Section 8.0 : Advanced Design of a Back Loaded Exponential Horn

In the previous section, a simple back loaded horn MathCad worksheet was described and results presented for exponential flare geometries. The worksheet was developed based on assumptions consistent with Thiele / Small[8,9,10,11] lumped parameter models. For example, the position of the driver and horn mouth were not specified and were assumed to be coincident. Also, the SPL was assumed to radiate only into 2π space ignoring the baffle step response produced by the baffle's physical geometry. The worksheet solved the simplified acoustic and electric circuit models to produce system SPL and impedance plots with slightly better accuracy compared to other lumped parameter freeware programs available on the Internet.

For a back loaded horn, these simplifying assumptions can become inaccurate and limit the quality of the design predictions. In this section, the simple model will be extended to include the size and shape of the horn mouth, the relative positions of the horn mouth and driver on the front baffle, the baffle step response, and the impact of a floor boundary condition. Since the majority of back loaded horn enclosures are floor standing speakers, the floor boundary condition cannot be ignored in design studies. In the following paragraphs, the MathCad worksheet from the preceding section will be referred to as the simple model while the new MathCad worksheet, derived in this section, will be referred to as the advanced model.

The advanced analysis approach is comprised of three steps. First the horn mouth's acoustic impedance is calculated factoring in the mouth size, mouth shape, and the floor boundary condition. Second, the simple model back loaded horn worksheet is run, using this calculated mouth acoustic impedance, and the volume velocities of the driver and at the horn mouth are calculated and saved. In the final step, the advanced model uses these calculated volume velocities to determine the system response including the effects of the relative position of the driver and the horn mouth on the front baffle, the baffle step, and all the floor reflections. The following paragraphs describe the calculations performed and present results for the generic driver used in the previous section.

<u>The Generic Driver</u> :

In Section 7, a generic driver was defined that is easily adjustable to produce different combinations of Thiele / Small parameters. The following driver parameters were defined and are intended to represent a typical eight inch diameter full range driver such as those produced by Lowther, Fostex, or AER. All of the results that follow are really intended for full range driver applications but should also be applicable to woofer or mid-bass drivers.

The generic full range driver is restated below based on key input properties and some derived properties. When looking at the relationships used to calculate the derived properties, please keep in mind that MathCad internally automatically converts frequency in Hertz to frequency in radians/sec. This property of MathCad leads to equations that may not look exactly like those familiar to the DIY speaker designer. In equations containing a frequency term, a 2π multiplier may be missing or added depending on the required units of the result.

Driver Thiele / Small Parameters : Generic Driver Derivation

$f_d := 50 \cdot Hz$ $\quad Q_{md} := 4$

$R_e := 8 \cdot ohm$ $\quad Q_{td} := 0.2$

$L_{vc} := 0 \cdot mH$ $\quad M_{md} := 14 \cdot gm$

$S_d := 205 \cdot cm^2$

Derived Thiele / Small Parameters

$$Q_{ed} := \left(\frac{1}{Q_{td}} - \frac{1}{Q_{md}}\right)^{-1} \qquad Q_{ed} = 0.2$$

$$C_{md} := \left(M_{md} \cdot f_d^{\,2}\right)^{-1} \qquad C_{md} = 7.2 \times 10^{-4} \frac{m}{newton}$$

$$V_{ad} := C_{md} \cdot \left(\rho \cdot c^2 \cdot S_d^{\,2}\right) \qquad V_{ad} = 43.0 liter$$

$$\eta_o := V_{ad} \cdot \left(2 \cdot \pi \cdot c^3 \cdot Q_{ed} \cdot f_d^{\,-3}\right)^{-1} \qquad \eta_o = 2.5\%$$

$$SPL := 112 + 10 \cdot \log\left(\eta_o\right) \qquad SPL = 96.0 \quad \text{dB}$$

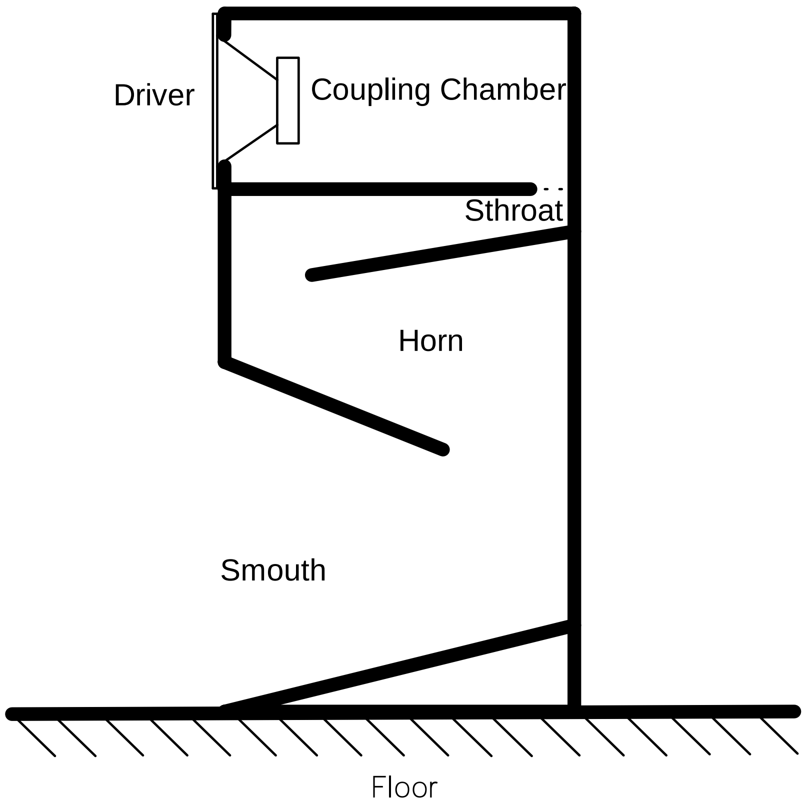
$$Bl := \left(\frac{f_d \cdot R_e \cdot M_{md}}{Q_{ed}}\right)^{0.5} \qquad Bl = 12.9 \frac{newton}{amp}$$

After the generic driver was defined in Section 8, baseline horn geometries were formulated and simulations run to calculate the on-axis anechoic SPL responses, the electrical impedances, and the impulse pressure responses.

Horn Geometry :

The back loaded exponential horn geometry that will be used to calculate the system SPL response is shown in Figure 8.1. The advanced model will account for the position of the driver, the position of the horn mouth, the impact of the baffle step response, and the influence of the floor boundary condition. It will be assumed that the speaker is out far enough into the room to neglect the influence of the back wall or any corner loading. These additional boundary conditions will be added later to the worksheet.

Figure 8.1 : Back Loaded Horn Geometry



where the Horn Geometry is defined by :

$S_0 = S_{throat}$ = throat area

$S_L = S_{mouth}$ = mouth area

L_{horn} = horn length

and the Coupling Chamber Geometry is defined by :

S_{DF} = coupling chamber area at the closed end

S_{LF} = coupling chamber area at the throat end

ξ = driver position ratio ($0 < \xi < 1$)

L_F = coupling chamber length

Baseline Exponential Horn Design :

The first simulation presented in Section 7 was referred to as the baseline design. Reviewing this baseline design, including the coupling chamber and fiber damping, the lumped parameter results are shown again in Figures 8.2, 8.3, and 8.4. The lower cut-off frequency f_c was specified as 50 Hz to match the driver's f_d. The throat area was assumed equal to the driver's S_d so that a length could be calculated. This geometry was designated as a consistent back loaded horn design.

The area of the horn mouth was calculated using Equation (5.3).

$$S_{mouth} = (1 / \pi) \times (c / (2 \times f_c))^2$$

$$S_{mouth} = (1 / \pi) \times (342 \text{ m/sec} / (2 \times 50 \text{ Hz}))^2$$

$$S_{mouth} = 3.723 \text{ m}^2 = 5771 \text{ in}^2 \text{ (}\sim 76 \times 76 \text{ inch square)}$$

$$S_{mouth} = 181.6 \times S_d$$

Using Equation (5.2), the flare constant was calculated next.

$$m = (4\ \pi\ f_c) / c$$

$$m = (4\ \pi\ 50 \text{ Hz}) / 342 \text{ m/sec}$$

$$m = 1.837 \text{ m}^{-1}$$

And finally, the horn's length was calculated, using Equation (5.1), after setting the throat area equal to S_d.

$$L_{horn} = \ln(S_{mouth} / S_{throat}) / m$$

$$L_{horn} = \ln(181.6 / 1) / 1.837 \text{ m}^{-1}$$

$$L_{horn} = 2.831 \text{ m} = 111.5 \text{ in}$$

The baseline design also included a coupling chamber that rolled off the horn's response at a higher cut-off frequency f_h of 100 Hz. The chamber volume V was calculated, in the same manner as demonstrated in Section 5, using Equation (5.4).

$$V = (342 \text{ m/sec} \times 0.021 \text{ m}^2) / (2\ \pi\ 100 \text{ Hz}) \times (1000 \text{ liters} / \text{m}^3) = 11.158 \text{ liters}$$

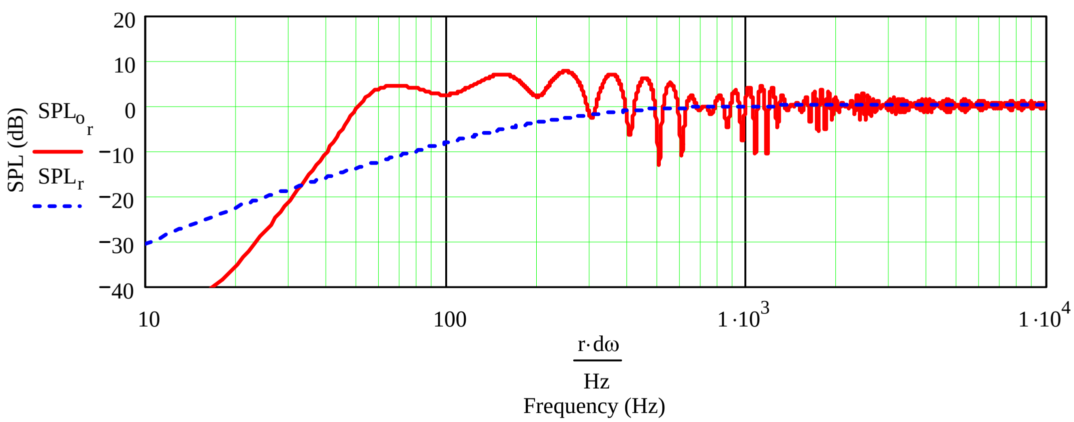
Looking at the dimensions shown above, not many DIYer's would build a horn with a mouth this big. The intent of simulating this geometry is to establish a baseline SPL response for a totally consistently sized back loaded exponential horn.

