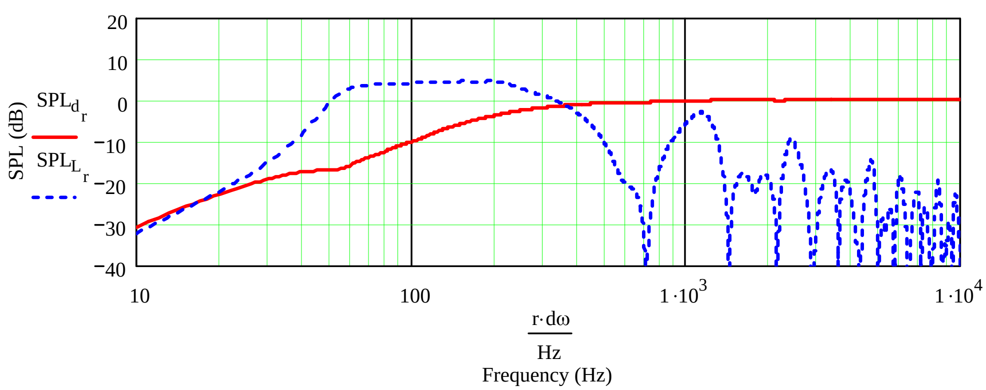
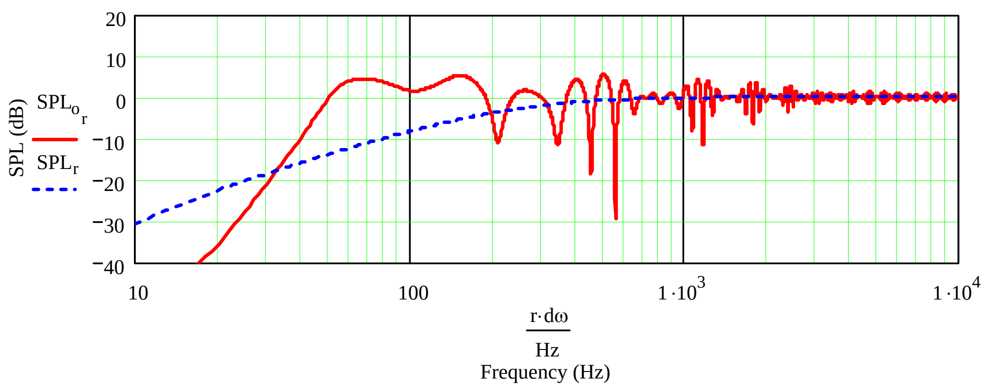
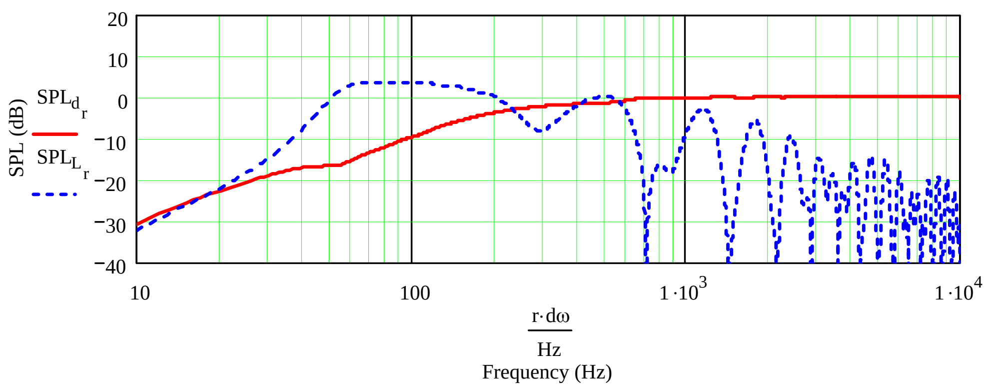


Figure 7.21 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber, Driver Position Ratio ξ = 1.0, and Fiber Damping

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Compromised Back Loaded Horn Geometries :

When the baseline back loaded horn geometry was defined on page 7, the size of the mouth was obviously bigger then typically found in designs shown on the Internet. This includes designs provided by driver manufacturers and also designs posted by DIY speaker designers/builders. To study the ramifications of reducing the mouth size, and generating a compromised horn/transmission line enclosure, a series of simulation results will be presented in the following figures. To simplify the results, and only show the impacts of changes to the mouth dimensions, the coupling chamber will again be treated as a lumped compliance. All of the following simulations use the baseline throat size, a higher cut-off frequency f_h of 100 Hz, and fiber stuffing. The mouth size and the horn length have been reduced to maintain the same exponential horn flare constant corresponding to 50 Hz.

Table 7.1 contains the baseline and the compromised back loaded horn geometries. The figures referenced in the table show the simulation results. The first row repeats the baseline horn configuration and the subsequent rows show the compromised configurations corresponding to $S_{mouth}/4$, $S_{mouth}/8$, $S_{mouth}/16$, and $S_{mouth}/32$.

Table 7.1 : Compromised Back Loaded Horn Study

Throat Area	Mouth Area	Length (in)	f_c (Hz)	Figures
1.0 x S_d	181.6 x S_d	111.5	50	7.22 a, b, and c
1.0 x S_d	45.4 x S_d	91.4	100	7.23 a, b, and c
1.0 x S_d	22.7 x S_d	81.4	141	7.24 a, b, and c
1.0 x S_d	11.4 x S_d	72.5	200	7.25 a, b, and c
1.0 x S_d	5.7 x S_d	61.3	282	7.26 a, b, and c

The simulation results in Figures 7.22 through 7.26 exhibit a number of trends. As the mouth size decreases, both the acoustic impedance and volume velocity plots show peaks associated with standing waves. As the mouth gets smaller, the peaks become more pronounced and extend higher in frequency. The acoustic impedance of the horn is no longer predominantly resistive above 50 Hz and has taken on the characteristics of a transmission line. The impact of the resonances are also seen as ripples in the SPL plots, additional peaks in the electrical impedance plots, and dips in the driver displacement plots. By the time the horn mouth is reduced by factors of 16 and 32, the horn is essentially behaving as a transmission line over the entire frequency range where it reinforces the driver's bass output.

Many back loaded horn designs found on the Internet have mouth areas in the same size range as the last two rows of Table 7.1. By calculating the actual mouth areas used in the last two rows of Table 7.1, the equivalent dimensions of a square mouth can be determined.

$$S_{mouth} = 11.4 \times 31.8 \text{ in}^2 = 362.2 \text{ in}^2 \text{ ----> } 19.0 \times 19.0 \text{ inch square}$$

$$S_{mouth} = 5.7 \times 31.8 \text{ in}^2 = 181.1 \text{ in}^2 \text{ ----> } 13.5 \times 13.5 \text{ inch square}$$

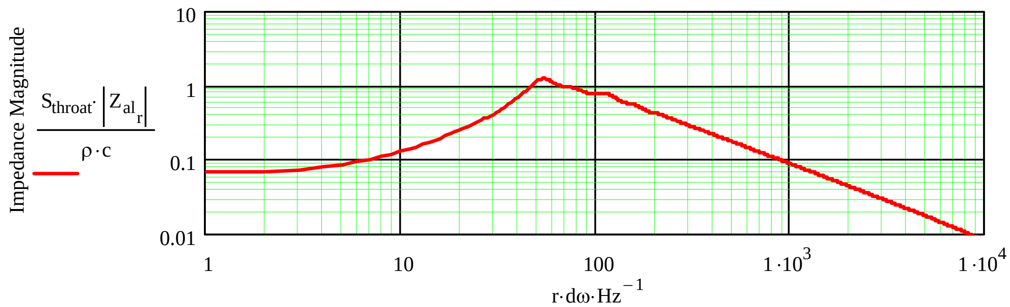
If you compare these mouth dimensions with probably the most famous bass horn design the Klipschorn®, which used corner reinforcement to reduce the required physical mouth area, there appears to be a large disconnect. Most back loaded horn designs being built today are really lightly stuffed transmission lines. This is easily tested by comparing the required mouth area for the claimed lower cut-off frequency, as calculated by Equation (5.3), to the physical mouth area used in the design. While there is nothing wrong with using a resonant transmission line to augment a driver's rolling off bass output, in my opinion calling it a back loaded horn is not completely technically correct.

Another downside to the compromised horn geometries, in the last two rows of Table 7.1, is the loss of efficiency in the low bass output. The resulting system SPL response is now almost flat, ignoring the nulls for a minute, so when these designs are located in a real room the baffle step problem will still need to be addressed. One of the very attractive response features in the slightly less compromised back loaded horn designs, in the first two rows in Table 7.1, is the 3 – 4 dB increase in bass output that will offset the 3 – 4 dB loss due to the baffle step phenomenon. The advantages of a true back loaded horn gradually disappear as the compromises increase and the usual problems associated with lightly damped transmission lines and baffle step response return.

