A Method for Designing a Compact Back Loaded Horn Loudspeaker System

Martin J. King
40 Dorsman Dr.
Clifton Park, NY 12065
MJKing57@aol.com

Introduction :

For the past 15+ years, I have been studying back loaded horn enclosure designs for full range drivers. This involved reverse engineering almost every back loaded horn design I have been able to find on the Internet. If a cross-section plot or picture is provided to scale internal dimensions, or even better detailed drawings are provided for other DIY builders to follow, then I have probably run a simulation to see how the design performs. Of particular interest were fully documented designs that also included measured electrical impedance and SPL frequency response plots to correlate against my MathCad worksheet predictions. When actual test data was provided the simulations usually correlated well and I have learned a lot of modeling techniques through these exercises.

In parallel with reverse engineering other people's designs, I have put a lot of effort into my own design studies with mixed results. It is not too hard to produce a paper design with a calculated response that looks really bad and not worth building. Or if the calculated response looks promising the size of the resulting enclosure is huge, again not worth building. The number of designs analyzed and stored on my hard drive has to be over one hundred.

After all these years of computer work, I am somewhat embarrassed to admit I have yet to build a back loaded horn of any kind. I have not found a design that stands out analytically as a great performer in a reasonably sized enclosure worth the effort required to build it. This is clearly bordering on a textbook case of analysis paralysis. The biggest problem is achieving adequate bass response in a reasonably sized and simple enclosure design.

About three years ago, I started to overcome some of the problems I had been struggling with for all these years. This was precipitated by newer versions of my MathCad back loaded horn worksheets along with a fresh perspective on how a back loaded horn really works at low frequencies in a typical listening room. The first design generated from this fresh look at the problem was built by a couple of different DIYers and the feedback was that the bass was too much. A relatively compact and simple enclosure design that produced too much bass, this was a problem that could be solved with a slight redesign. The intent of this document is to lay out the rational for a back loaded horn design process and to propose a speaker enclosure that is relatively small, simple to build, and works well with several different Fostex full range drivers.

Definition of a Horn :

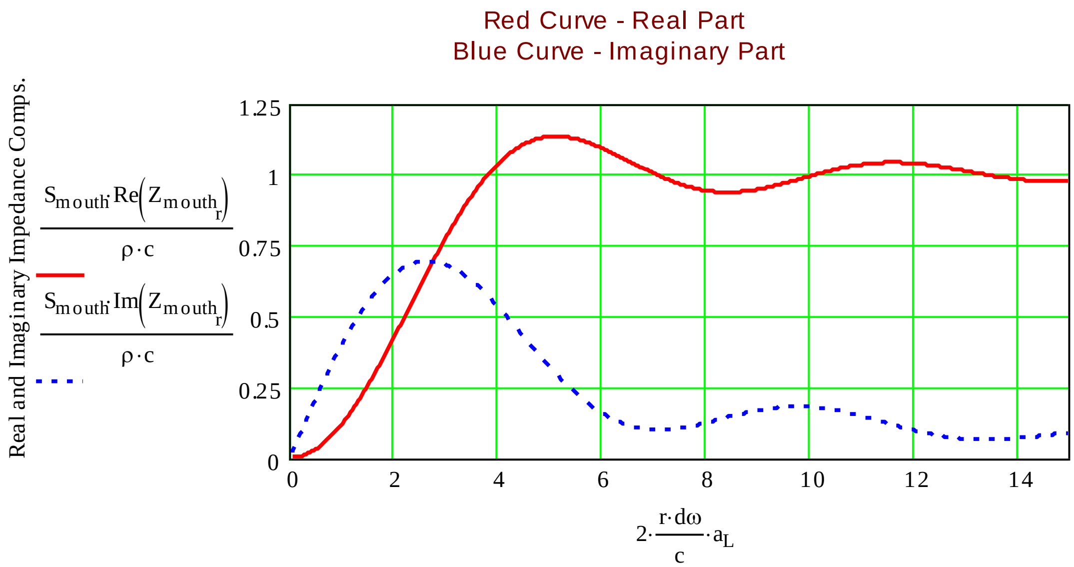
The first step is to define what a horn is in terms of its acoustic and physical properties. Consulting Wikipedia[(1)], the following definition is provided.

"A horn is a tapered sound guide designed to provide an acoustic impedance match between a sound source and free air. This has the effect of maximizing the efficiency with which sound waves from the particular source are transferred to the air. Conversely, a horn can be used at the receiving end to optimize the transfer of sound from the air to a receiver."

The easy part of this definition is the expanding geometry, the physical property. Everybody can visualize this type of geometry and in many people's minds this is the sole property that defines a back loaded horn enclosure. But the tougher part of the definition is the acoustic property, an impedance match between the source and the air in the room, which is not so easy to visualize. The second part of the definition is determined by the effective size of the horn's mouth, the resulting acoustic impedance, and the frequency range being reproduced.

Figure 1 shows the real (red curve) and imaginary (blue dashed curve) parts of the assumed acoustic impedance used in most derivations of the equations of motion for a horn. This acoustic impedance is applied as a boundary condition at the horn's mouth. These simplified curves are derived assuming a rigid circular piston vibrating in an infinite baffle. It turns out that for a square piston in an infinite baffle, with the same area, the calculated acoustic impedance as a function of frequency is very similar. More accurately calculated acoustic impedance curves, that include the specific mouth shape (rectangular aspect ratio in particular) and finite baffle edge sources, are used in newer MathCad worksheets but the curve in Figure 1 should be familiar and adequately demonstrates the significant points.