Substituting the dimensions and areas into the lumped parameter back loaded horn MathCad worksheet the acoustic impedance, the volume velocity ratio, the SPL, the electrical impedance, the driver displacement, and the impulse response were calculated. Adding 0.5 lb/ft^3 of fiber stuffing to the coupling volume, and 0.125 lb/ft^3 of fiber stuffing to the first one third of the horn's length helped reduce the peaks and nulls. Figures 8.2, 8.3, and 8.4 show the SPL traces for driver position ratios ξ = 0.0, ξ = 0.5, and ξ = 1.0 respectively. In each of the figures, a 3 to 4 dB boost of the lower bass response has been achieved. Nulls still exist but they have been reduced significantly. This is the starting configuration for the following advanced back loaded horn simulations.

Figures 8.2, 8.3, and 8.4 present the back loaded horn system SPL response (solid red curve) along with the driver in an infinite baffle response (dashed blue curve) as a reference in the top plot. In the bottom plot, the driver (solid red curve) and horn mouth (dashed blue curve) contributions to the back loaded horn system SPL response are shown. This is the color scheme used throughout this section in the different plots presented. Please keep this convention in mind when reviewing the plots that follow.

Figure 8.2 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber, Driver Position Ratio ξ = 0.0, and Fiber Damping

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

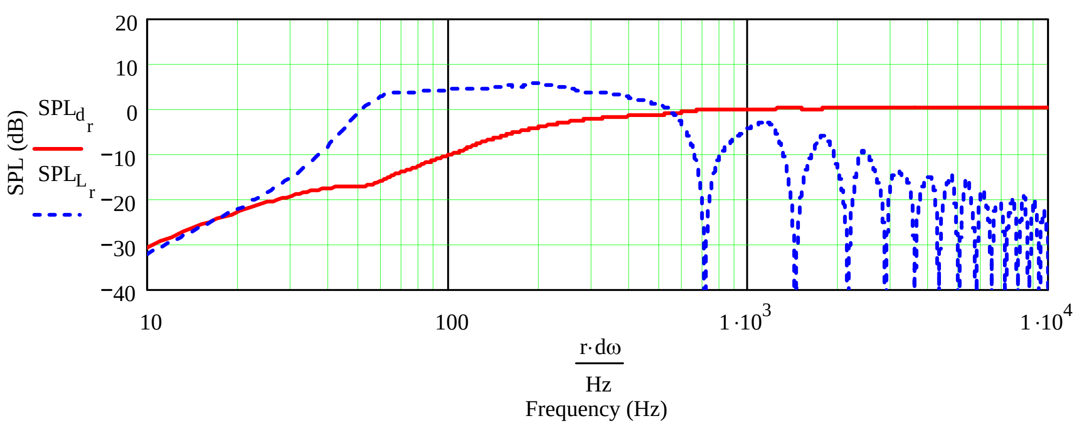
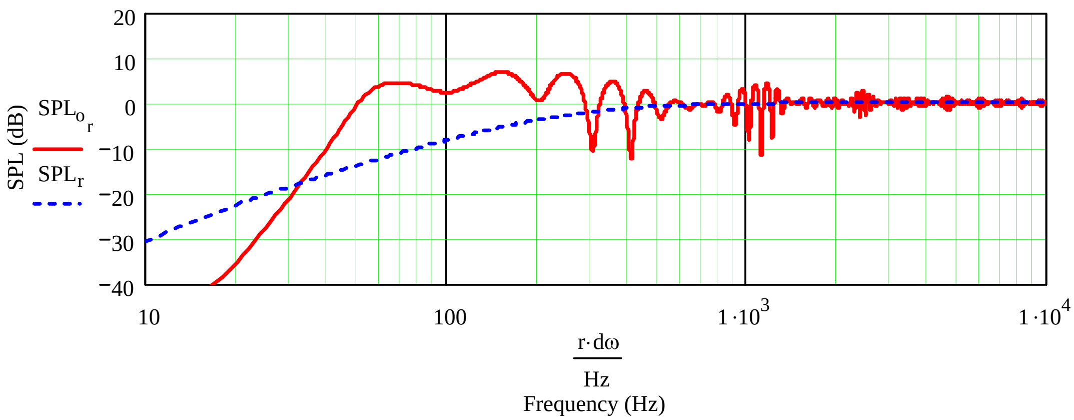


Figure 8.3 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber, Driver Position Ratio ξ = 0.5, and Fiber Damping

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

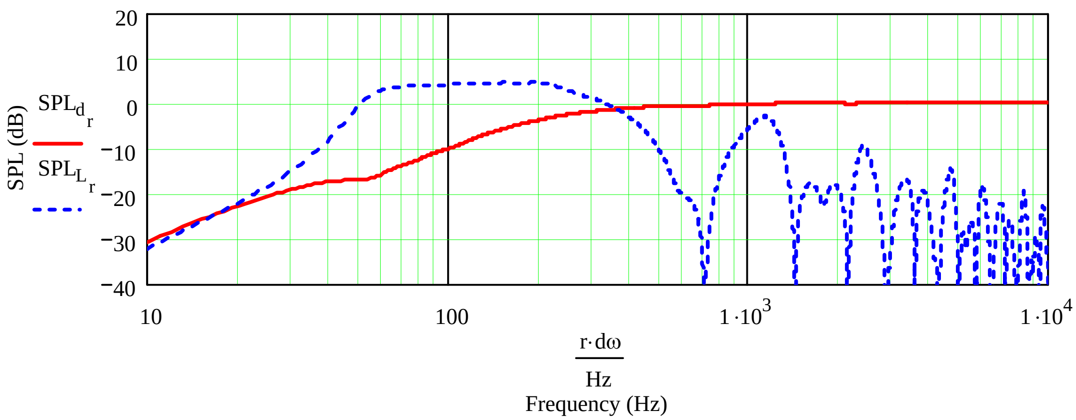
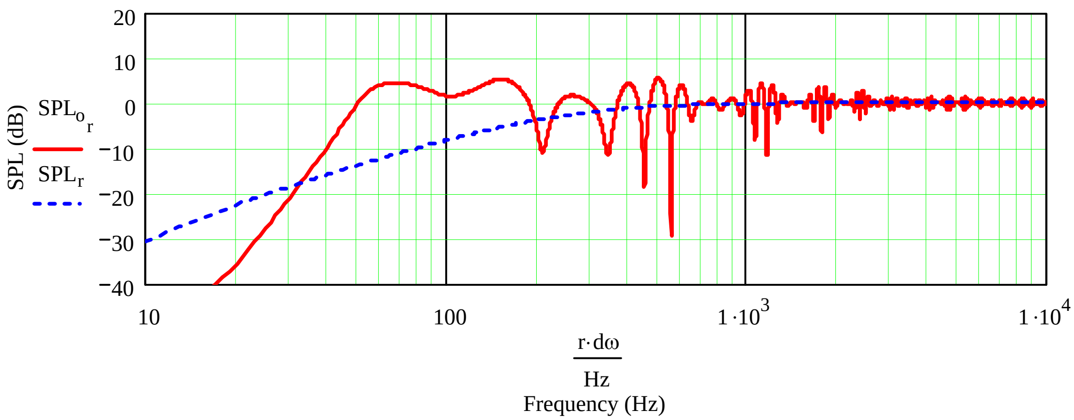
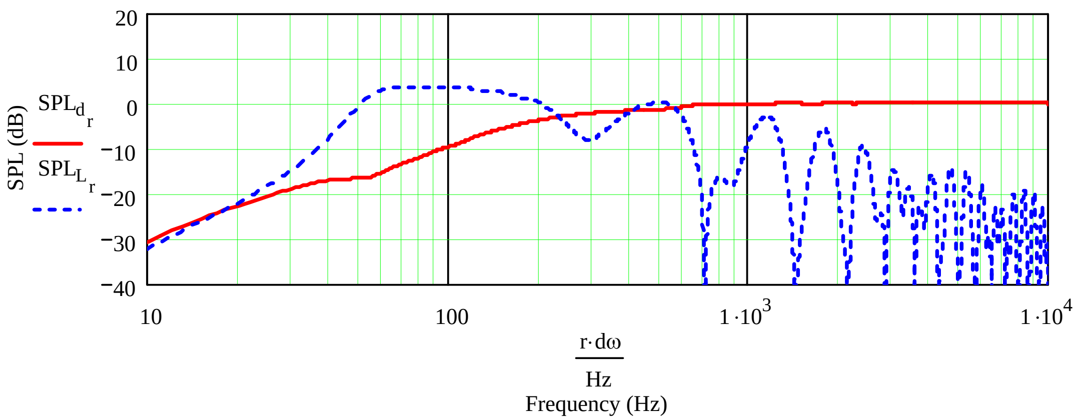


Figure 8.4 : Back Loaded Exponential Horn Response – Baseline Configuration with a Coupling Chamber, Driver Position Ratio ξ = 1.0, and Fiber Damping