Figure 7.22 a : Back Loaded Exponential Horn Response – Baseline Configuration Acoustic Impedance and Volume Velocity Ratio

Driver Summary	Horn Summary
$f_d = 50.0Hz$	$f_l = 50.0Hz$
$R_e = 8.0ohm$	$f_h = 100.0Hz$
$Q_{td} = 0.2$	$S_{throat} = 1.0S_d$
$V_{ad} = 43.04liter$	$S_{mouth} = 181.61S_d$
$Bl = 12.93tesla \cdot m$	$L_{horn} = 111.47in$

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m³/sec Driver Excitation

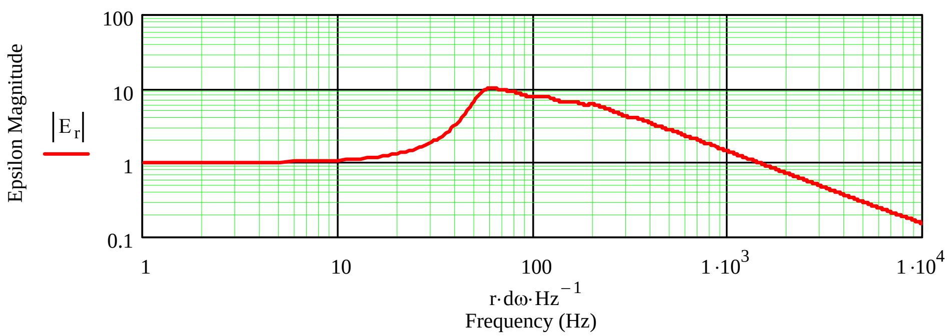
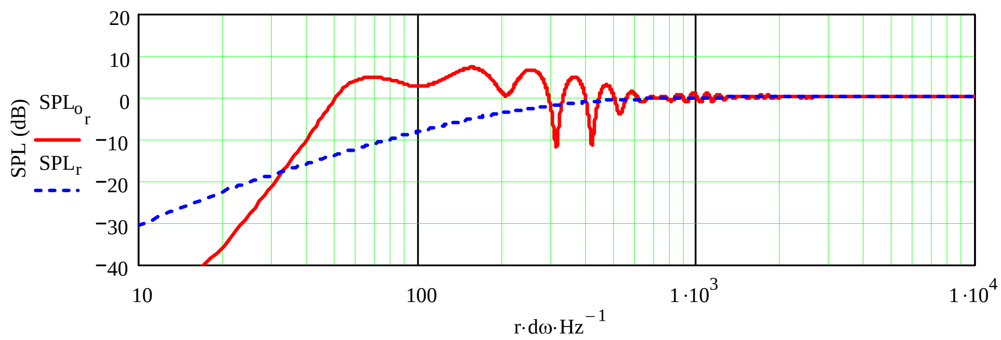
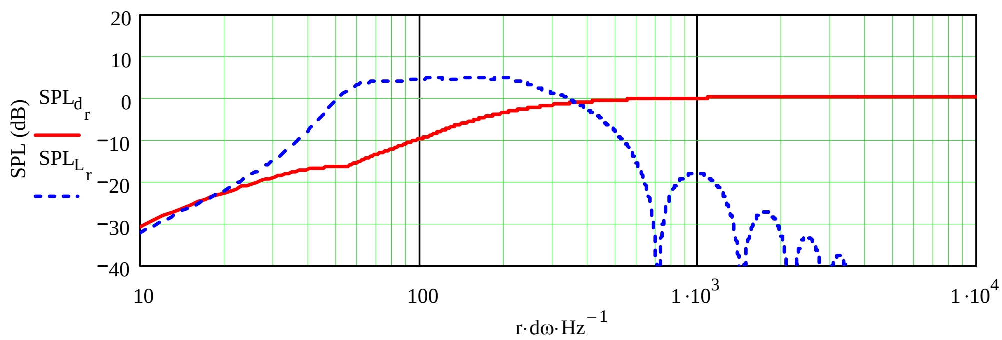


Figure 7.22 b : Back Loaded Exponential Horn Response – Baseline Configuration
Sound Pressure Levels and Electrical Impedance

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

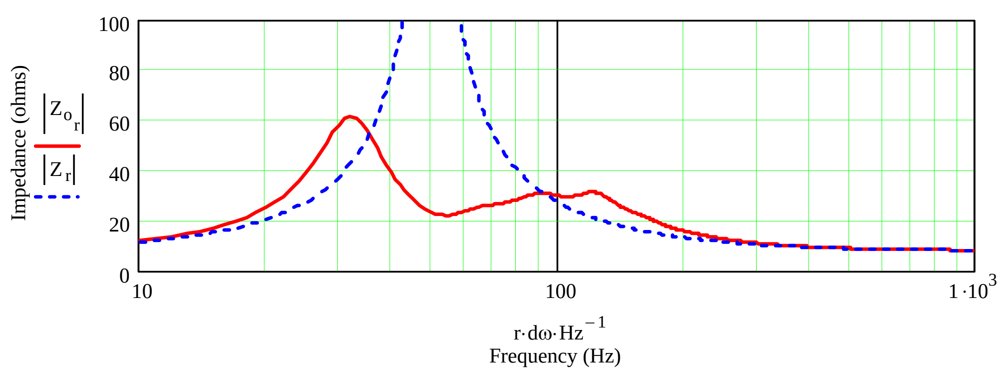
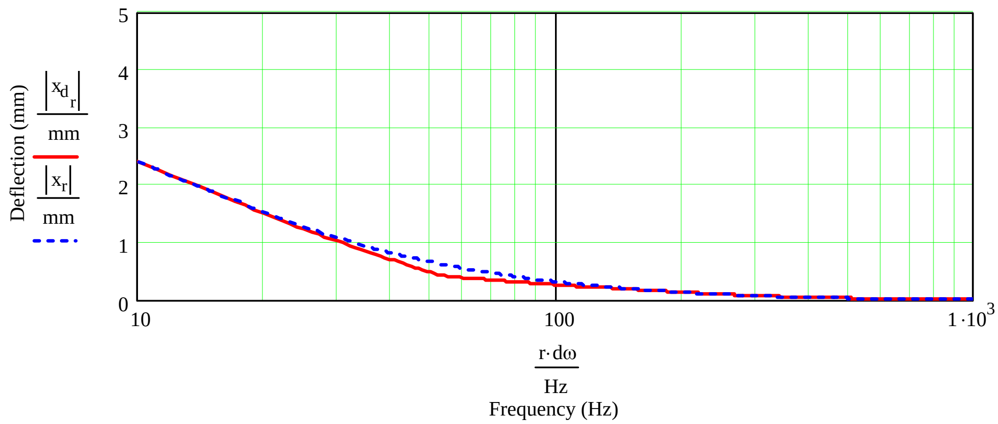


Figure 7.22 c : Back Loaded Exponential Horn Response – Baseline Configuration
Driver Displacement and Impulse Response

Woofer Displacement



System Time Response for an Impulse Input

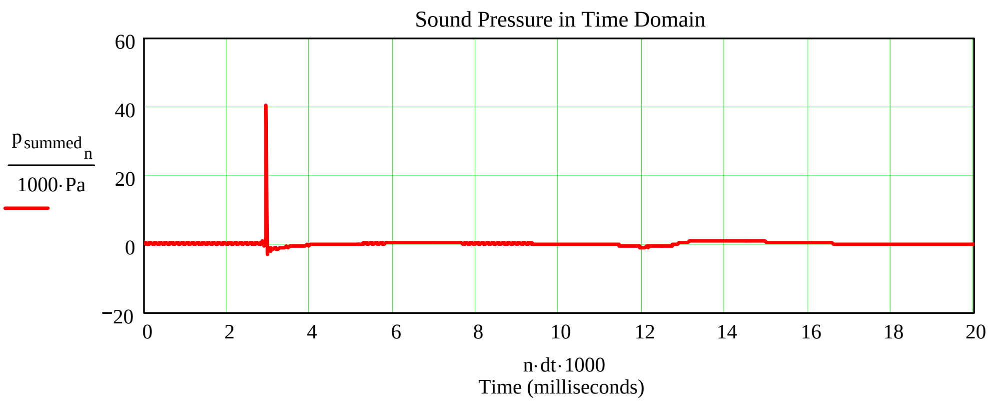
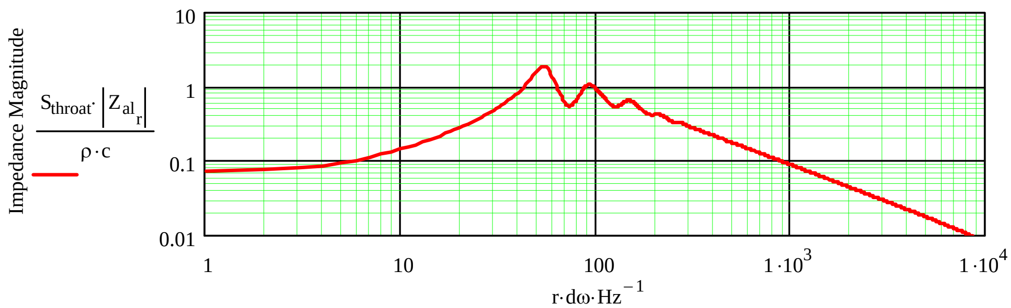


Figure 7.23 a : Back Loaded Exponential Horn Response – Mouth Area / 4
Acoustic Impedance and Volume Velocity Ratio

Driver Summary	Horn Summary
$f_d = 50.0Hz$	$f_l = 50.0Hz$
$R_e = 8.0ohm$	$f_h = 100.0Hz$
$Q_{td} = 0.2$	$S_{throat} = 1.0S_d$
$V_{ad} = 43.04liter$	$S_{mouth} = 45.40S_d$
$Bl = 12.93tesla \cdot m$	$L_{horn} = 91.41in$

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m ^{3}/sec Driver Excitation