Figure 1 : Square or Circular Horn Mouth Acoustic Impedance



At low values of $2 \cdot k \cdot a_L$, where k is equal to ω/c ($r \cdot d\omega/c$ in the plot above) and a_L is the piston radius, the acoustic impedance is mostly imaginary due to the mass loading produced by the air directly in front of the mouth. Low frequency sound waves traveling along the expanding geometry are reflected back generating standing waves at discrete frequencies determined by the path length and expansion rate. Standing waves are a property of transmission line enclosures and not of horn enclosures.

As $2 \cdot k \cdot a_L$ increases, the real part of the impedance also increases leading to energy transfer from the mouth into the room. The real part of the acoustic impedance acts as a damper for any standing waves dissipating energy into the room that otherwise would have been reflected back into the enclosure. This efficient resistive coupling of the mouth to the room is the second part of what defines a horn.

The frequency at which sound waves start to be efficiently transmitted into a room is usually defined at $2 \cdot k \cdot a_L = 2$ as shown in Figure 1. At this frequency the real part of the acoustic impedance is approaching 0.5 so the SPL generated by the real part of the acoustic impedance is 6 dB below the broad band SPL output.

$$S_{mouth} \times \mathrm{Re}(Z_{mouth}) / (\rho \times c) \sim 0.5$$

$$20 \times \log(0.5) = -6 \text{ dB}$$

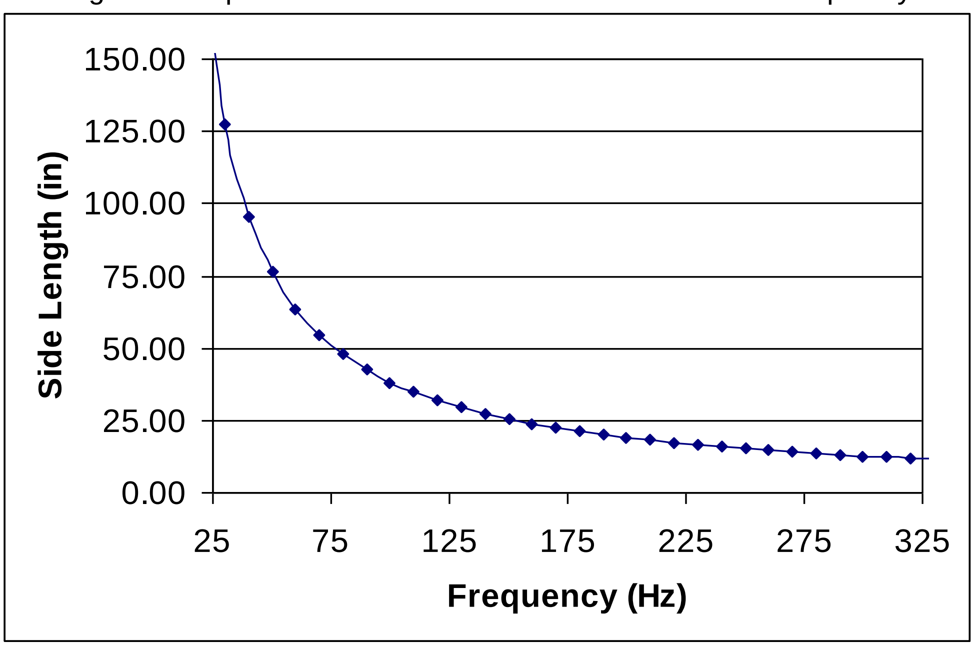
Returning to the "Horn Physics" article[(2)], an expression for the mouth area at $2 \cdot k \cdot a_L = 2$ was derived and presented as Equation 5.3. Assuming a square cross-section, the length of the sides ($S_L = b \times b$) can now be calculated as a function of frequency.

$$S_L = \frac{\left(\frac{c}{2 f_c}\right)^2}{\pi}$$

$$b = \frac{c}{2 f_c \sqrt{\pi}}$$

Using the equation above, a curve can be drawn as a function of frequency that defines the effective square mouth size required for a back loaded horn enclosure to act like a true horn.

Figure 2 : Square Mouth Size as a Function of Cut-Off Frequency



The curve in Figure 2 demonstrates that for a back loaded horn with a square mouth to reach down to 100 Hz, the effective mouth's sides must be over 30 inches long. To achieve 50 Hz the effective mouth must be approximately 75 inches square.

Clearly for a back loaded horn enclosure to behave as a true horn all the way down to the lowest bass frequencies the effective mouth size must be huge.

What this means for the modest sized back loaded horn enclosures found on the Internet is that they are dominated by transmission line behavior at low frequencies and really do not start to act like horns until they reach a frequency of a few hundred hertz. Trying to use equations derived from classic horn theory to set flare rates, mouth areas, coupling volumes, or throat areas for a compact back loaded horn operating at low frequencies is really misdirected. If things work well acoustically it is by luck and not by proper application of horn physics.

Recognizing the transmission line behavior at low frequencies, and designing with this in mind, the behavior of back loaded horn enclosures should be easier to understand and optimize. Then at higher frequencies, horn physics will take over and the design methodology should shift recognizing this change in acoustic behavior. At mid range frequencies the acoustic impedance at the throat will start to become resistive and interact with the coupling volume to roll off the horn's output. Even though the TL behavior dominates at low frequencies, I will still refer to this type of enclosure as a back loaded horn because horn physics comes into the mix at mid range frequencies to control and attenuate the mouth's SPL output.