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



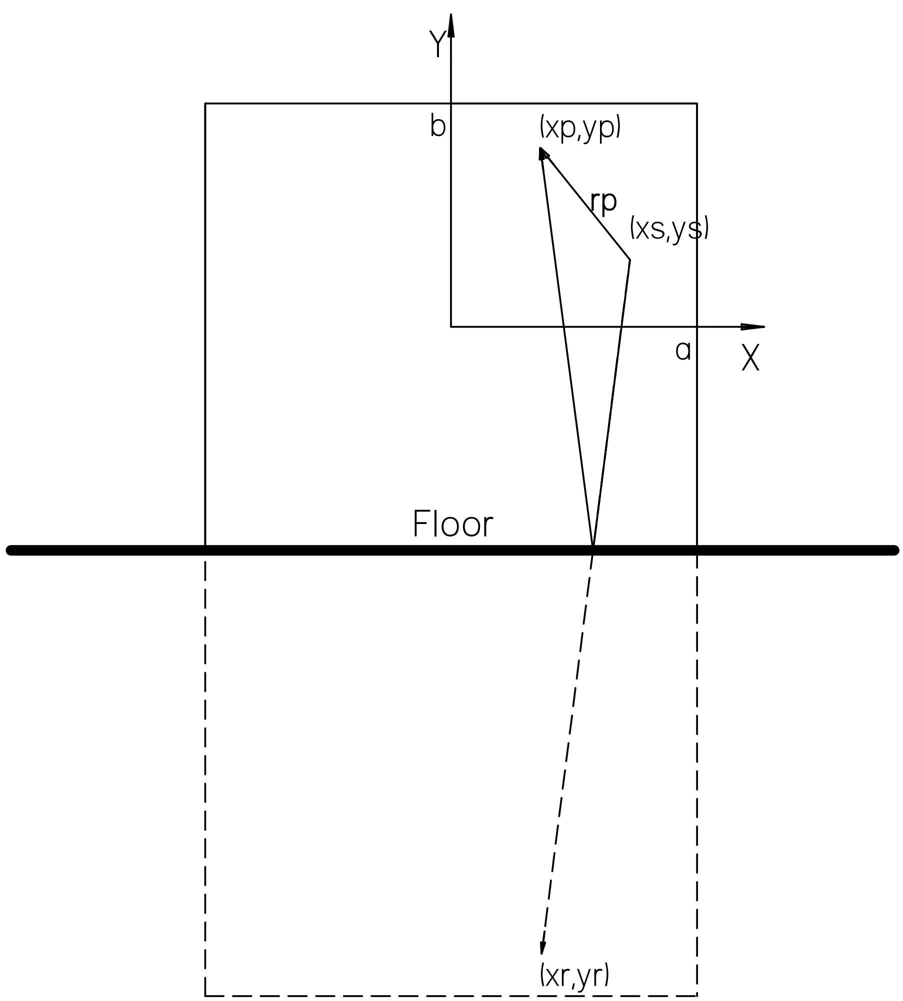
Woofer and Mouth Far Field Sound Pressure Level Responses



<u>Acoustic Impedance of the Horn Mouth Including the Floor Boundary Condition</u> :

In Section 3, the acoustic impedance of a rectangular mouth in an infinite baffle was calculated. In Figure 3.2, the geometry used to set up the integration was shown along with two calculation points. The first point (xs,ys) represented a simple source while the second point (xp,yp) represented a position at which the pressure from the simple source was calculated. Thinking of the rectangular piston as an assemblage of many small simple sources, radiating into 2π space, the incremental pressure that one simple source exerts on all of the remaining simple sources can be calculated and then summed to determine its contribution to the total acoustic impedance. Repeating this for each remaining simple source leads to the total acoustic impedance of the horn mouth. Adding a floor boundary condition introduces a reflective surface and reflected position (xr,yr) under the floor. The reflected position (xr,yr) represents a second contribution to the pressure calculated at (xp,yp) from the simple source at (xs,ys). The floor boundary condition and the key points are shown in Figure 8.5.

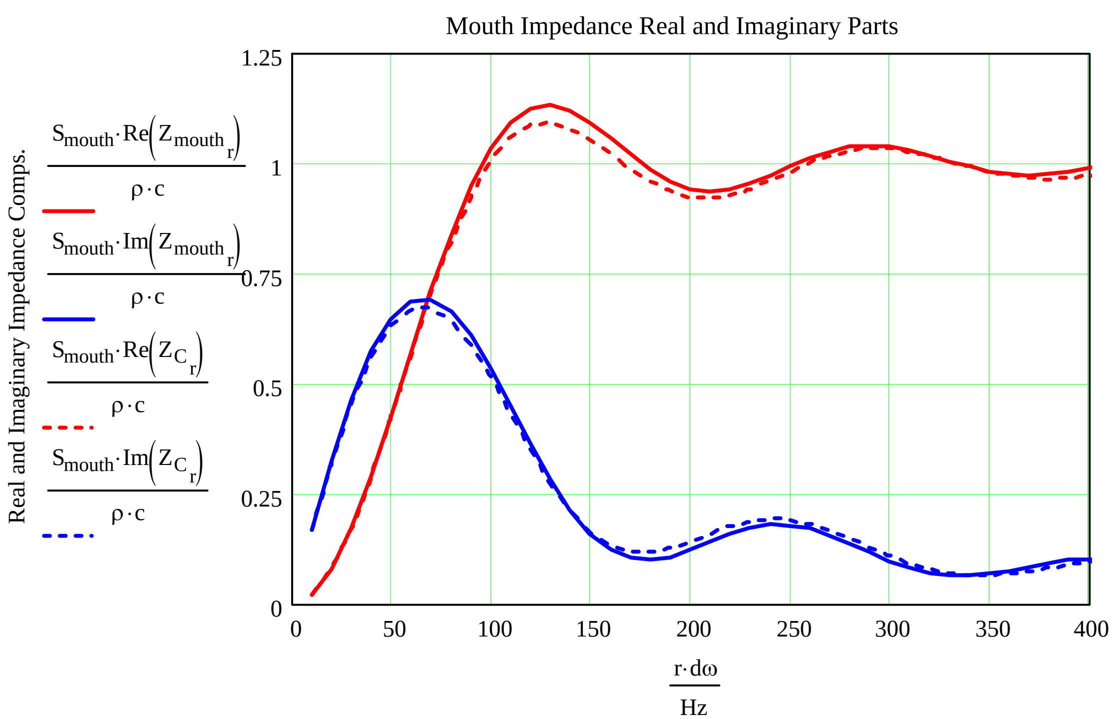
Figure 8.5 : Rectangular Geometry Definition Including The Floor Boundary Condition



Since the point (xp,yp) now has two pressures acting on it, the direct pressure from (xs,ys) and a pressure resulting from the floor reflection, it is not too difficult to visualize that the acoustic impedance approximately doubles at low frequencies compared to the same mouth area without the floor boundary condition. If this is true, then the area of the mouth can be half of the area required without the presence of a floor boundary condition. To explore this possibility, a series of acoustic impedance calculations were performed and are presented in the following figures.

In Figure 8.6, the acoustic impedance for the baseline mouth geometry is shown along with the closed form acoustic impedance calculated for the equivalent circular piston in an infinite baffle. The area of the baseline mouth was calculated above to be 5771 in^2 which is equivalent to a 76 inch by 76 inch square.

Figure 8.6 : Baseline Horn Mouth Acoustic Impedance
Solid – Circular Cross-Section
Dashed – Rectangular Cross-Section
Red Curve – Real Parts
Blue Curve – Imaginary Parts



Comparing Figure 8.6 with Figure 3.4, the lower cut-off frequency was determined in Section 3 using the expression

$$S_{mouth} \times Re(Z_{mouth}) / (\rho \times c) \sim 0.5$$

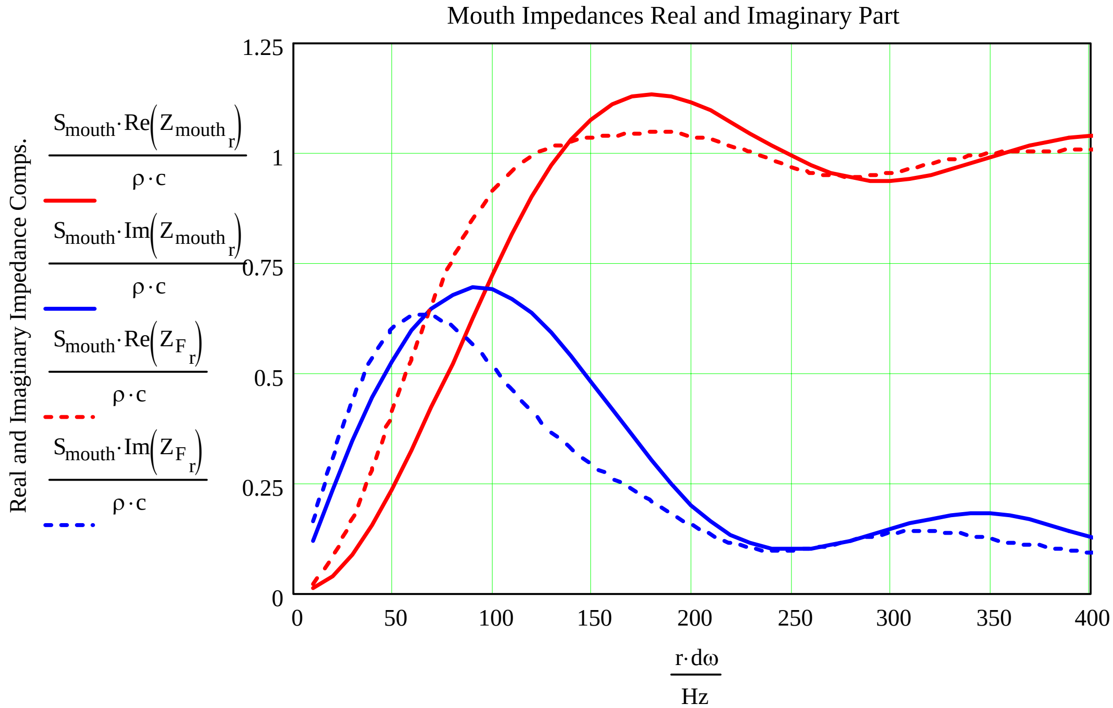
which occurs at approximately 50 Hz. Also notice that the square mouth and the circular mouth produce almost identical acoustic impedance curves. These are the baseline acoustic impedance curves for a mouth tuned to a lower cut-off frequency of 50 Hz.

Cutting the mouth area in half and adding the floor reflection is the next logical step. So the mouth area becomes

$$S_{mouth} = 1/2 \times 5771 \text{ in}^2 = 2886 \text{ in}^2 \text{ (~ 54 x 54 inch square)}$$

This is still a very large mouth, probably also not attractive to most DIYer's, but with the floor reflection it represents a consistent horn design. The acoustic impedance for the reduced mouth is plotted in Figure 8.7 with and without the floor reflection.

Figure 8.7 : Horn Mouth Acoustic Impedance Including Floor Boundary Condition
Solid – Without Floor Boundary Condition
Dashed – With Floor Boundary Condition
Red Curve – Real Parts
Blue Curve – Imaginary Parts



The solid curves in Figure 8.7 are the acoustic impedance of the smaller mouth which has a lower cut-off frequency of approximately 70 Hz. After adding the floor boundary condition the acoustic impedance is plotted as the dashed curves in Figure 8.7. The lower cut-off frequency, with the floor boundary condition, is 50 Hz therefore the design can be treated once again as consistent.

Comparing the dashed curves in Figure 8.7 with the curves in Figure 8.6 shows that the acoustic impedances are very similar. The primary difference is the magnitude of the ripple in the real and imaginary parts that results when the floor boundary condition is applied. The floor boundary condition effectively changes the shape of the mouth from a square to a rectangle. Similar shaped curves can be seen in Figure 3.5 for horn mouths of equal area but different aspect ratios.