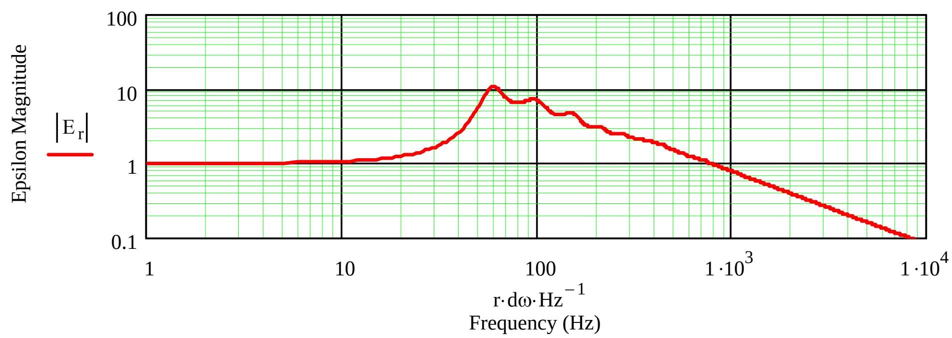
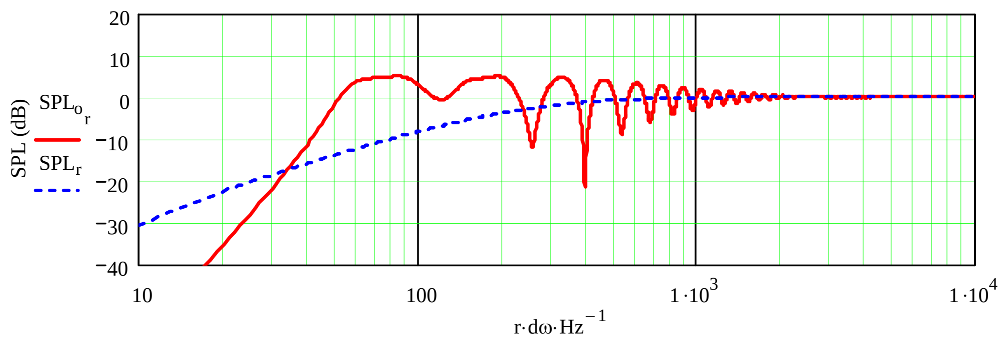
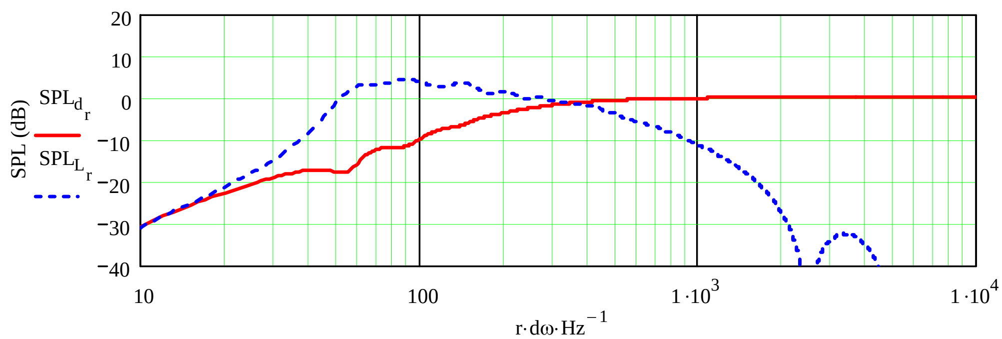


Figure 7.23 b : Back Loaded Exponential Horn Response – Mouth Area / 4
Sound Pressure Levels and Electrical Impedance

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

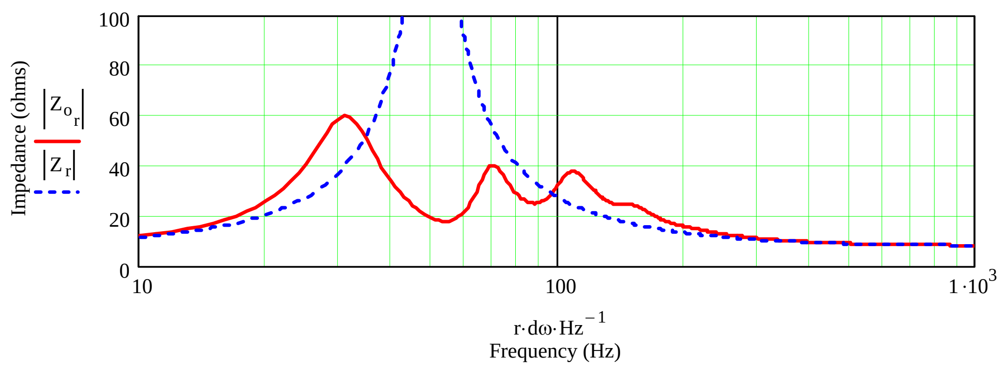
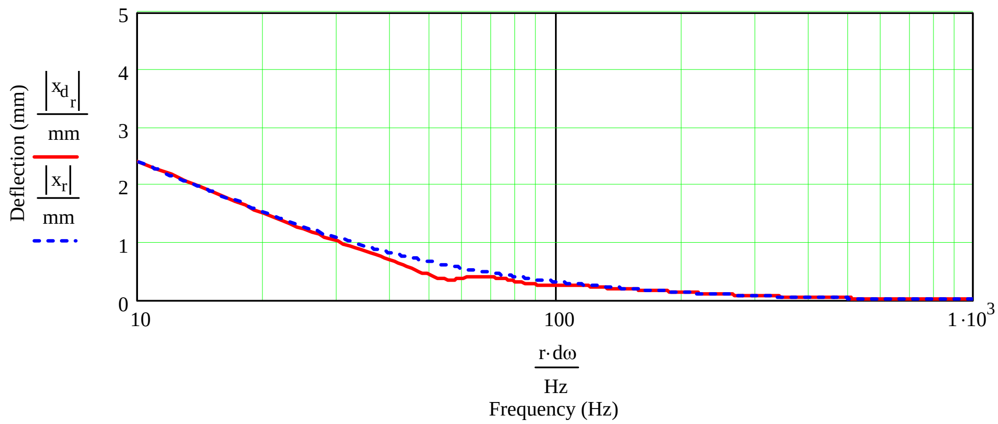


Figure 7.23 c : Back Loaded Exponential Horn Response – Mouth Area / 4 Driver Displacement and Impulse Response

Woofer Displacement



System Time Response for an Impulse Input

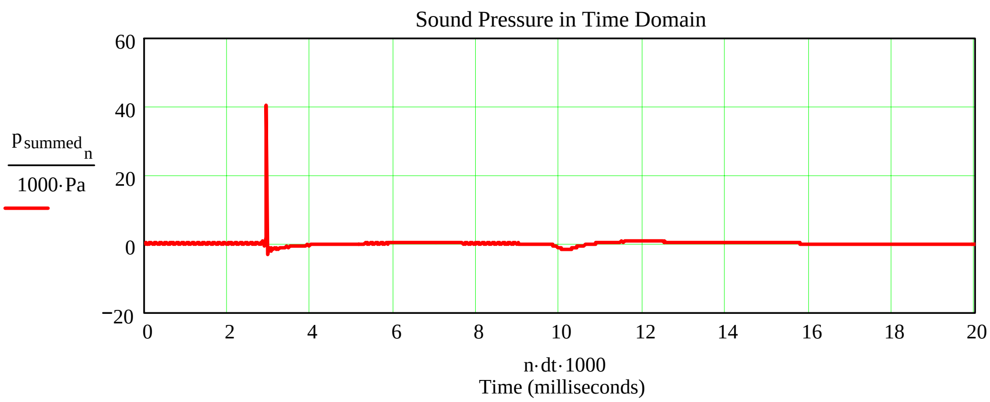
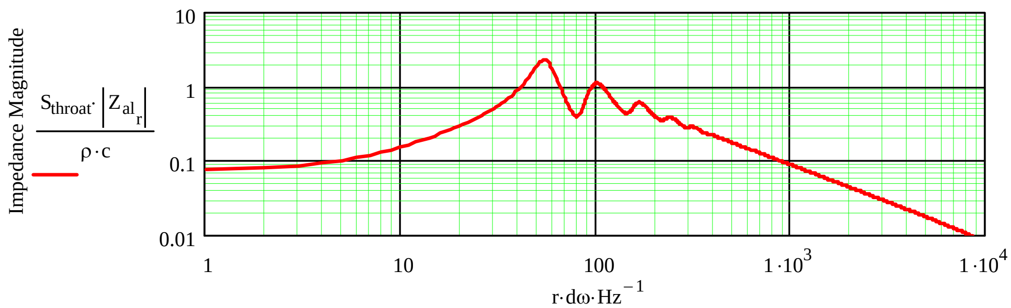


Figure 7.24 a : Back Loaded Exponential Horn Response – Mouth Area / 8
Acoustic Impedance and Volume Velocity Ratio

Driver Summary	**Horn Summary**
$f_d = 50.0\text{Hz}$	$f_l = 50.0\text{Hz}$
$R_e = 8.0\text{ohm}$	$f_h = 100.0\text{Hz}$
$Q_{td} = 0.2$	$S_{throat} = 1.0S_d$
$V_{ad} = 43.04\text{liter}$	$S_{mouth} = 22.70S_d$
$Bl = 12.93\text{tesla}\cdot\text{m}$	$L_{horn} = 81.38\text{in}$

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m ^{3}/sec Driver Excitation