Influence of Room Boundaries :

Room boundaries will have two effects on the performance of a back loaded horn enclosure at low frequencies. First, sound waves emanating from the mouth reflecting off nearby boundaries return and provide additional pressure loading of the mouth changing the acoustic impedance. The effective mouth area will appear to be larger than the physical mouth area. Second, every room boundary cuts the volume that the horn mouth is radiating into leading to an increased sound pressure level at the listening position. This second effect is equivalent to changing the "ang" input variable in the HornResp freeware program(3). These two results are important considerations during the design of compact back loaded horn enclosures.

Looking a little closer at the change in acoustic impedance at the back loaded horn's mouth, nearby room boundaries create reflections of the outgoing sound waves. These reflections should be thought of as emanating from phantom horn mouths on the other side of each room boundary. Each room boundary can be visualized as a mirror producing additional phantom horns equidistant behind or below the wall or floor respectively. The pressure seen at the mouth is the sum of the mouth's self pressure loading and the pressure loading arriving from each of the nearby phantom mouths. The closer the horn's mouth is to the boundary, the more significant the influence of these reflections on the acoustic impedance. The net result will be a lowering of the frequency at which the real part of the acoustic impedance passes through 0.5 in Figure 1; the effective mouth will appear to be acoustically bigger than its physical dimensions.

These phantom horns, reflected on the other side of room boundaries, can also be visualized as additional sound sources that are summed at the listening position. At low frequencies where the wavelengths of sound are long, the SPL from the phantom sources will add constructively to the direct SPL from the actual speaker. However, as the frequency rises and the wavelength of sound decreases, the SPL from the phantom sources will start to arrive out of phase with the SPL from the actual speaker producing severe dips. The best designs will take advantage of the sound reinforcement at low frequencies and roll off the mouth's SPL output before the destructive interference starts to occur at higher frequencies.

Looking back at my previous documents addressing back loaded horn designs, the original goal was for the horn geometry to be so efficient that when its low frequency SPL output was combined with the driver's SPL output the need for any form of baffle step compensation would be eliminated. In a no compromise system(4) (5) where size is not an issue, this approach is probably still valid. Unfortunately as nice as this concept sounds, I no longer believe that this is possible for a reasonably sized back loaded horn enclosure design.

If at this point you buy into the transmission line analogy at lower frequencies, then placing the back loaded horn speaker out in the room will lead to the same problem of baffle step losses at low frequencies associated with sealed or ported enclosure designs. The expanding transmission line geometry will help produce a little more bass SPL output compared to a straight or tapered transmission line. But the key concept is using the room boundaries to both acoustically load the back loaded horn's mouth and generating low frequency reflections that arrive at the listening position in phase reinforcing the bass region of the SPL frequency response.

Based on the discussion in the last few paragraphs, it should not be a big surprise that a back loaded horn design with the mouth on the rear baffle will probably be preferred over a mouth located on the front baffle. A mouth on the rear baffle of a back loaded horn pushed into a room corner will maximize the mouth's acoustic impedance boundary condition and provide an extended frequency range for the constructive reinforcement. The low frequency performance can be adjusted by changing the distance to the corner; smaller distances will produce larger acoustic returns.

Goals for a Back Loaded Horn Enclosure :

The primary goal of a back loaded horn enclosure is to produce a significant amount of bass output to reinforce the driver's falling SPL response at low frequencies. Typically a back loaded horn enclosure is the first choice of DIYers with high efficiency full range drivers driven by low powered tube amps. A simple set of goals for a back loaded horn enclosure design are listed below.

1. Produce significant broad band bass SPL output to reinforce the driver's SPL output at low frequencies.
2. Roll off the SPL output from the horn's mouth in the 100 – 300 Hz range eliminating destructive interference at the listening position
3. Control driver low frequency displacement, many efficient full range drivers have Xmax values of 1.0 mm or less.
4. If the driver displacement can be controlled / attenuated at low frequencies then the impedance peak associated with a driver's resonant frequency can be flattened.

Almost all of the back loaded horn designs I found on the Internet and analyzed in detail did not meet all of these goals. Typically the output from the mouth was peaky and still contributing well up into the mid range frequencies. The peaky output from the mouth was caused by discrete standing wave resonances along the horn's length. The peaky output did nothing to attenuate the driver's output resulting in constructive and destructive interference at the listening position and dramatic swings in the system's SPL response. Looking at the electrical impedance typically showed a series of peaks in the magnitude plot indicative of standing wave resonances. The enclosures acted more like undamped transmission lines which is really what they were despite the back loaded horn label applied by the designer.

Designing a Hybrid Transmission Line / Back Loaded Horn Enclosure :

Recognizing that a back loaded horn enclosure is acting as a transmission line at low frequencies, and does not exhibit any horn like behavior until it reaches the mid range, a design method based on transmission line acoustics can be applied. The first step is to size the effective mouth consistent with a horn tuned between 100 and 500 Hz, this frequency range is where the mouth's output should be rolled off. The mouth's physical sizing needs to take into account any of the room boundaries where there is direct contact; for example the floor when the mouth is located at the bottom of the front or rear baffle or even better downward firing. From Figure 2, tuning a mouth for 300 Hz indicates a square area with 12.75 inch long sides. If the mouth is placed at the floor, this area can potentially be cut in half to achieve approximately the same tuning frequency.