An additional reduction of the mouth area can be made if the room's side walls are also used as boundary conditions. This is easily done but will not be pursued at this time. The goal of this section is the design of a floor standing exponential back loaded horn that is located out in the listening room away from the walls. From this point forward the floor boundary condition will be used to reduce the size of the required horn mouth while still producing a consistent horn design.

New Simplified Results :

By taking advantage of the floor boundary condition, the size of the horn's mouth can be cut in half. The lower cut-off frequency f_c is again specified as 50 Hz to match the driver's f_d. The throat area is set equal to the driver's S_d so that a new length can be calculated. This is still defined as a consistent back loaded horn design.

The area of the horn mouth was calculated using Equation (5.3) modified by a factor of one half.

$$S_{mouth} = (1/2) \times (1 / \pi) \times (c / (2 \times f_c))^2$$

$$S_{mouth} = (1/2) \times (1 / \pi) \times (342 \text{ m/sec} / (2 \times 50 \text{ Hz}))^2$$

$$S_{mouth} = 1.862 \text{ m}^2 = 2885 \text{ in}^2 \text{ (~ 54 x 54 inch square)}$$

$$S_{mouth} = 90.8 \times S_d$$

Using Equation (5.2), the flare constant was calculated next.

$$m = (4 \pi f_c) / c$$

$$m = (4 \pi 50 \text{ Hz}) / 342 \text{ m/sec}$$

$$m = 1.837 \text{ m}^{-1}$$

And finally, the horn's length was calculated, using Equation (5.1), after setting the throat area equal to S_d.

$$L_{horn} = \ln(S_{mouth} / S_{throat}) / m$$

$$L_{horn} = \ln(90.8 / 1) / 1.837 \text{ m}^{-1}$$

$$L_{horn} = 2.454 \text{ m} = 96.6 \text{ in}$$

A coupling chamber that rolls off the horn response at a higher cut-off frequency f_h of 100 Hz is still used. The chamber volume V calculated, in the same manner as demonstrated in Section 5, using Equation (5.4) is unchanged from 11.158 liters.

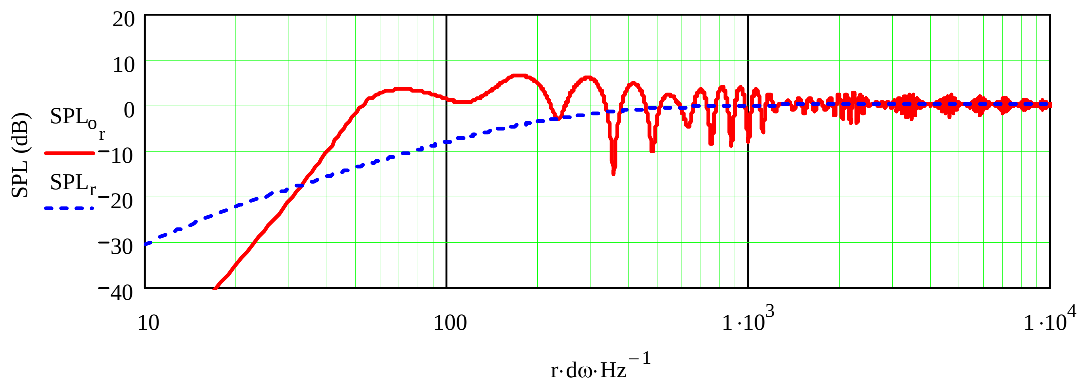
Internal dimensions for the coupling chamber are defined assuming a cross-sectional area of twice the throat area, which in this case is also twice the driver's cone area S_d. The length of the coupling chamber is calculated, from the 11.158 liter volume, to be 10.715 inches (or 0.272 m).

Again 0.5 lb/ft^3 of fiber stuffing is added to the coupling volume and 0.125 lb/ft^3 of fiber stuffing to the first one third of the horn's length to help reduce the peaks and nulls. The simple model was rerun after adjusting the acoustic impedance to account for the floor boundary condition. The results of this simulation are presented in Figure 8.8 for a driver position ratio $\xi = 0.5$.

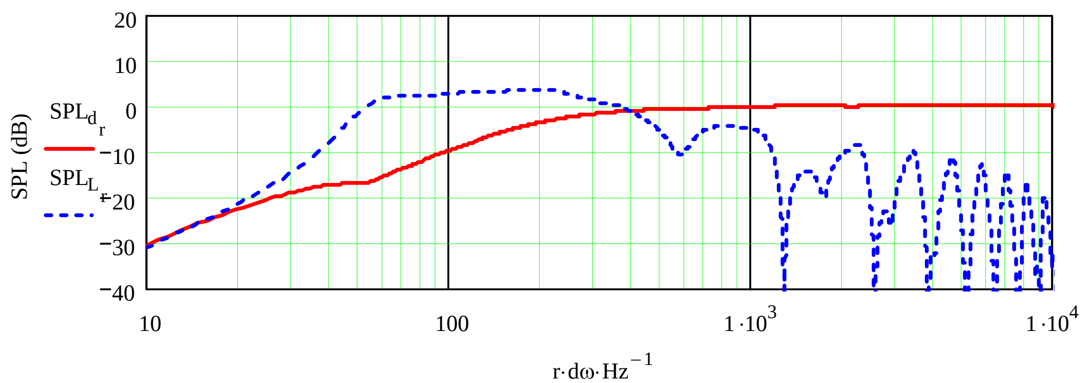
Comparing Figure 8.8 to the traces shown in Figure 8.3 demonstrates that the two exponential back loaded horns produce almost identical responses. The small differences are generated by the shorter length horn required when the floor boundary condition is taken into account.

Figure 8.8 : Back Loaded Exponential Horn Response – Revised Baseline Configuration with a Coupling Chamber, Driver Position Ratio $\xi = 0.5$, Fiber Damping, and Floor Boundary Condition
SPL at 1 m for a 1 Watt Input

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses



Advance Back Loaded Horn Model :

Figure 8.5 showed the reflection that occurs at the floor boundary condition and the impact on the horn mouth's acoustic impedance. Reflections also occur for the sound being radiated from the driver and horn mouth of the back loaded exponential horn enclosure geometry shown in Figure 8.1. Figure 8.9 depicts the different paths that sound traveling from the driver and the horn mouth follow to arrive at the listening position.

Figure 8.9 : Sound Paths from the Driver and the Horn Mouth to the Listening Position