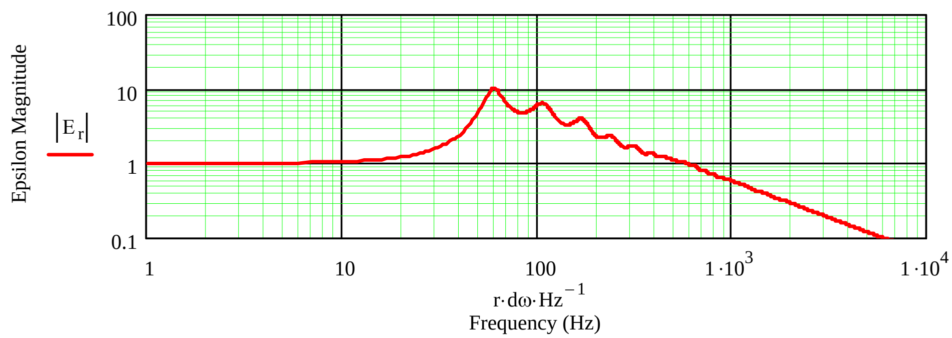
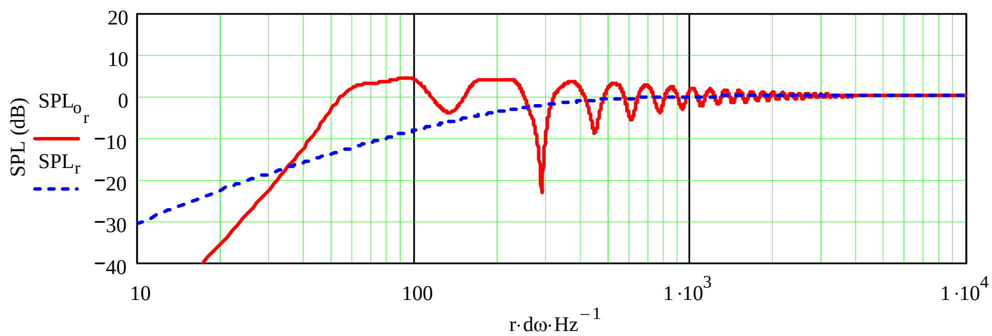
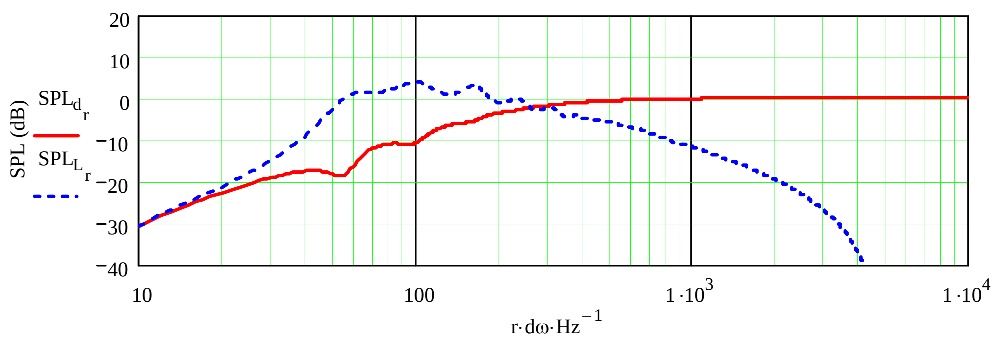


Figure 7.24 b : Back Loaded Exponential Horn Response – Mouth Area / 8
Sound Pressure Levels and Electrical Impedance

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

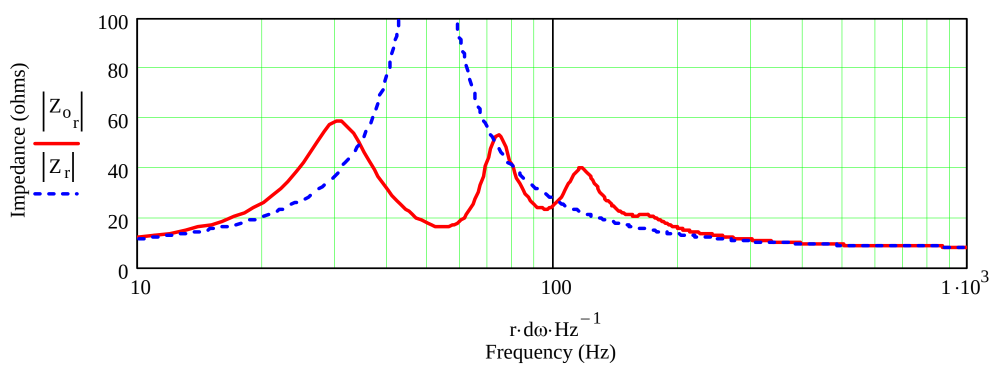
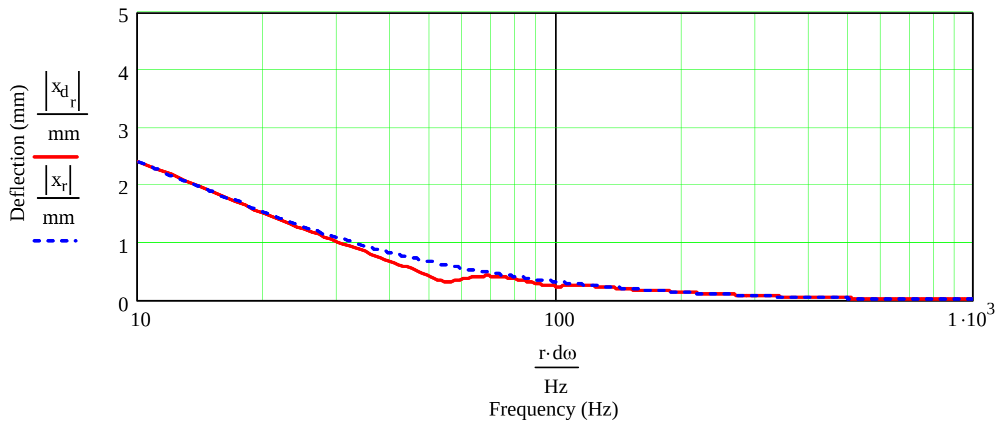


Figure 7.24 c : Back Loaded Exponential Horn Response – Mouth Area / 8 Driver Displacement and Impulse Response

Woofer Displacement



System Time Response for an Impulse Input

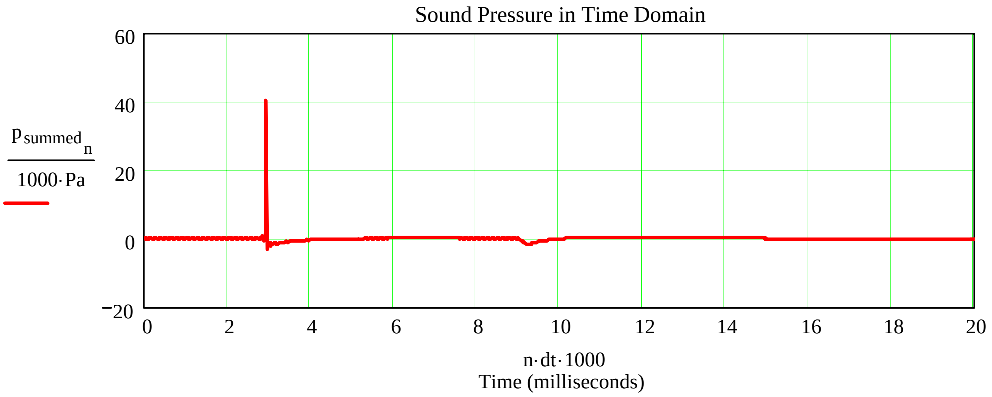
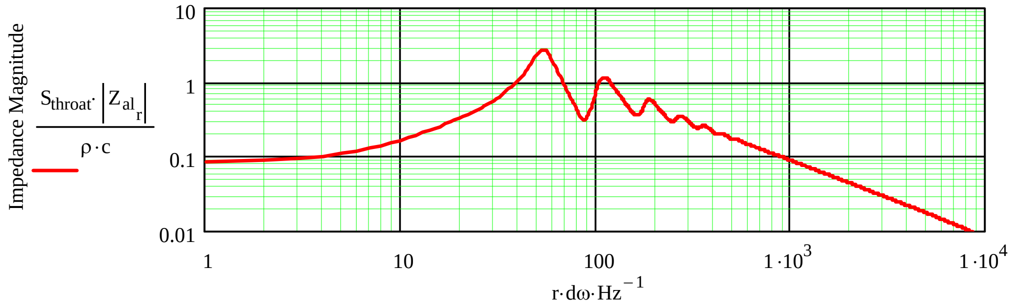


Figure 7.25 a : Back Loaded Exponential Horn Response – Mouth Area / 16
Acoustic Impedance and Volume Velocity Ratio

Driver Summary	Horn Summary
$f_d = 50.0Hz$	$f_l = 50.0Hz$
$R_e = 8.0ohm$	$f_h = 100.0Hz$
$Q_{td} = 0.2$	$S_{throat} = 1.0S_d$
$V_{ad} = 43.04liter$	$S_{mouth} = 11.35S_d$
$Bl = 12.93tesla \cdot m$	$L_{horn} = 72.46in$