Once the mouth has been sized, the throat area and the length of the horn need to be iterated to achieve the desired fundamental tuning frequency and acoustic gain. The fundamental tuning frequency can be seen as the first peak in the acoustic impedance plot that is shown in each MathCad worksheet. Since the corner loading is going to reinforce the output at this frequency, the tuning of the transmission line's first quarter wavelength resonance can be set below the driver's resonant frequency. Juggling the driver's Qts, the expanding line's tuning frequency, and the amount of corner loading the only way to get optimized final horn geometry is by iterating the simulations.

Once the horn geometry has been determined, the coupling volume should be sized. Equation 5.4 from the horn physics article[2] can be used to calculate the minimum coupling volume.

$$V = \frac{c\, S_0}{2\, \pi\, f_h}$$

This equation represents the minimum size for a coupling volume that will start to roll off the mouth's SPL at the desired horn tuning frequency. The coupling volume can be made larger with little impact. The bigger lever for attenuating the mouth's output is the coupling volume's longest dimension and the driver position. The longest dimension and driver position will set the quarter wave resonance between the driver and the closed end of the transmission line creating a deep null in the mouth's SPL output. Between tuning the coupling volume to act as a stub tube, and the coupling volume naturally rolling off the horn's output as frequency increases, the back loaded horn's contribution to the system SPL response can essentially be eliminated above a selected frequency. Sizing of the mouth to establish the frequency for the onset of horn behavior, sizing the coupling volume, and determining the internal layout of the coupling volume are variables that need to be optimized so everything coincides with the frequency at which the mouth's SPL output is to be attenuated.

Sound simple? I have followed these basic steps many times and designed back loaded horns with expansion profiles ranging from a straight linear taper to a complicated expansion like an exponential or hyperbolic profile. Every time the calculated output starts to look promising, the question of how to fold the geometry and fit it into a simple box that is both compact and easy to build kills the design.

Approaching the problem from the opposite direction has become the key. I now start by setting the overall box external dimensions and selecting a folding pattern. The height, width, and depth of the enclosure are the first inputs along with the material thickness. A simple folding pattern is assumed, for example a series of straight sections that expand in size at each bend. This information is used to construct a cross-section plot of the proposed back loaded horn enclosure. Two new MathCad worksheets have been formulated for a front and rear firing horn mouth using this method. Each worksheet requires the bare minimum number of dimensional inputs to define the enclosure geometry and then the worksheet draws the cross-section and automatically populates the detailed section input fields. If the horn needs to be bigger, smaller, longer, or shorter changing one of the defined dimensions usually does the trick. Since the sections input is automatically populated, iterating a design to optimize the system SPL response is very quick and intuitive. The drudgery of detailed section by section input editing has been eliminated from the process.

Figure 3 shows the cross-sections assumed in these two new MathCad worksheets. In both cross-sections the driver outline is shown in red and the enclosure walls in black. The driver is located in the coupling volume and the model accounts for the longest dimension, in these cases the horizontal direction. Directly behind the coupling volume is a vibration absorber tuned to the frequency where the horn mouth output is to be rolled off. A throat is located just below the driver position with the folded horn connecting the throat to the mouth. The mouth area required for a given tuning frequency can be cut in half by taking advantage of the floor's reflective boundary condition. The total lengths of both horns in Figure 3 are within three inches (less than 4% of the total path length).

To completely define the geometry of these two enclosures approximately twelve dimensional inputs are needed. The first four are the height, width, depth, and wall thickness which set the external size of the enclosure. Then to complete the definition of the internal geometry the dimensions of the coupling volume, the vibration absorber, two or three values to define the horn path widths, and the mouth height are specified. With a minimal number of inputs, the complete sections table is populated by calculating all of the detailed entries from these basic dimensions. The final variable is placement and location of fiber stuffing, fiber stuffing is needed to control the transmission line resonances at low frequency. Stuffing the coupling volume, the vibration absorber, and the first few lengths of the folded path is usually all that is required to smooth the SPL response. Figure 4 shows the standard and hopefully familiar area profile plot for the mouth on the rear baffle version of the worksheets.

Figure 5 contains the views depicting room placement for the compact back loaded horn with a mouth located on the rear baffle. This is the design for which the remainder of the results will be presented and discussed. Notice the enclosure is located very close to the room corner to take advantage of all the reflections produced by the floor and the two walls. The responses will be calculated at a 1 meter distance, on the axis of the driver, as shown by the small purple square in the three plots.