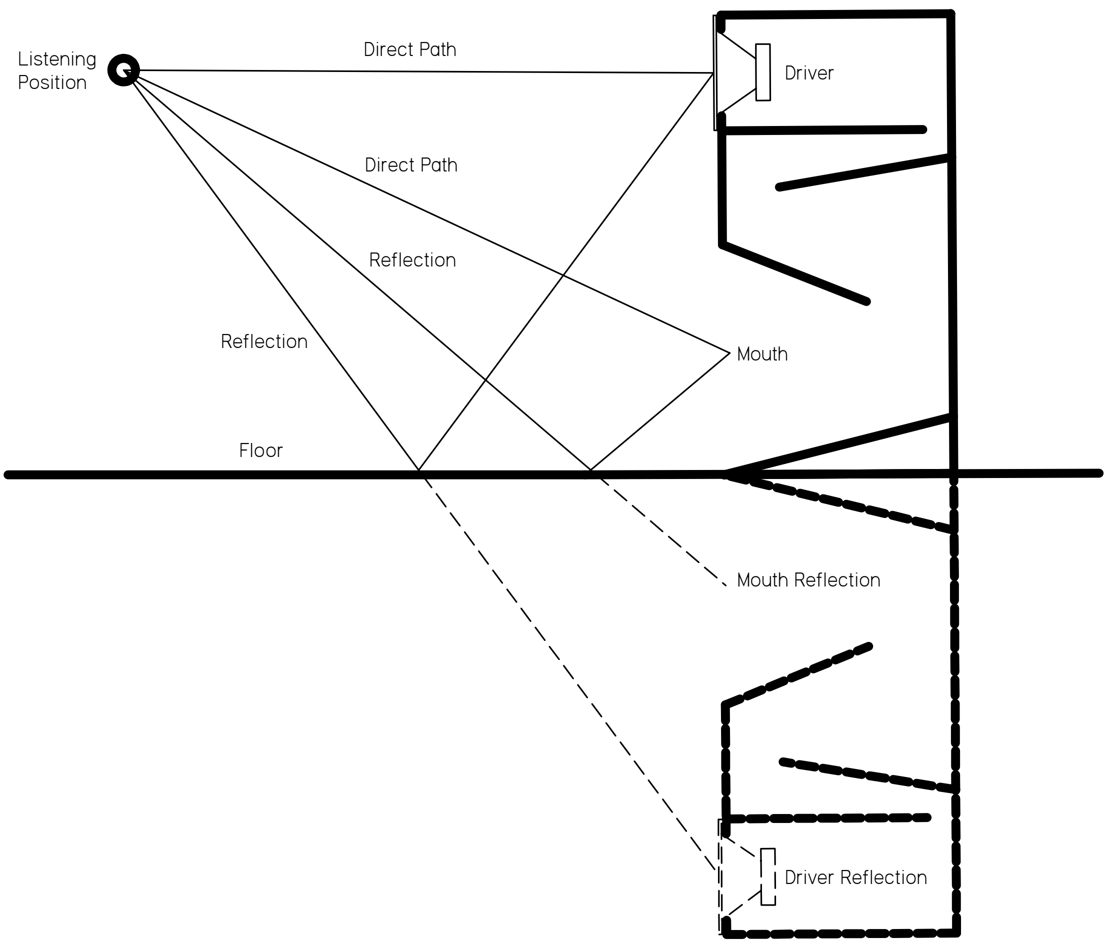


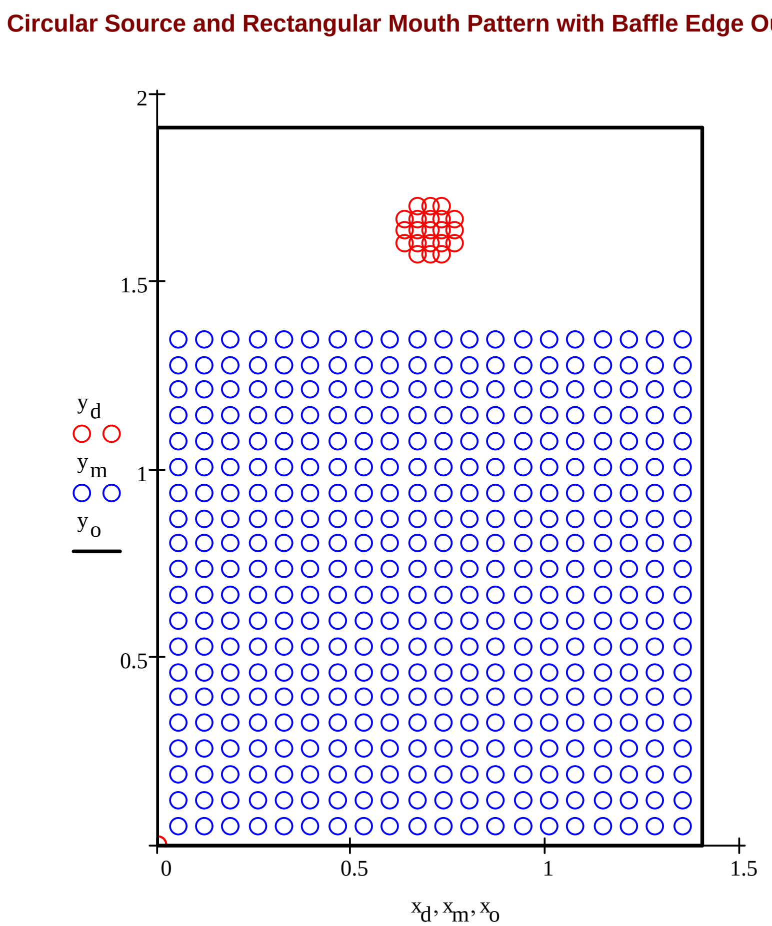
Figure 8.9 includes a number of improvements to the original simple model that will be applied in the advanced model. As stated earlier, the volume velocity of the driver and the horn mouth are output from the simple model and then imported into the advanced model to calculate the system SPL response. The system response model includes different direct paths from the driver and the horn mouth to the listening position due to the location of each on the front baffle. Also included are the reflected paths from the driver and horn mouth generated by the floor boundary condition. Although not explicitly shown in Figure 8.9, the baffle step response and the directional nature of each source will also be included in the calculations of the summed system SPL response. The advanced model is used to calculate more accurate SPL output but has no impact

on the electrical impedance or the driver displacement plots calculated using the simple model.

Figure 8.10 shows the front baffle used in the advanced model. The technique used to perform all of the calculations divides the driver and the mouth into a large number of simple sources and then sums the individual simple source responses. The driver is represented by the small red simple sources in Figure 8.10 while the mouth is broken up into the collection of small blue simple sources. The baffle is depicted as the solid black lines outlining the rectangular perimeter. The floor is represented by the x axis and the reader should visualize a mirror image collection of simple sources below the floor to account for reflections of the sound waves. The length units on the x and y axes are meters.

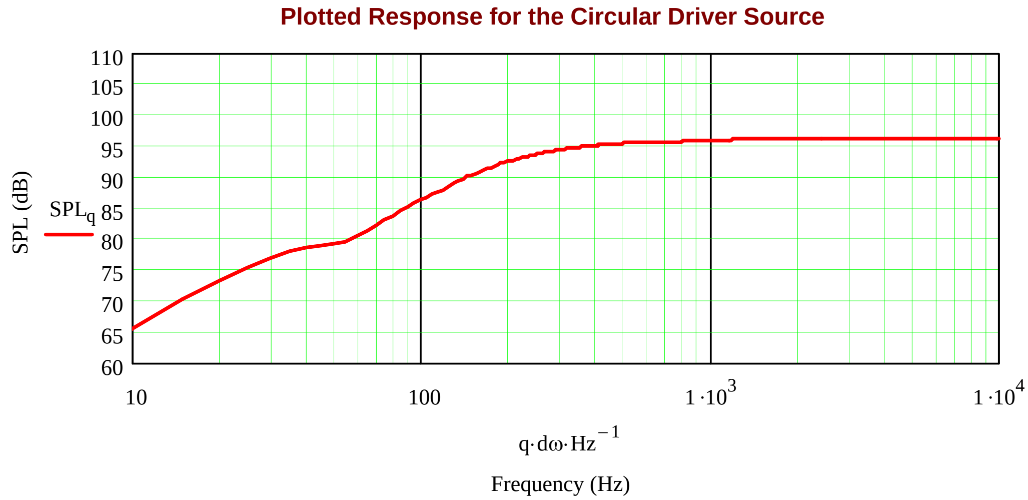
For each driver or mouth simple source, a baffle step response calculation is also performed. Each baffle step response also includes a floor reflection. To improve the accuracy of the calculations at high frequencies, the numbers of simple sources assumed for the driver, the mouth, and the baffle edge were increased until the SPL solution converged over the frequency range of interest. This can lead to very long calculations that can extend for several hours, and sometimes even overnight, if accurate high frequency data is required.

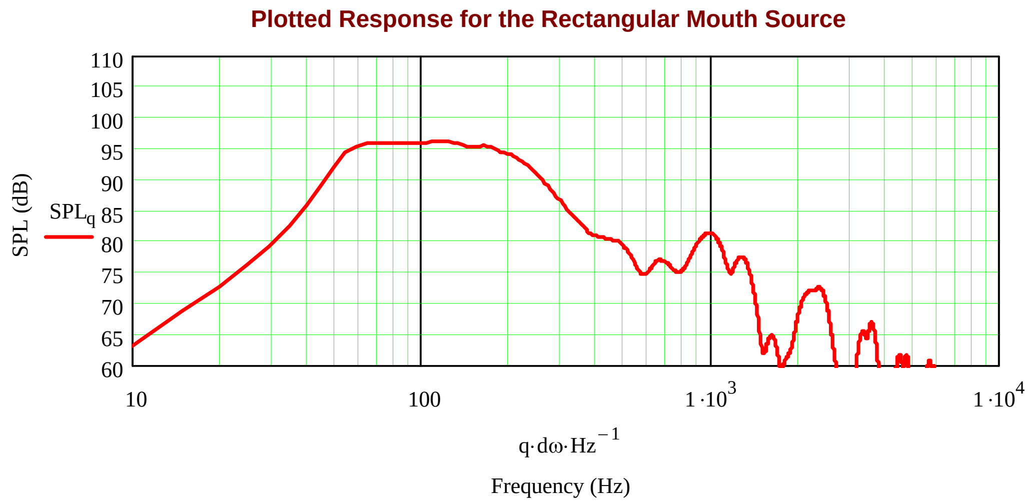
Figure 8.10 : Driver, Horn Mouth, and Front Baffle Edge Definitions



The driver, horn mouth, and edge definition shown in Figure 8.10 was analyzed to determine the combined system SPL response at a listening position 1 meter in front of the driver's center. One feature of the new advanced worksheet is the capability to show separately the impact of each contribution on the total system SPL response. Figure 8.11 shows the driver and mouth SPL response at the listening position without any other contributions. These results can be compared directly to those shown in the lower plot of Figure 8.8 for the simple model.

Figure 8.11 : Driver and Horn Mouth SPL Responses in 2 pi Space at 1 m for a 1 watt Input

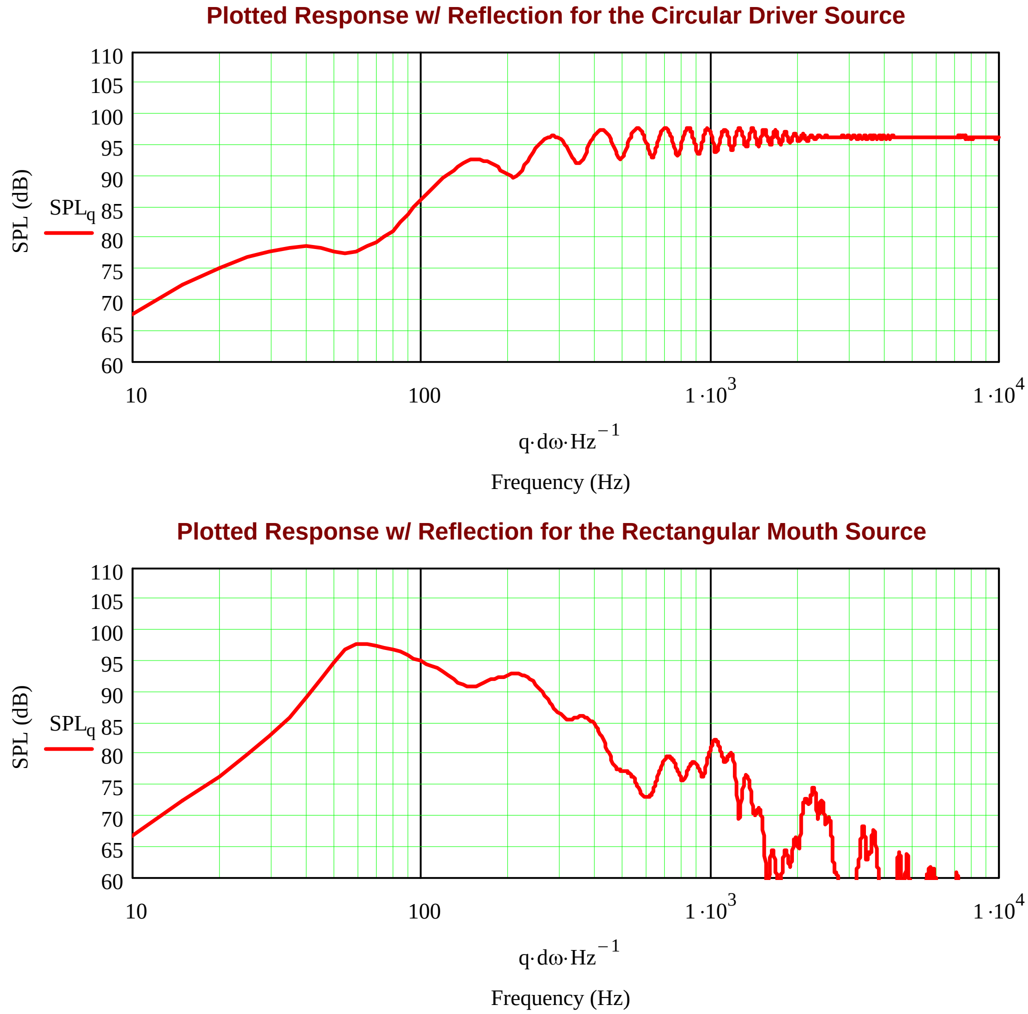




There are some differences between the mouth SPL responses shown in Figures 8.8 and 8.11. The differences result from the listening position not being on the axis of the mouth, so a directivity term exists as demonstrated in Figure 4.3. Also the distance from the mouth to the listening position is slightly greater than 1 meter. But overall, the plots above double check with the plots in Figure 8.8.