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m³/sec Driver Excitation

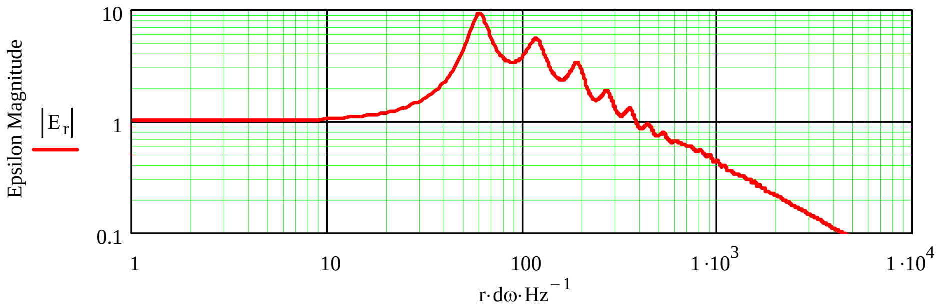
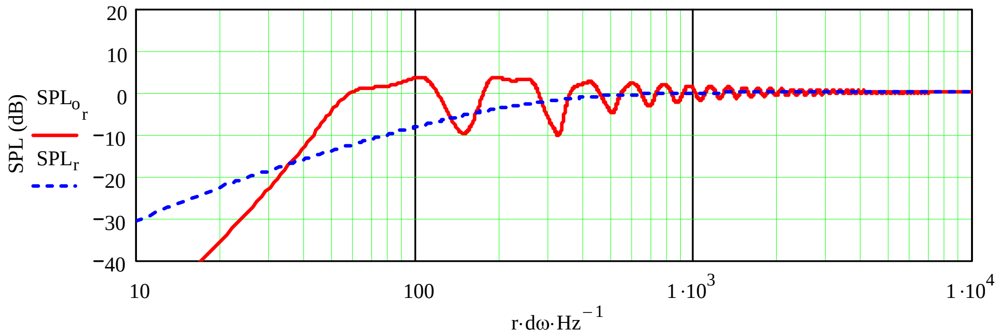
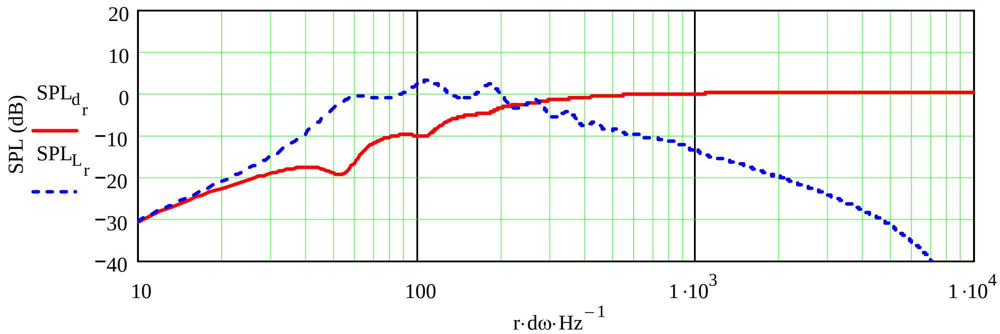


Figure 7.25 b : Back Loaded Exponential Horn Response – Mouth Area / 16
Sound Pressure Levels and Electrical Impedance

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

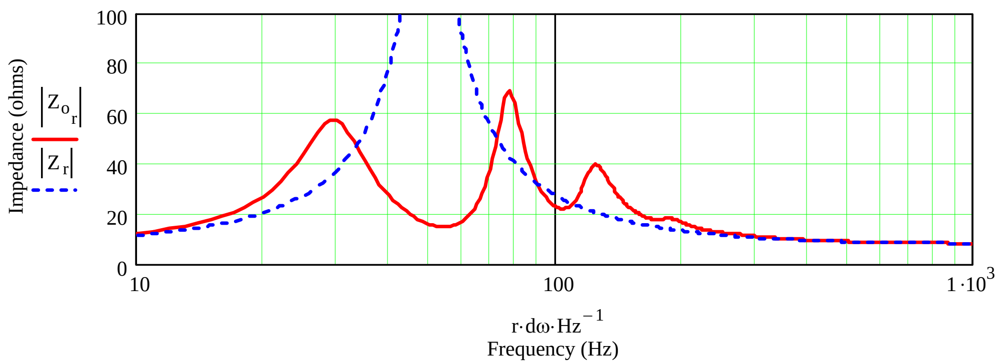
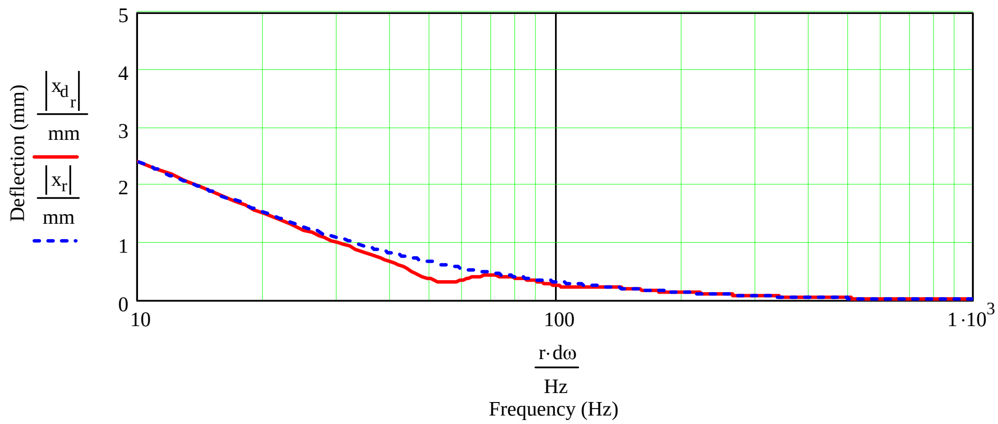


Figure 7.25 c : Back Loaded Exponential Horn Response – Mouth Area / 16 Driver Displacement and Impulse Response

Woofer Displacement



System Time Response for an Impulse Input

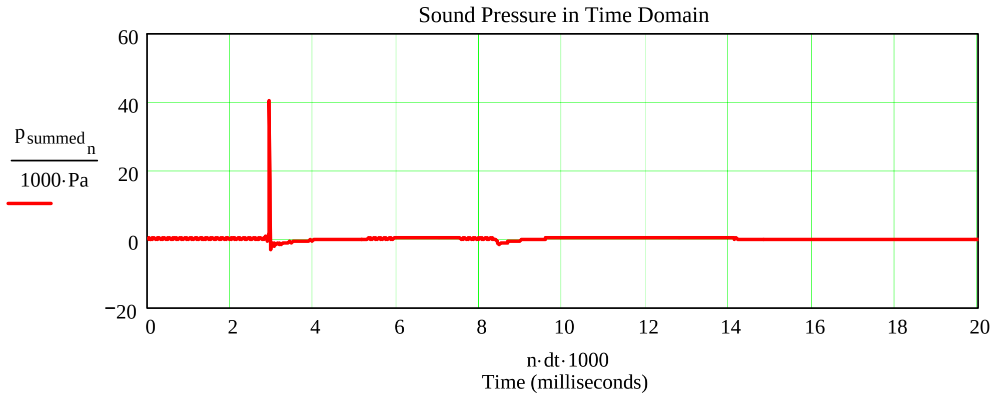
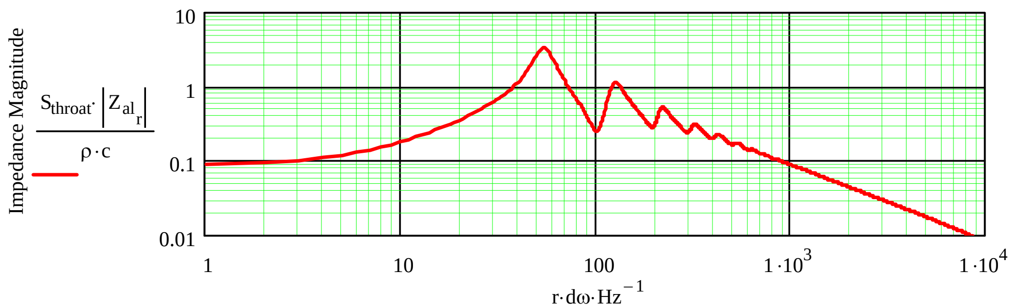


Figure 7.26 a : Back Loaded Exponential Horn Response – Mouth Area / 32
Acoustic Impedance and Volume Velocity Ratio

Driver Summary	Horn Summary
$f_d = 50.0Hz$	$f_l = 50.0Hz$
$R_e = 8.0ohm$	$f_h = 100.0Hz$
$Q_{td} = 0.2$	$S_{throat} = 1.0S_d$
$V_{ad} = 43.04liter$	$S_{mouth} = 5.68S_d$
$Bl = 12.93tesla \cdot m$	$L_{horn} = 61.31in$