Figure 3 : Compact Back Loaded Horn Cross-Sections with the Mouth on the Front (left) and Rear (right) Baffle

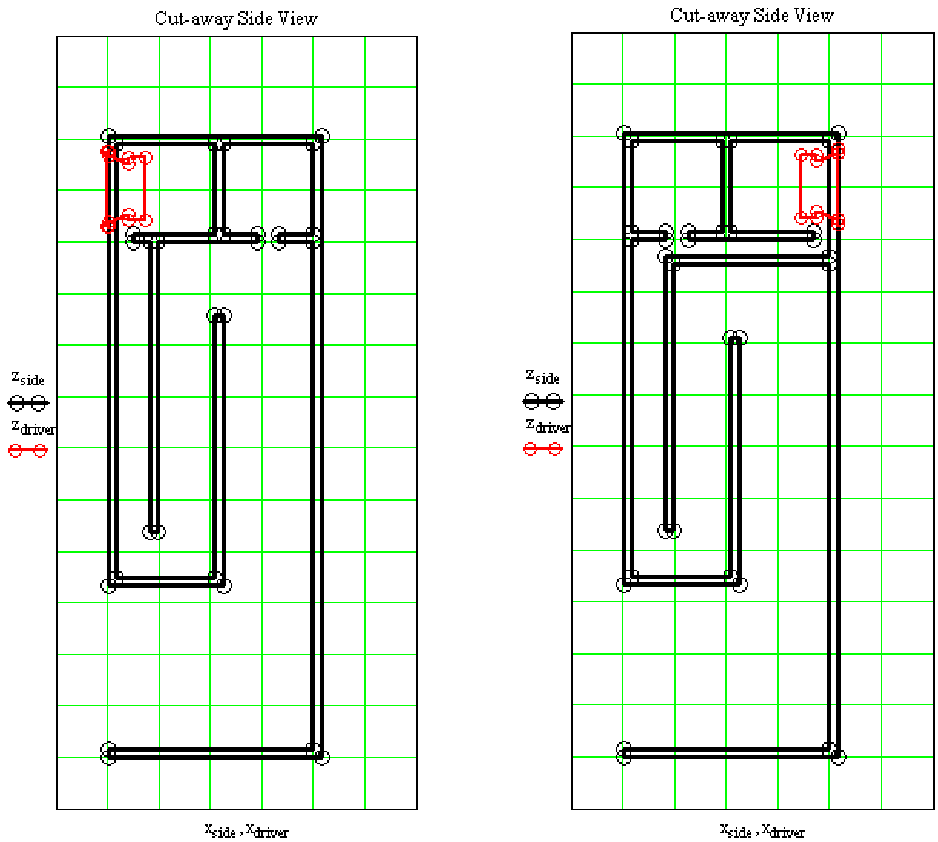


Figure 4 : Compact Back Loaded Horn Profile for the Mouth on the Rear Baffle

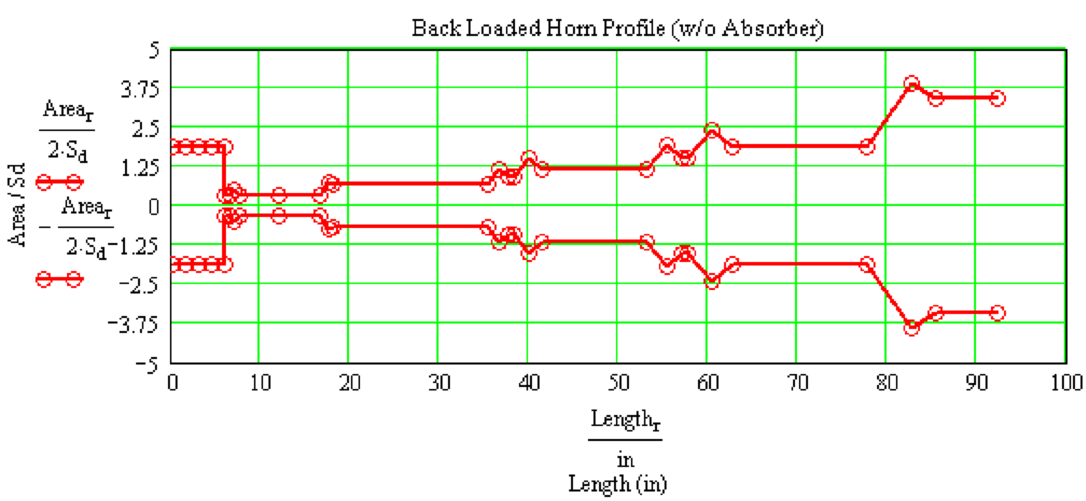
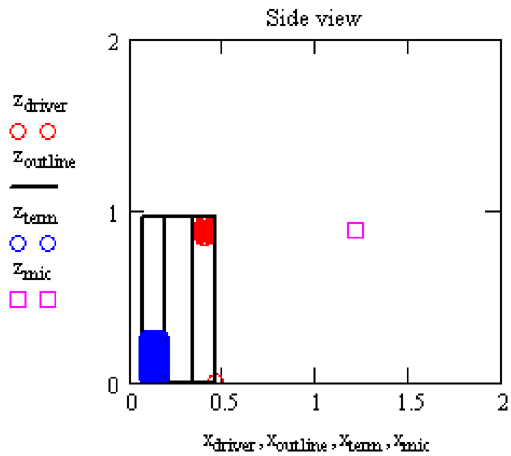