Next, the impact of the floor reflection is added. Figure 8.12 shows the driver and mouth SPL response including the reflection that occurs at the floor boundary.

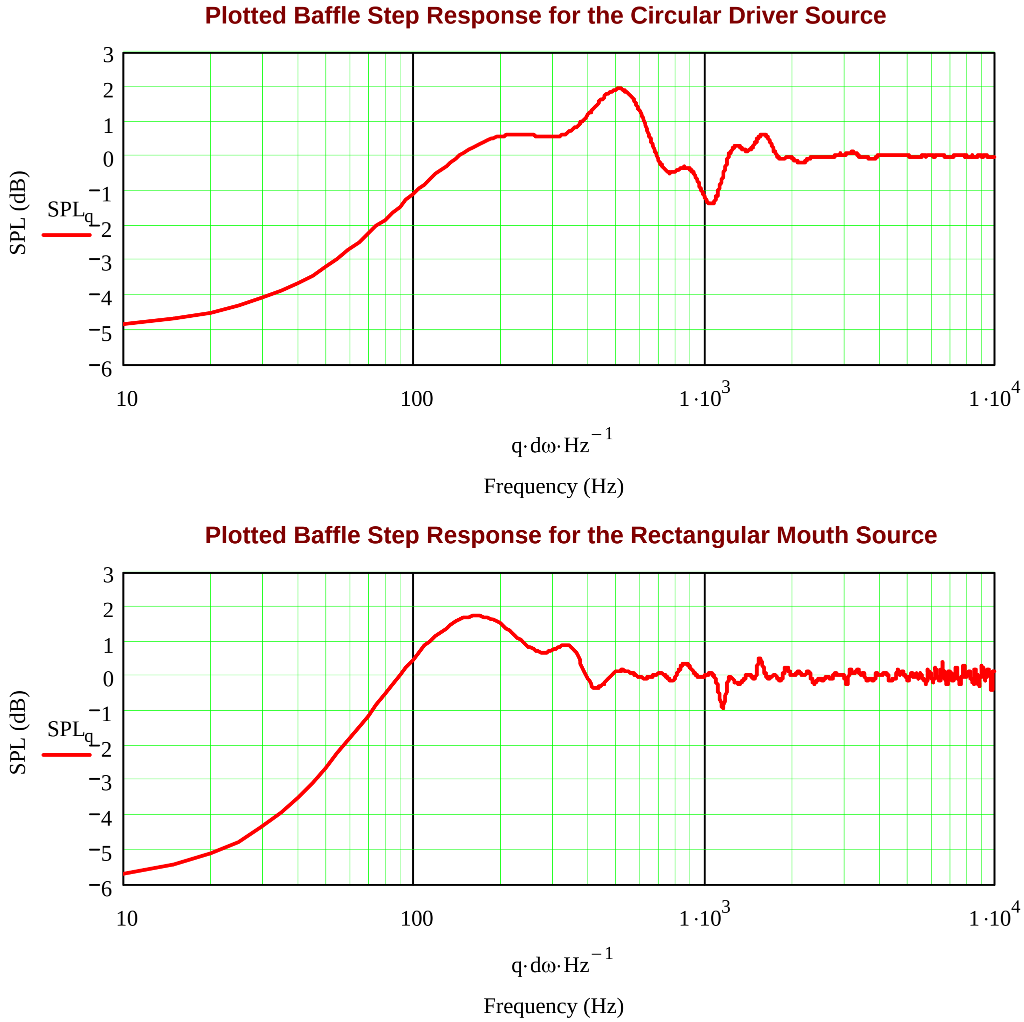
Figure 8.12 : Driver and Horn Mouth SPL Responses Including the Floor Boundary Condition at 1 m for 1 watt Input



The floor reflection is evident as dips in the SPL responses. The frequency at which the dips occur is determined by the difference in the distance between the source and the listening position and the distance from the source to the floor, where it is reflected, to the listening position. When this difference is an odd multiple of a half wavelength, at a given frequency, a dip is generated.

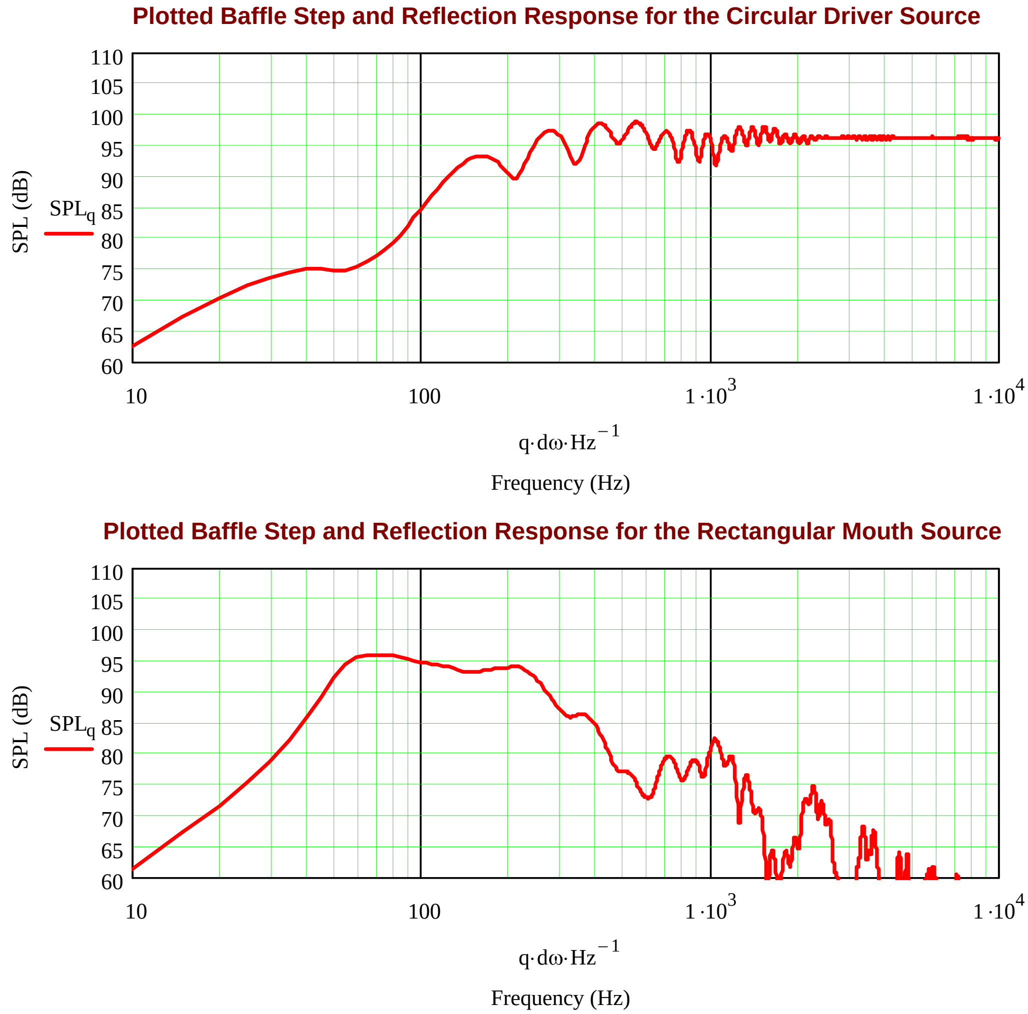
The baffle step response is also calculated for the driver and the mouth. Figure 8.13 shows the baffle step response for each source.

Figure 8.13 : Baffle Step SPL Response for the Driver and the Horn Mouth



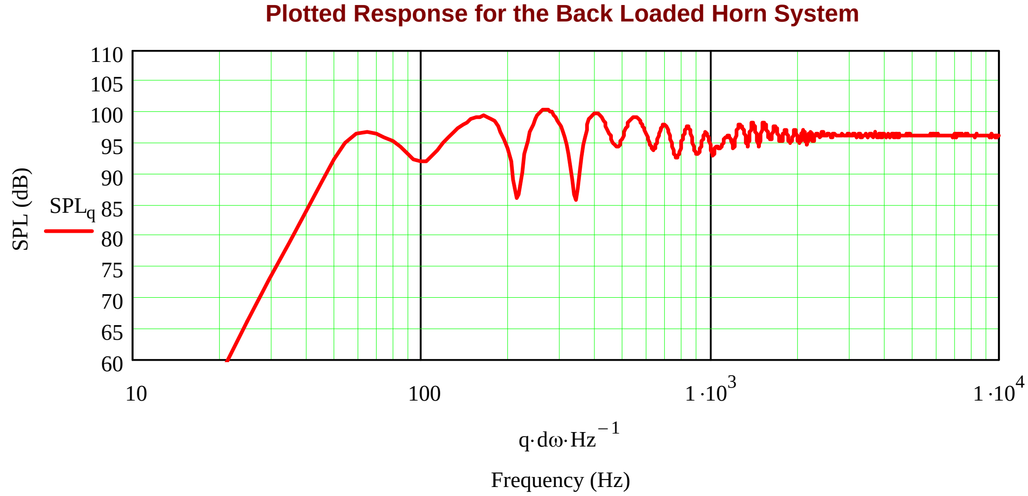
Finally the summed driver and mouth SPL response is calculated and shown in Figure 8.14. These plots include the direct sound from the driver or mouth, the reflected sound from the driver or mouth generated by the floor boundary condition, and the baffle step response determined by the shape and size of the front baffle and the shape and size of the driver and the mouth.

Figure 8.14 : Summed SPL Response for the Driver and the Horn Mouth at 1 m for a 1 Watt Input



Combining the two SPL results, presented in Figure 8.14, produces the system SPL response, at a 1 m distance for a 1 watt input, on the axis of the driver. This result is shown in Figure 8.15

Figure 8.15: Combined System SPL Response at 1 m for a 1 Watt Input



Examining the system SPL response curve in Figure 8.15 and comparing it with the driver and mouth SPL response curves in Figures 8.14 produces a number of interesting observations. Remember that the ripple seen in the driver SPL response was created by the reflected pressure waves at the floor boundary condition and therefore are not a property of the back loaded horn speaker design. Above 400 Hz, the system response is the same as the driver response since the horn mouth is no longer contributing significantly to the SPL response. In the system SPL response, there are three deep nulls at 100 Hz, 215 Hz, and 345 Hz. Tables 8.1 and 8.2 present the driver and mouth SPL magnitudes and phases at these three frequencies. At each frequency, the deep null is generated by similar SPL magnitudes arriving at the listening position with phase differences of approximately 180 degrees.