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m^3/sec Driver Excitation

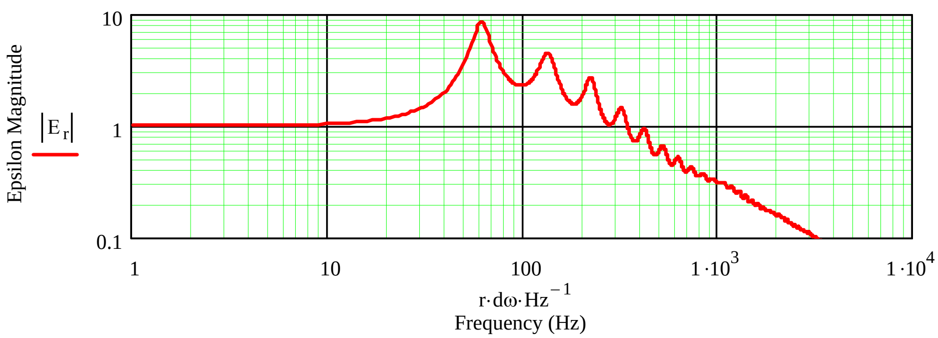
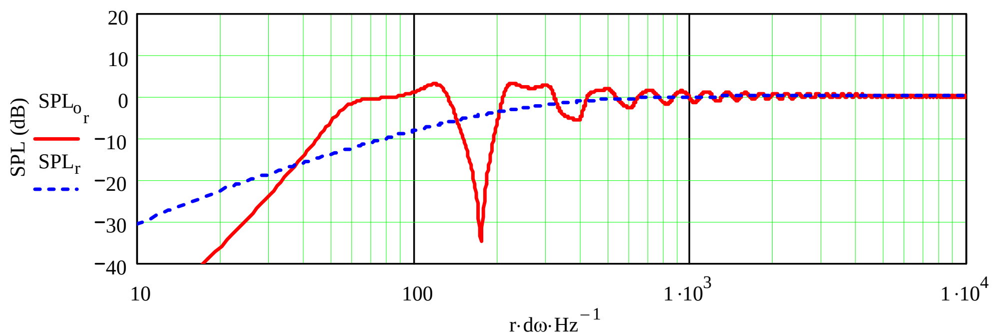
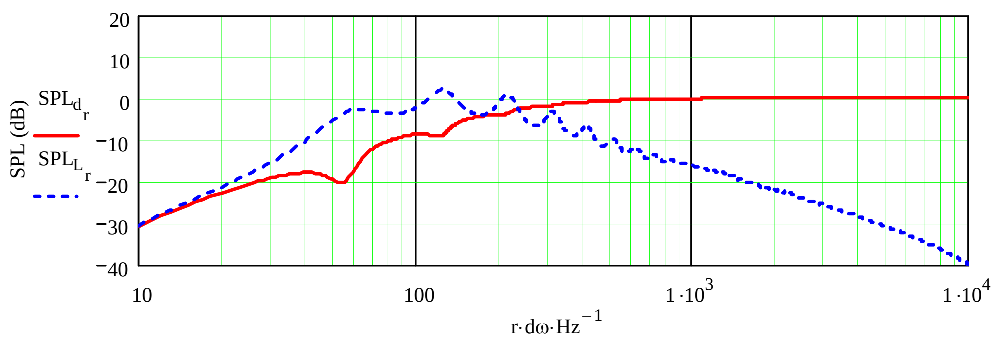


Figure 7.26 b : Back Loaded Exponential Horn Response – Mouth Area / 32
Sound Pressure Levels and Electrical Impedance

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

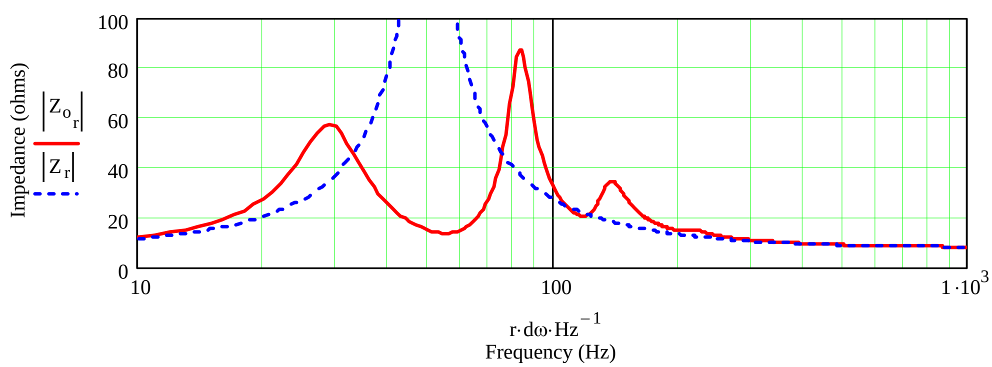
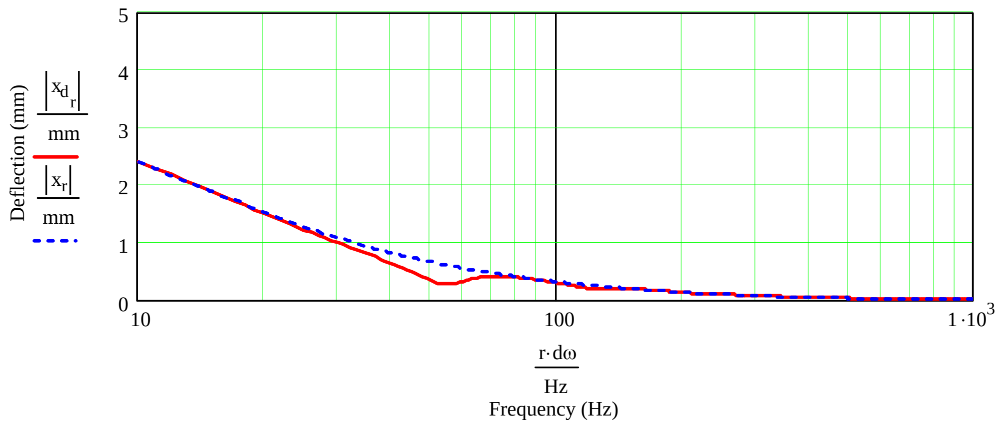
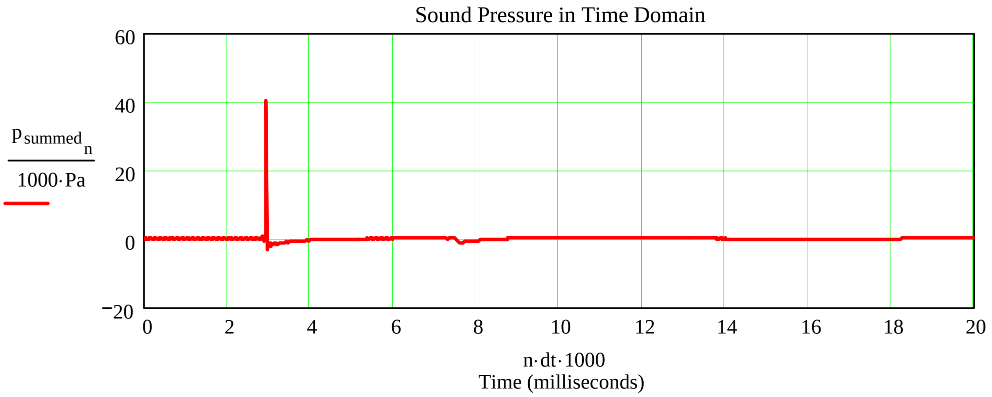


Figure 7.26 c : Back Loaded Exponential Horn Response – Mouth Area / 32
Driver Displacement and Impulse Response

Woofer Displacement



System Time Response for an Impulse Input



Changing the Driver's Q_{td} Parameter :

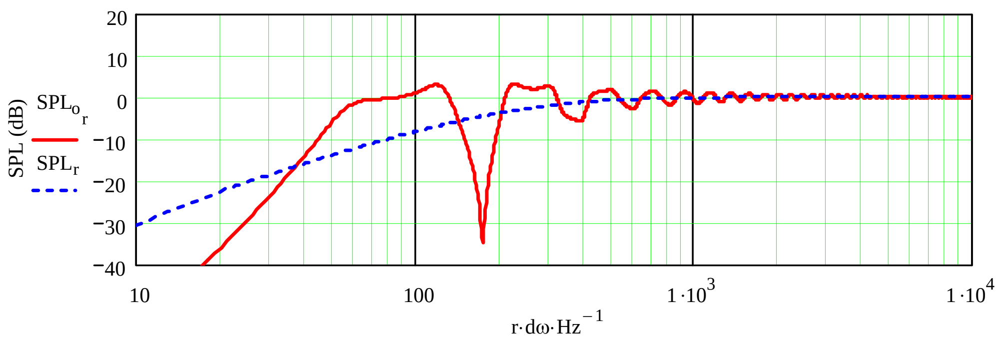
Traditionally, drivers with Q_{td} values between 0.15 and 0.25 are the ones recommended for back loaded horns. These drivers have a very large and powerful magnet, resulting in a high Bl parameter, and are extremely efficient with SPL's approaching 98 – 100 dB at 1 m for 1 watt input. It is typical to see strong comments on the Internet forums stating that a particular driver was designed strictly for horn loading when a question is raised about the potential for using a bass reflex or transmission line enclosure design. It is stated as a fact that cannot be questioned. This is a common theme when discussing some of the Lowther (PM2A, DX4, or EX4 models) or Fostex (FE-206E or FE-208 Sigma models) drivers.

The compromised horn geometry associated with the last row of Table 7.1 and Figure 7.26 was selected to simulate different driver Q_{td} values. Figures 7.27, 7.28, 7.29, 7.30, and 7.31 present an abbreviated set of simulation results corresponding to Q_{td} values of 0.2, 0.4, 0.6, 0.8, and 1.0 respectively. As the driver's Q_{td} value increases, the low frequency driver output increases as does the horn's output. The combined system SPL result increases with higher Q_{td} values restoring the low frequency 3 – 4 dB hump that could be used to offset the 3 – 4 dB dip associated with the baffle step response. While in general higher Q_{td} drivers are not looked upon as being particularly high fidelity, the use of a higher Q_{td} driver in a severely compromised back loaded horn may have some real benefits.