Figure 5 : Room Corner Placement of the Compact Back loaded Horn

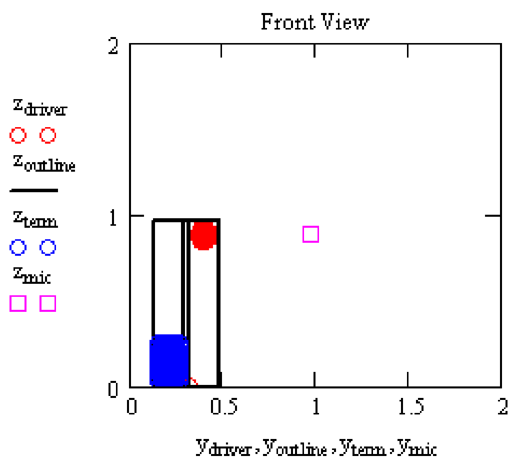
Three Dimensional View

Axis Length (m) axis := 2 <---- Change value of "axis" to rescale plots

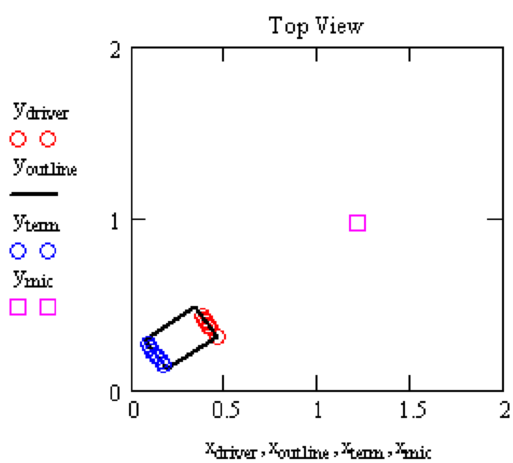
Room Corner is the Origin



Side View - looking out from side wall



Front View - looking towards rear wall



Top View - looking down from ceiling

The Final Design Calculation Results :

I selected the new Fostex FF125WK full range driver to study the back loaded horn design being presented. I chose this driver after hearing it mounted in one of my open baffle speaker systems. It also shares a common hole cut out and screw pattern with quite a few other Fostex drivers of the same diameter (FE-126E, FE-126En, FF125K, FX120, and F120A) that I have in my collection. The enclosure dimensions were iterated to balance the trade-offs between enclosure geometry and room corner positioning.

Figure 6 presents a dimensioned layout of the final compact back loaded horn enclosure. All material is 0.5 inches thick. Both the coupling volume and the vibration absorber chamber are modeled with 0.75 lb/ft^3 of polyester fibers. The entire horn up to the last bend before the 5.5" wide front section is modeled with 0.375 lb/ft^3 of polyester fibers. The 5.5" and 10" wide lengths of the horn enclosure are left empty. I assumed inexpensive loose polyester fiber that can be bought in bulk from a craft or fabric store.

Figures 7, 8, and 9, show the SPL response, electrical impedance, and driver cone deflection plots respectively for a 1 watt input (referenced to an 8 ohm voice coil resistance). These plots show a number of features of the design some of which are evident in all three plots.

Starting with Figure 7 the SPL response plot, it is interesting to look at the driver output (red dashed curve) and horn mouth output (blue dashed curve) for the frequency range around 200 Hz. The curves resemble what one would expect to see from a passive crossover between a woofer and a midrange driver. The driver output below 200 Hz is recessed and the horn mouth output above 200 Hz is dropping rapidly. There is very little interaction between the driver and the horn output above and below this crossover point leading to a smooth system SPL response. Most back loaded horn designs on the web exhibit a series of peaks in the horn mouth output leading to a series of dips in the driver output and a ragged combined system SPL response.

The attenuated driver output below 200 Hz means the driver's cone motion is being controlled by the back pressures generated in the enclosure. Lower driver deflection means lower electrical impedance near the driver's resonant frequency. Both of these impacts are seen in Figures 8 and 9 respectively.

The plot in Figure 10 depicts the SPL frequency response plot for the same back loaded horn speaker design without the influence of the corner loading. Without the corner, the low frequency response is six to eight decibels below the 90 dB/W/m efficiency of the FF125WK full range driver. The advantages of corner loading the back loaded horn design are very evident after comparing Figures 7 and 10.

As mentioned above, there is a family of Fostex drivers that will also fit in this back loaded horn design. The drivers have similar resonant frequencies and a range of Qts values. All of the drivers work in this enclosure with the FE-126En providing a little brighter / hotter calculated SPL response and the FX120 producing a more laid back calculated SPL response. For each driver the SPL response can also be adjusted by pulling the enclosure further out into the room or by pushing it back further closer to the corner. The enclosure looks to be a robust design that is very adjustable through driver selection and room corner placement.

Figure 6 : Compact Back Loaded Horn Enclosure Drawing
Material Thickness = 0.5 inches