Table 8.1 : Driver SPL Magnitude and Phase

Frequency	Magnitude	Phase
100 Hz	84.5 dB	16.8 deg
215 Hz	89.7 dB	-155.4 deg
345 Hz	92.0 dB	40.8 deg

Table 8.2 : Mouth SPL Magnitude and Phase

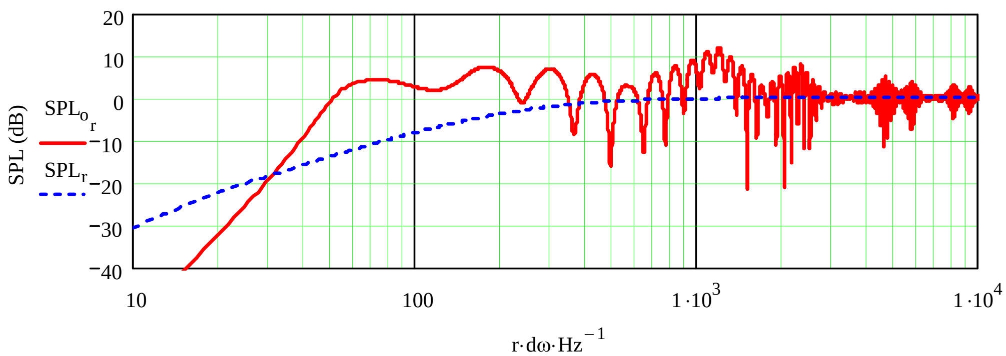
Frequency	Magnitude	Phase
100 Hz	94.7 dB	175.7 deg
215 Hz	94.0 dB	29.8 deg
345 Hz	86.2 dB	-145.2 deg

The summed system response, shown in Figure 8.15, was calculated at a 1 m distance. A 1 m distance is less than the dimensions of the horn mouth and the front baffle. Also, actual listening positions would probably be much further from the speaker system. Therefore, Figure 8.15 is really a near field response and the SPL generated by the driver and the mouth have not really merged completely.

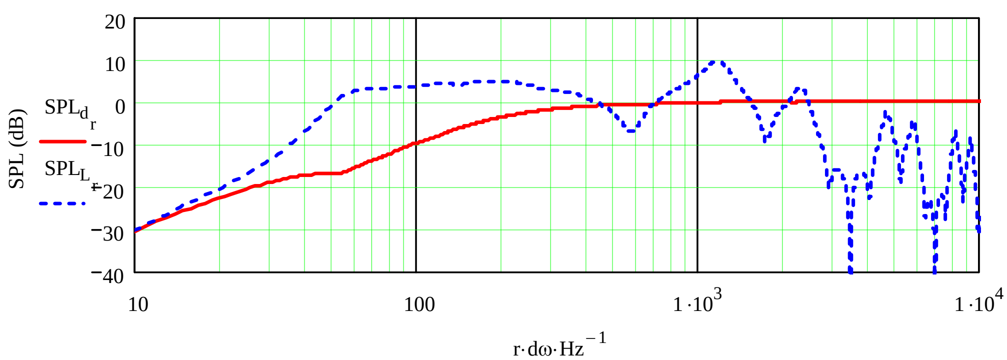
If the listening position were moved further from the back loaded exponential horn speaker system, the SPL generated by the driver and the mouth would combine and produce a system response more typical of actual listening positions. Assume the listening position was moved to 3 m away on the axis of the driver. The simplified model calculated SPL results are shown in Figure 8.16. Comparing the results in Figure 8.16, calculated at a 3 m distance, with the results in Figure 8.8, calculated at a 1 m distance, shows that the output form the horn mouth is still significant above 1000 Hz producing a summed response that is very ragged.

Figure 8.16 : Back Loaded Exponential Horn Response – Revised Baseline Configuration with a Coupling Chamber, Driver Position Ratio ξ = 0.5, Fiber Damping, and Floor Boundary Condition
SPL at 3 m for a 1 Watt Input

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses

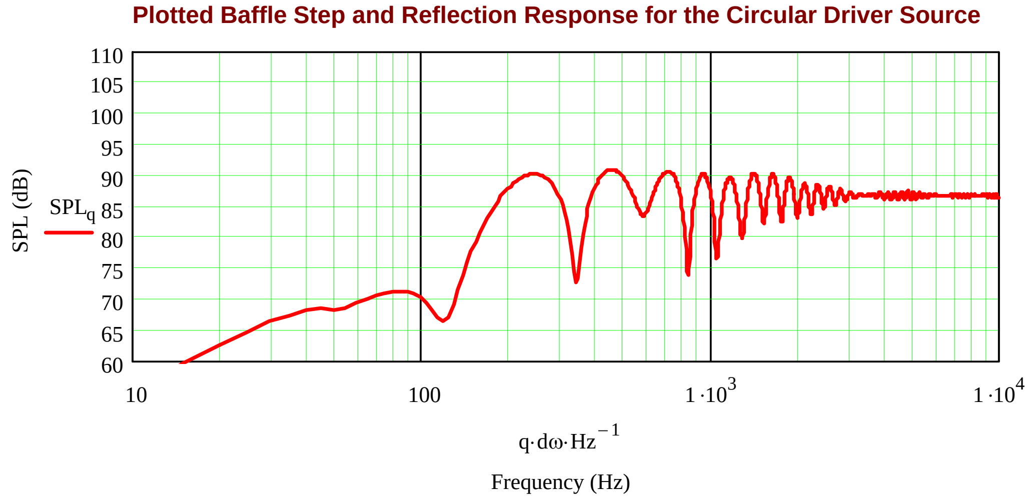


Woofer and Mouth Far Field Sound Pressure Level Responses



The SPL results calculated by the advanced model for the driver, the mouth, and the combined system are shown in Figures 8.17 and 8.18 respectively. Comparing Figure 8.17 with Figure 8.14 a few interesting differences can be seen. In the driver SPL response, the ripples associated with floor bounce are much deeper including a deep first null at 100 Hz. In the mouth SPL response the bass region is very similar between the two plots but the higher frequency peaks have become taller at the 3 m listening position because the directivity angle has decreased. The taller peak at approximately 1200 Hz in the mouth SPL response combined with the larger floor bounce ripple in the driver SPL response produce a system SPL response that has significant peaks and nulls extending higher in frequency.

Figure 8.17 : Summed SPL Response for the Driver and the Horn Mouth at 3 m for a 1 Watt Input



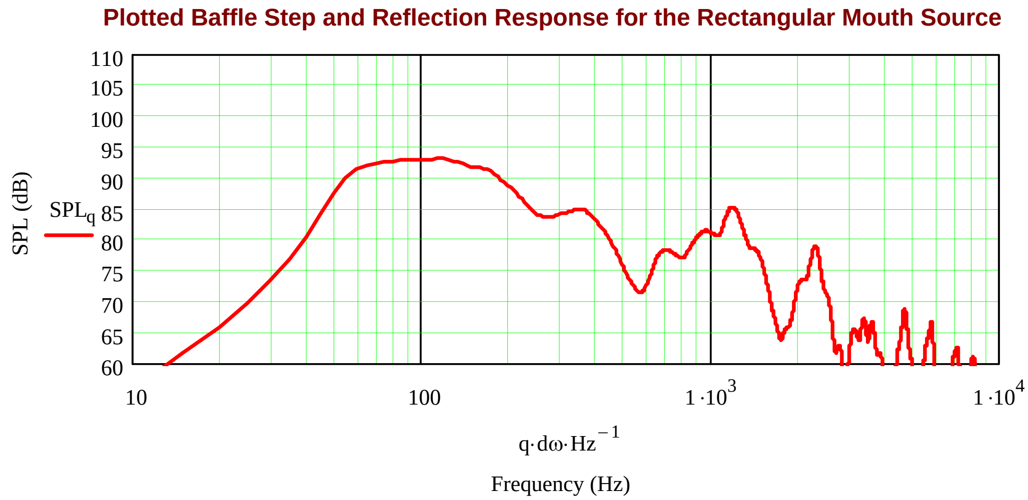
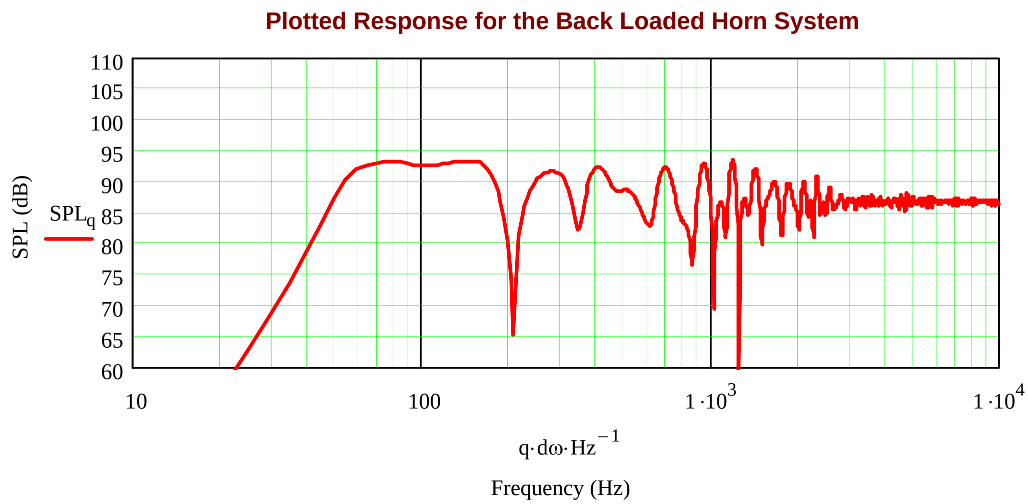


Figure 8.18 : Combined System SPL Response at 3 m for a 1 Watt Input



Reexamining the system SPL response curve in Figure 8.15, and comparing it with the system SPL response curve in Figures 8.18, generates a number of key observations. The deep null at 100 Hz, created by the floor bounce, has allowed the horn's mouth to completely supply the system bass output. The bass is efficient and smooth from the -3 dB point at approximately 50 Hz up to the first null at 210 Hz. In the system SPL response, there are two deep nulls at 210 Hz and 350 Hz caused by destructive interference. Tables 8.3 and 8.4 present the driver and mouth SPL magnitudes and phases at these frequencies. At each frequency, the deep null is generated by similar SPL magnitudes arriving at the listening position with phase differences of approximately 180 degrees.