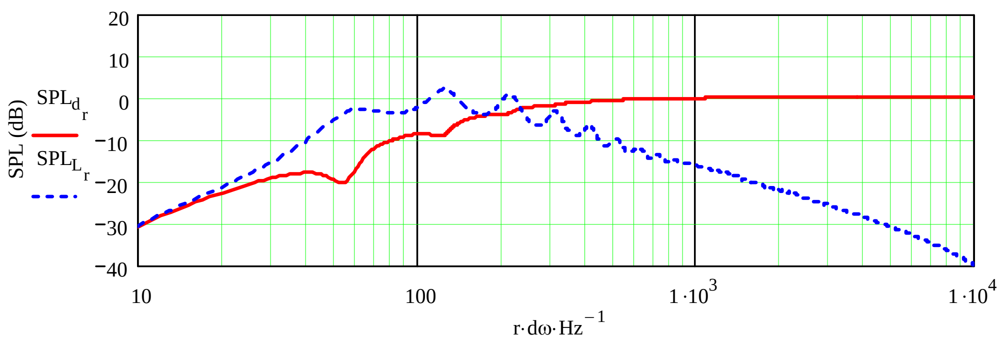
If the back loaded horn was a consistent design, then the resistive impedance would have produced an elevated response proportional to the driver's response as was seen in the front loaded horn discussion of this topic. The resistive impedance of the horn acts to damp the high Q_{td} driver's ringing response producing a very nice SPL frequency response and impulse response in the time domain.

Figure 7.27 : Back Loaded Exponential Horn Response – Mouth Area / 32
Sound Pressure Levels and Electrical Impedance for Q_{ts} = 0.2

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

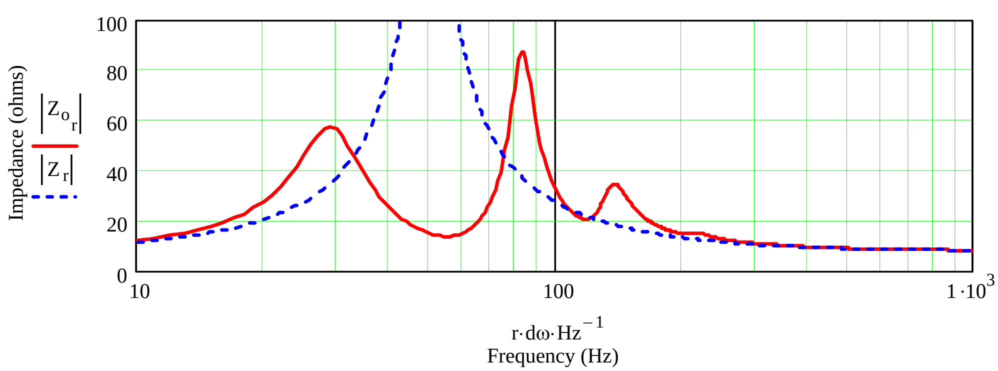
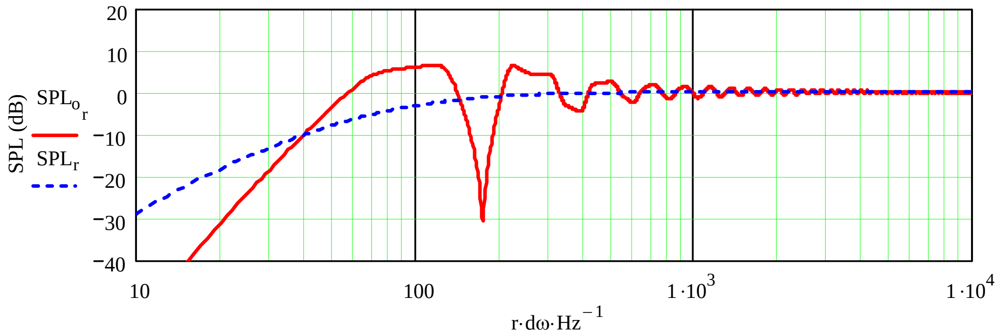
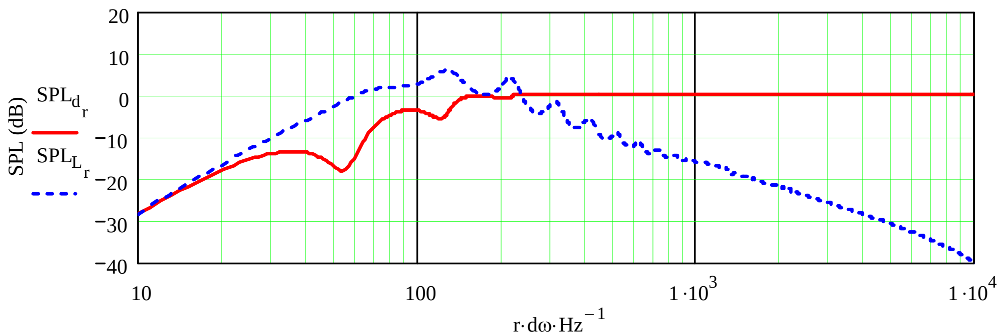


Figure 7.28 : Back Loaded Exponential Horn Response – Mouth Area / 32
Sound Pressure Levels and Electrical Impedance for Q_{ts} = 0.4

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

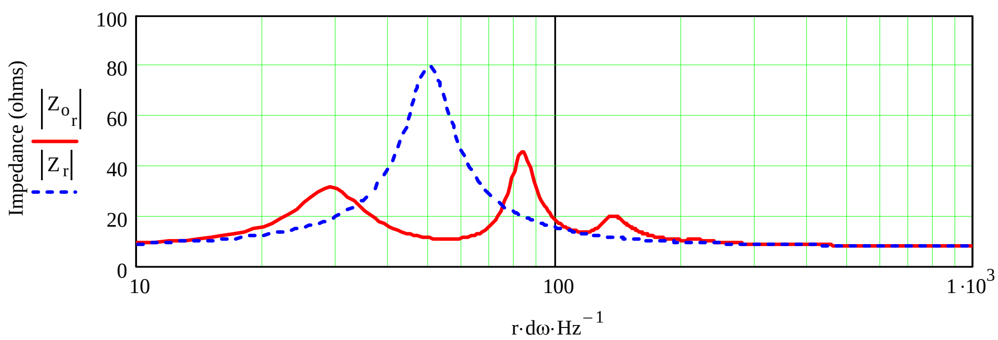
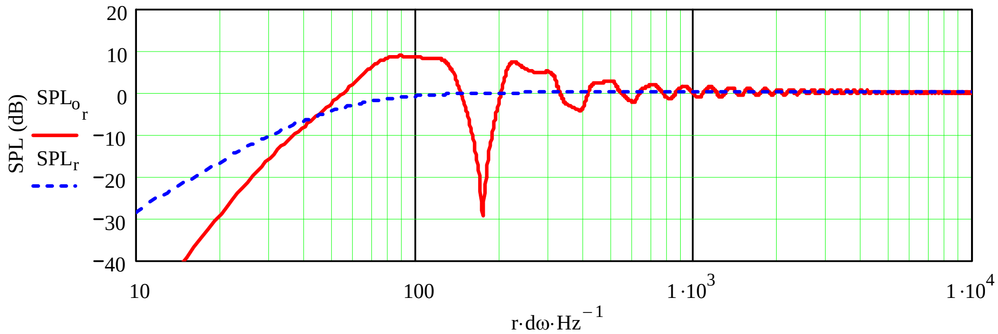
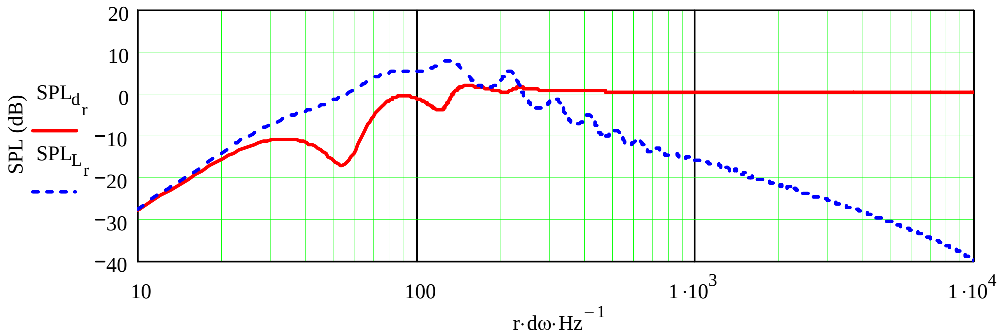


Figure 7.29 : Back Loaded Exponential Horn Response – Mouth Area / 32
Sound Pressure Levels and Electrical Impedance for Q_{ts} = 0.6