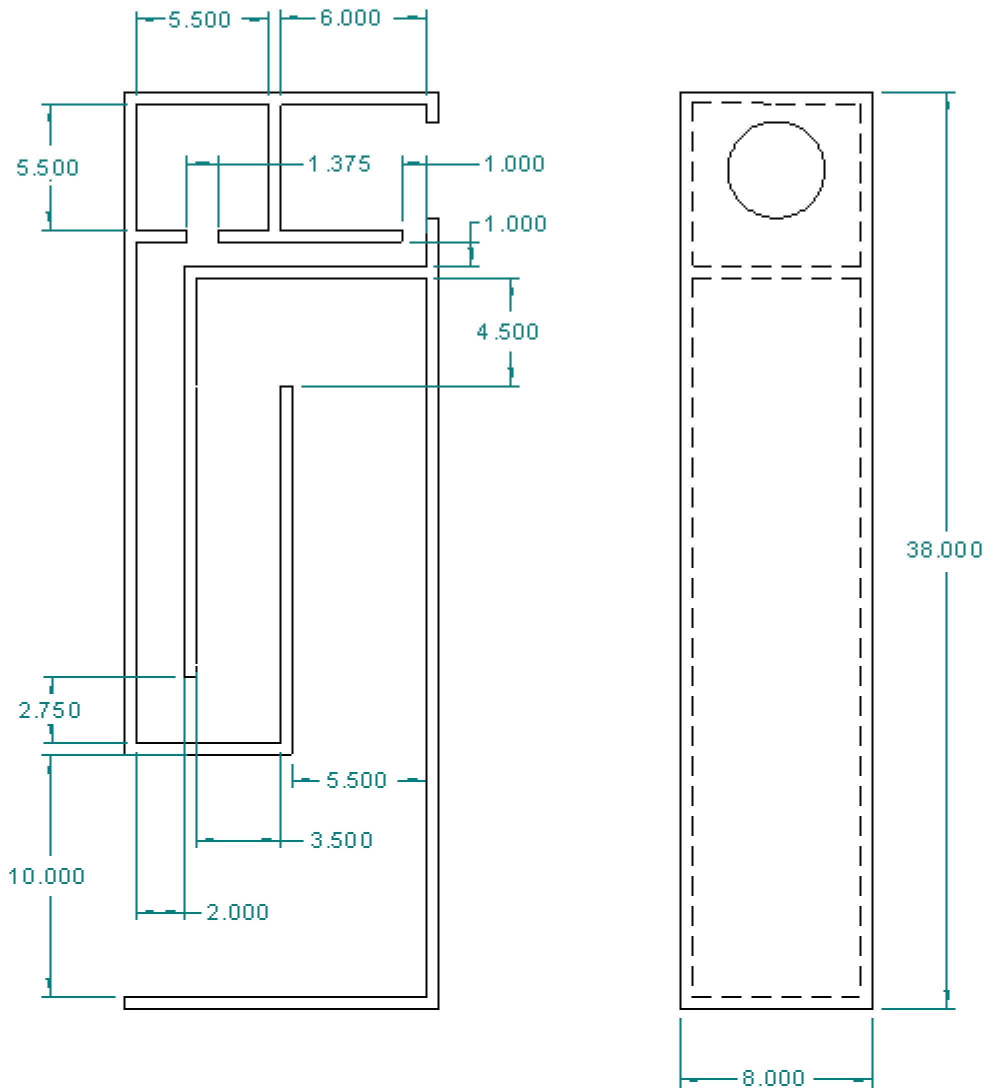


Figure 7 : SPL Response at 1 m on the Axis of the Driver

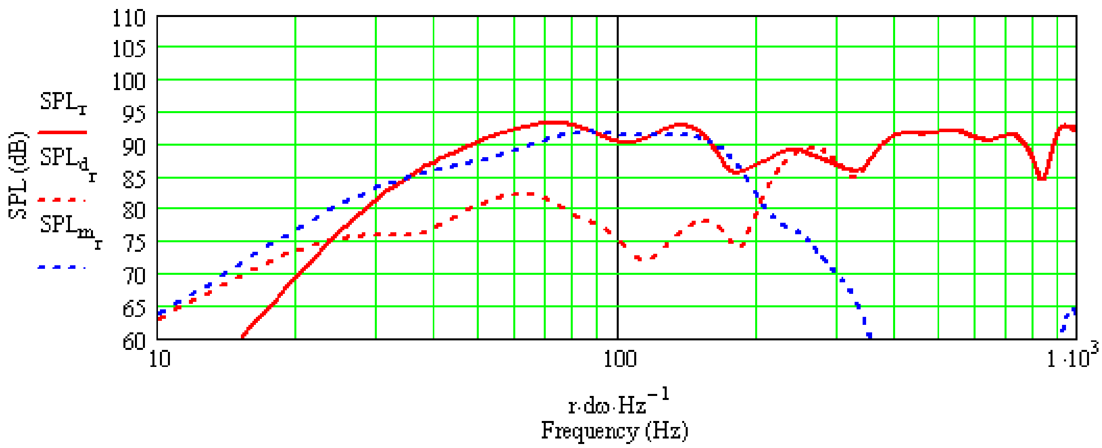


Figure 8 : Electrical Impedance Magnitude

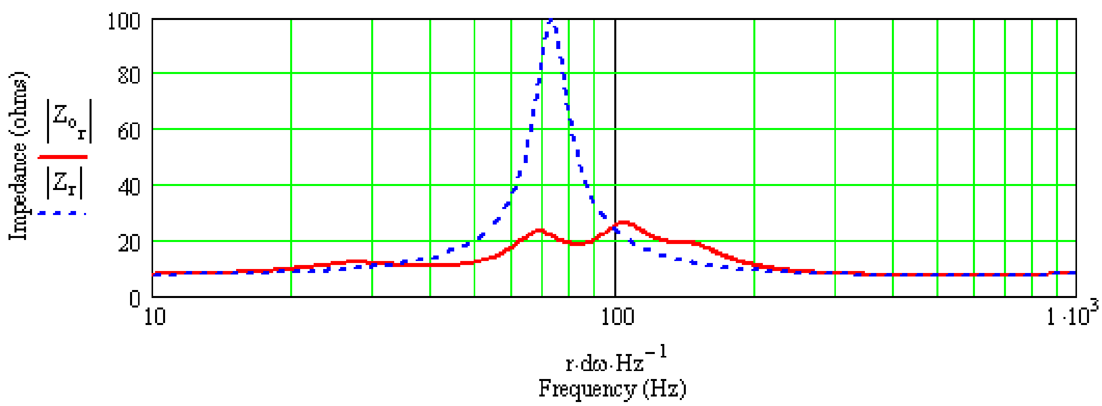


Figure 9 : Cone Displacement

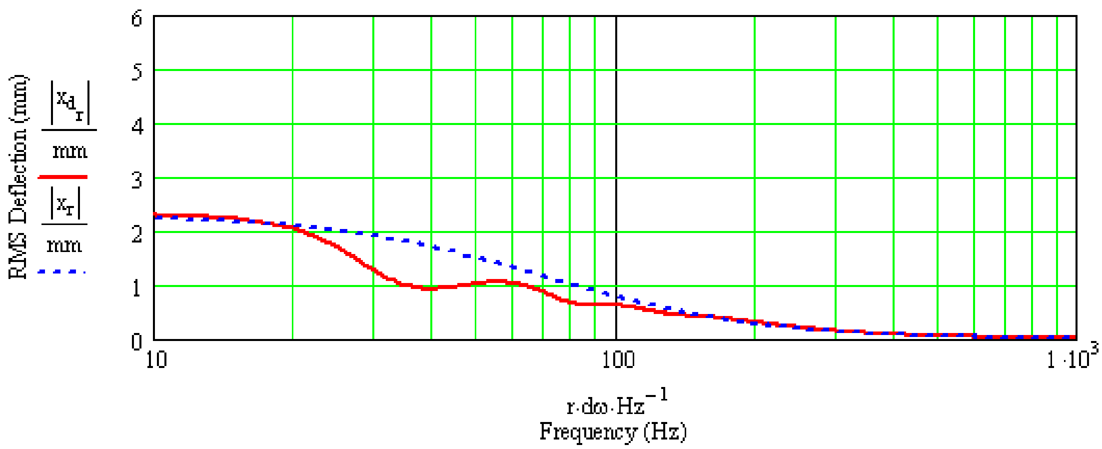
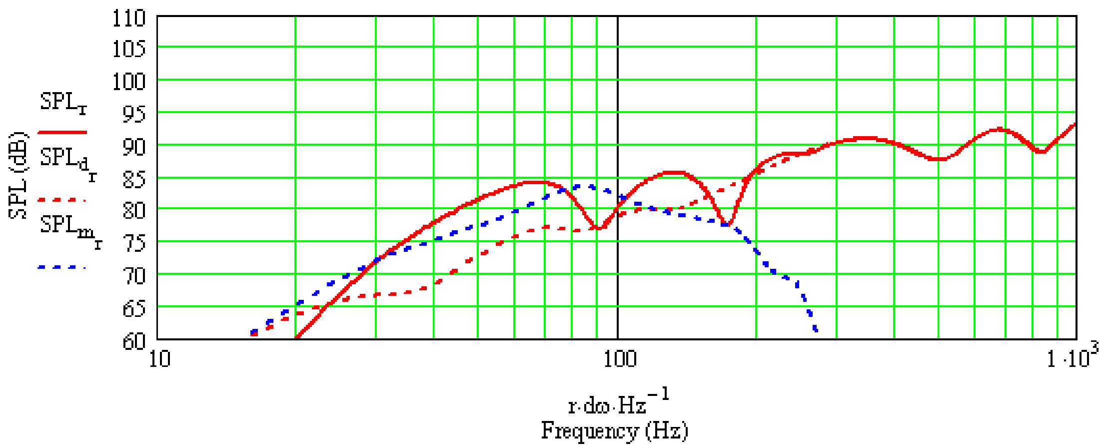


Figure 10 : SPL Response at 1 m on the Axis of the Driver w/o the Room Corner



Conclusions :

Figure 2 pretty much tells the entire story behind back loaded horn design. Expanding geometry **and** mouth size determine the acoustic behavior of back loaded horn enclosures. My contention is that almost all designs found on the Internet labeled back loaded horns are really transmission lines that transition to horn behavior at mid range frequencies. As such, designing the enclosure using horn sizing equations and not using transmission line theory is neglecting the physics of the problem and relying more on luck to produce an enclosure that performs as desired. Using transmission line methods and room boundaries to reinforce the bass frequencies is one method for accurately and optimally designing this type of speaker enclosure.

Future Work :

The worksheet models shown in Figure 3 are the first two in a series of preformatted back loaded horn design worksheets. Using a limited number of geometric variables to describe a specific design and pre populating the section input fields allows for quick and accurate iteration of the calculated performance. This allows the designer to rapidly focus in on an optimized geometry. The next step in the development of these two models is to create an option for the path to continuously taper between the throat and the mouth. The final step will be to extend these models to simulate the symmetric double mouthed back loaded horns that have become very popular over the past few years.

The longer range goal is to create additional back loaded horn worksheets that closely integrate with room corners so that the effective mouth area starts to approach the requirements shown in Figure 2 for low frequency horn behavior. These worksheets will have calculated mouth acoustic impedances that account for the boundary reflections and push the transmission line to horn transition much lower in frequency. I am hoping that these additional worksheets will allow bass horn performance in moderately sized enclosures that are not too complicated to build. Quick and easy simulations that produce accurate SPL frequency responses should open up many new opportunities for high performance back loaded horn enclosure designs.

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