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

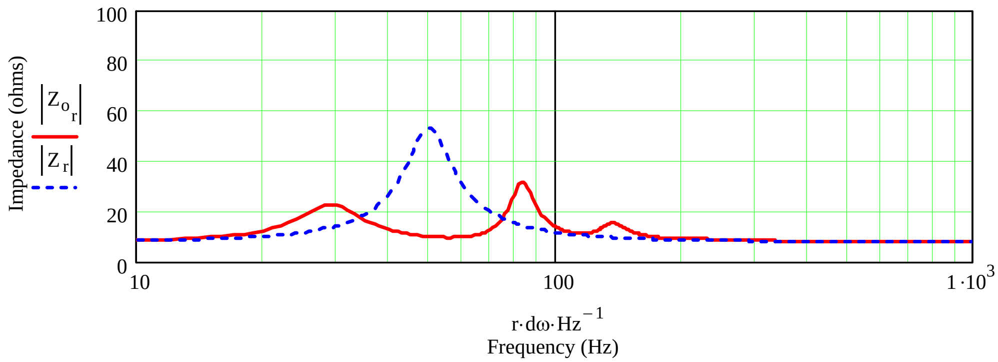
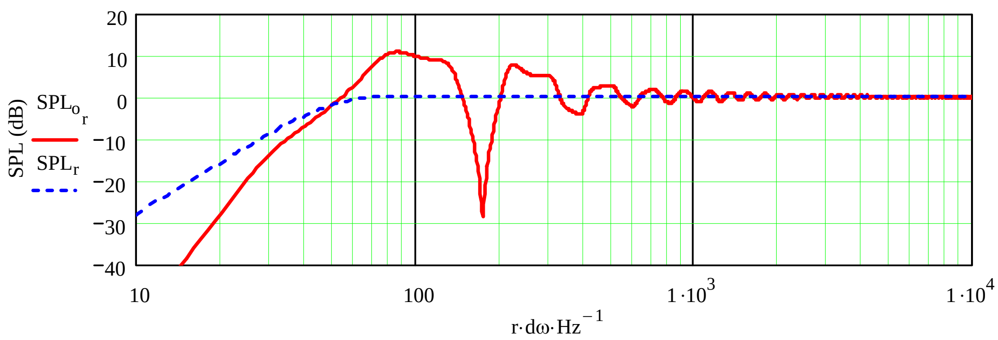
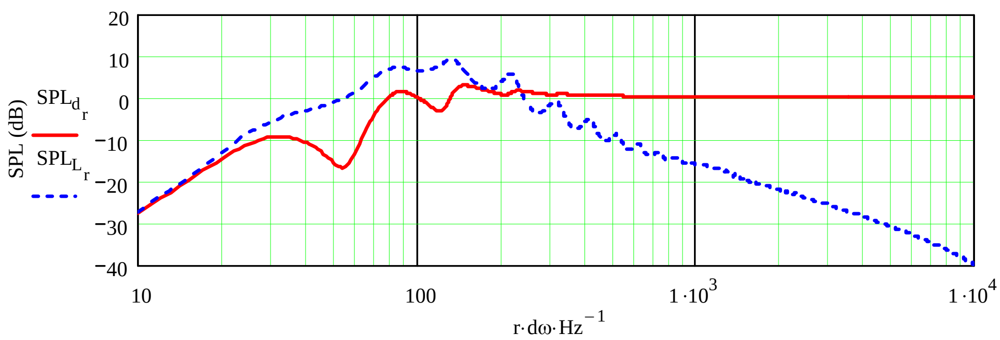


Figure 7.30 : Back Loaded Exponential Horn Response – Mouth Area / 32
Sound Pressure Levels and Electrical Impedance for Q_{ts} = 0.8

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance

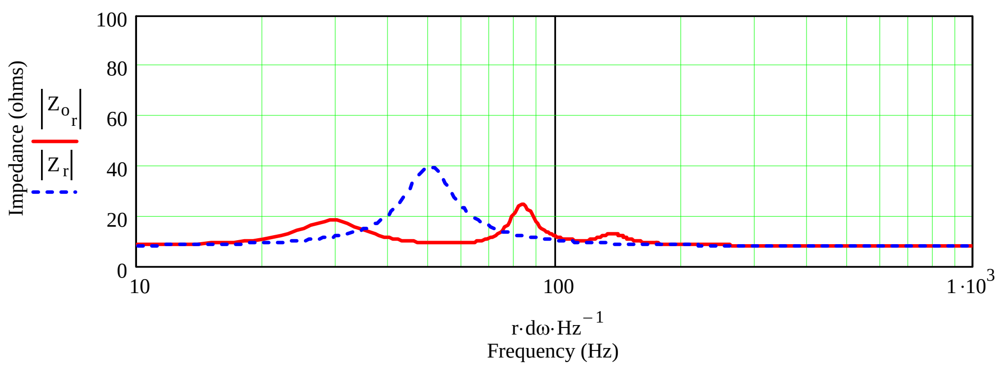
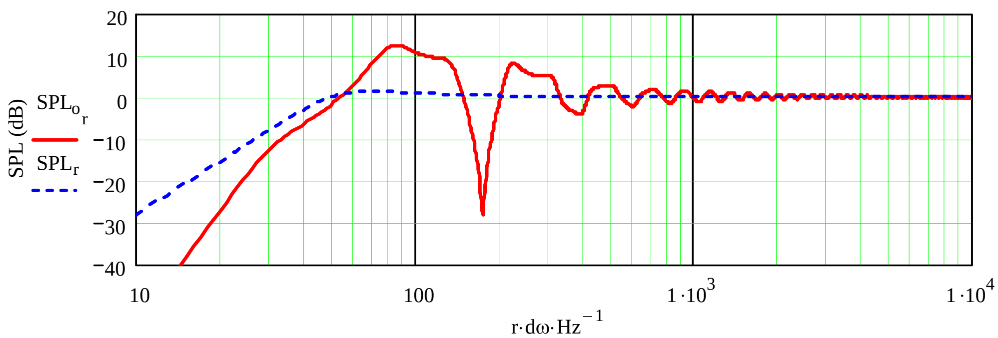
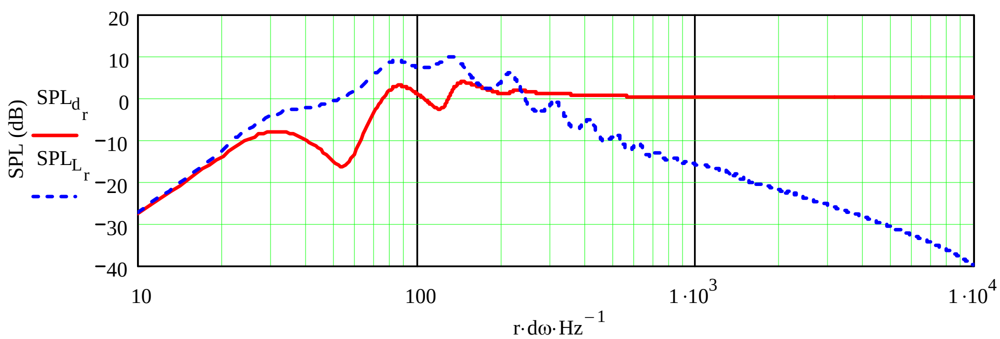


Figure 7.31 : Back Loaded Exponential Horn Response – Mouth Area / 32
Sound Pressure Levels and Electrical Impedance for Q_{ts} = 1.0

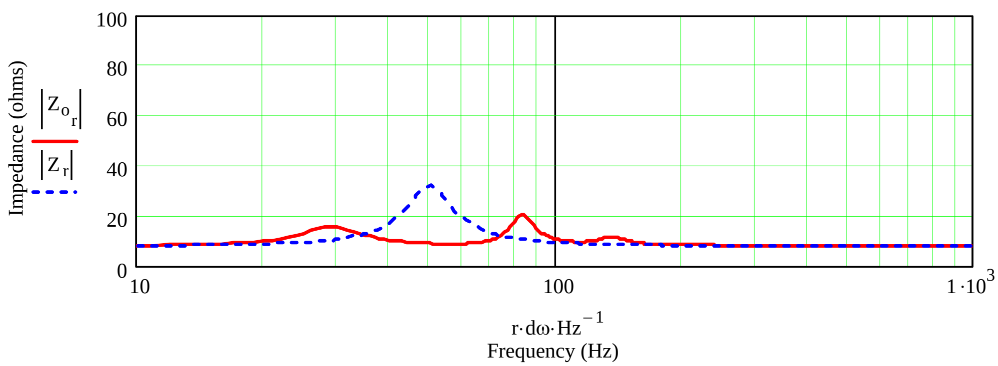
Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Back Loaded Horn System and Infinite Baffle Impedance



Summary :

Rather then wade through many pages of mathematical derivations, this section again used some simple sample problems to gain insight into how to design a back loaded exponential horn system. One key result discussed was the phase of the consistently sized horn's output relative to the output from the front of the driver. Since the horn presents resistive impedance to the back of the driver above the lower cut-off frequency, the mouth's output will be in phase with the back of the driver cone after accounting for the distance traveled getting to the mouth. A dipole situation results with the length of the horn corresponding to the distance between the two sources. This dipole leads to a set of peaks and dips corresponding to frequencies where the two signals arrive at the listening position exactly in phase or 180 degrees out of phase respectively.

A second key result showed the importance of the coupling chamber in rolling off the horn's response at the higher cut-off frequency f_h so that the back loaded horn provides bass boost to the drivers rolling off low frequency response. This bass boost is significant enough to avoid the need for an electrical filter or EQ to correct for the baffle step response loss at low frequencies. However, one of the design challenges for the coupling chamber is to control, or eliminate, the half wavelength standing waves between the closed end of the chamber and the horn's throat.

A simulation program can be extremely useful for iterating the geometry parameters to get an optimized result. Sizing the back loaded horn and the corresponding coupling chamber to work well together is not an easy task. Due to the large mouth size for a low frequency horn, a compromised design is probably the only acceptable choice and careful analysis is required to manage the trade-offs between transmission line and horn behavior.

When sizing the back loaded horn geometry, it is interesting to note that the driver's size and Thiele / Small parameter do not enter into the calculations. The lower cut-off frequency f_c determines the mouth cross-sectional area. The throat area and horn length are determined by the desired SPL boost above the lower cut-off frequency f_c. So given two similar drivers, that vary only in diameter, the same size horn mouth would result for the same low cut-off frequency f_c. The same exponential back horn could be used for an eight inch and a six inch diameter driver.

The next step in this study is to extend the simple MathCad modeling of back loaded exponential horns in an attempt to decrease the required enclosure size and produce more accurately calculated SPL system responses.