Table 8.3 : Driver SPL Magnitude and Phase

Frequency	Magnitude	Phase
210 Hz	88.6 dB	147.5 deg
350 Hz	73.4 dB	37.3 deg

Table 8.4 : Mouth SPL Magnitude and Phase

Frequency	Magnitude	Phase
210 Hz	88.0 dB	-32.0 deg
350 Hz	85.0 dB	-144.1 deg

The summed system response, shown in Figure 8.15, was calculated again at a 3 m distance which is more typical of a listening distance in a home audio set-up. At the 3 m distance, the driver and horn mouth combined to produce bass SPL output of approximately 92 dB. At frequencies above 200 Hz, the average system SPL is closer to 87 dB which is 5 dB lower. The bass output from the horn's mouth is too efficient to produce a smooth SPL curve over the entire frequency range even after factoring in the baffle step loss at lower frequencies. A further reduction of the horn mouth size seems possible. In the next section, a compromised transmission line / horn system will be presented.

Compromised Transmission Line / Horn Loaded Model :

In all of the preceding discussions of back loaded exponential horn design, the size of the mouth was the first parameter calculated based on a defined lower cut-off frequency f_c. By setting the size of the mouth consistent with f_c, the acoustic impedance is resistive and energy is efficiently transmitted from the horn's mouth into the listening room and not reflected back into the horn. Without the reflection of sound back into the horn, standing wave resonances are not excited and the SPL response is smooth without the peaks and nulls generally associated with transmission line enclosures.

The open end of a transmission line enclosure is typically much smaller than a horn's mouth resulting in acoustic impedances that are primarily mass loading. This mass loading reflects the sound waves back into the transmission line resulting in an inefficient transfer of energy into the listening room. Transmission line enclosures rely on quarter wavelength standing waves to produce large volume velocities at the open end. These strong resonances, large volume velocities, and low resistive component of the acoustic impedance inefficiently generate the bass SPL output required to augment the driver's rolling off low frequency SPL response.

The other important property of a consistent back loaded exponential horn design is the acoustic impedance at the throat. Since the acoustic impedance at the mouth is resistive above the lower cut-off frequency f_c, so is the acoustic impedance at the throat. The resistive nature of the throat's acoustic impedance allows the selection of a coupling chamber volume that rolls off the horn output at a prescribed higher cut-off frequency f_h. The horn mouth's SPL output can be limited to a narrow range of frequencies where the horn is required to augment the driver's falling low frequency SPL output. In contrast, the transmission line's acoustic output can only be slightly controlled by fiber damping and contributes to the system SPL over a very wide range of frequencies.

In the previous pages of this document, the floor boundary condition was used to cut the required size of the horn mouth by a factor of two. Even with this significant reduction in the required mouth area, the mouth is still much larger then typical DIYer's would want to build. Consider a further reduction in the mouth area; the horn is no longer a consistent design. At the same time, keep the mouth area large enough to maintain the resistive nature of the throat impedance near the higher cut-off frequency f_h. This would occur if the reduced mouth's cut-off frequency is greater than the previously specified f_c but still less than or approximately equal to f_h. The result would be transmission line behavior at the original lower cut-off frequency f_c transitioning to horn behavior as the higher cut-off frequency f_h is approached. This is a compromised back loaded horn design that acts as both a transmission line and a back loaded horn over the low frequency range of operation.

To demonstrate this design for back loaded exponential horns, a sample problem has been formulated. Dimensions have been selected to yield a standing wave transmission line design at the lower cut-off frequency f_c that transitions to a horn loading as the higher cut-off frequency f_h is reached. This is not the only possible geometry but just one that demonstrates the acoustic response properties of this enclosure design approach.

Assume a mouth area equal to a 14 inch square and a throat area of 0.5 x S_d, both areas are smaller then what was used in the previous examples. The length was iterated to produce a first transmission line resonance at 50 Hz. The resulting length was 93 inches. This geometry represents a compromised transmission line / exponential horn. These dimensions are the result of iterating to tune a convenient geometry to a 50 Hz fundamental while still producing a reasonable flat extended bass SPL response.

$$S_{mouth} = (1/2) \times (1 / \pi) \times (c / (2 \times f_c))^2$$

$$0.126 \text{ m}^2 = 14 \text{ in} \times 14 \text{ in} = (1/2) \times (1 / \pi) \times (342 \text{ m/sec} / (2 \times f_c))^2$$

$$f_c = 192 \text{ Hz}$$

Based on the reduced mouth geometry, the horn's mouth cut-off frequency is 192 Hz. The horn's flare constant m was calculated, from Equation (5.1), using the defined mouth and throat areas and setting the horn's length to 93 inches.

$$L_{horn} = \ln(S_{mouth} / S_{throat}) / m$$

$$93.0 \text{ in} = 2.363 \text{ m} = \ln(6.2 / 0.5) / m$$

$$m = 1.066 \text{ m}^{-1}$$

Using Equation (5.2), the horn's cut-off frequency corresponding to the flare constant m can be calculated and compared to the transmission line's fundamental frequency of 50 Hz. These two frequencies are not consistent in a compromised design.

$$m = (4 \pi f) / c$$

$$1.066 \text{ m}^{-1} = (4 \pi f) / 342 \text{ m/sec}$$

$$f = 29 \text{ Hz} < 50 \text{ Hz}$$

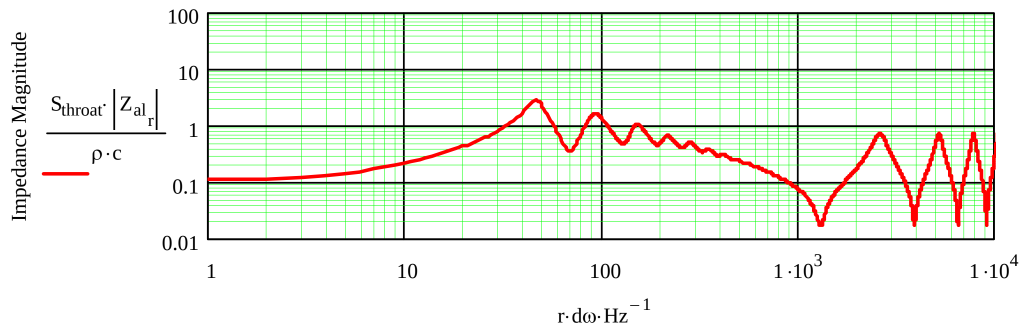
A coupling chamber that rolls off the horn response at a higher cut-off frequency f_h of 150 Hz was also selected. The coupling chamber volume was iterated to produce a reasonably flat bass response. The chamber influences both the transmission line and the back loaded horn response. The chamber volume V was calculated, in the same manner as demonstrated in Section 5, using Equation (5.4).

$$V = (342 \text{ m/sec} \times 0.021 \text{ m}^2) / (2 \pi \, 150 \text{ Hz}) \times (1000 \text{ liters} / \text{m}^3) = 3.719 \text{ liters}$$

The simple model was used to initially look at the compromised horn's performance. Figure 8.18 shows the calculated acoustic impedance at the horn's throat and the volume velocity ratio between the horn's throat and mouth. Notice that both plots showed a multiple peaking response similar to a transmission line starting at the fundamental frequency of 50 Hz and extending up to approximately 200 Hz. By 200 Hz, the mouth is starting to produce a horn-like behavior allowing the coupling chamber to roll off both the acoustic impedance and the volume velocity ratio.

Figure 8.18 : Back Loaded Exponential Horn Response – Compromised Configuration with a Coupling Chamber, Driver Position Ratio ξ = 0.5, Fiber Damping, and Floor Boundary Condition
Horn Throat Acoustic Impedance and Volume Velocity Ratio

Resulting Acoustic Impedance for the Back Loaded Horn



Volume Velocity at the Mouth of the Back Loaded Horn for a 1 m ^{3}/sec Driver Excitation

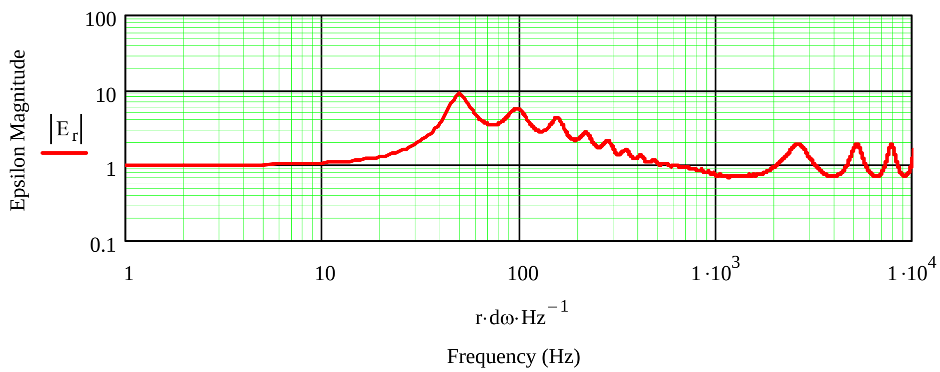
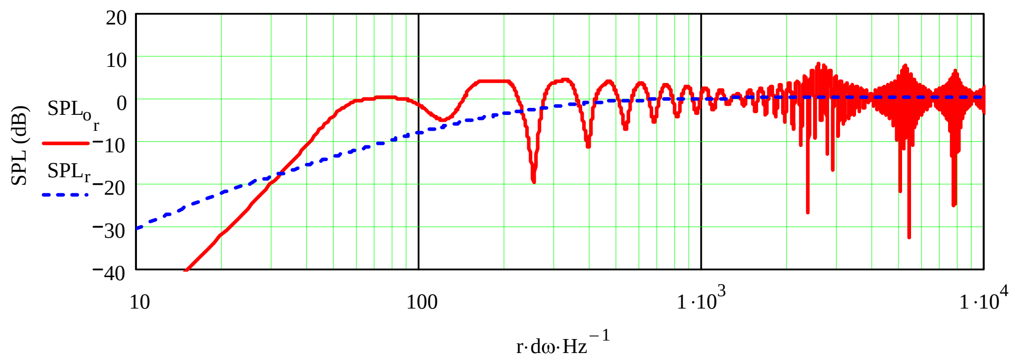


Figure 8.19 presents the back loaded horn system SPL response (solid red curve) along with the driver in an infinite baffle response (dashed blue curve) as a reference in the top plot. In the bottom plot, the driver (solid red curve) and horn mouth (dashed blue curve) contributions to the back loaded horn system SPL response are shown. Obviously the plotted response is nowhere near the desired smooth and flat SPL response. But looking at the different peaks and nulls, the cancellation of the driver SPL response by the horn mouth SPL response is evidence of 180 degree phase differences at the listening position at reasonably equally spaced frequency values. Above 1000 Hz, standing waves in the coupling chamber start to affect the mouth SPL output producing a series of tightly spaced peaks and nulls in the system SPL response.

Figure 8.20 shows the electrical impedance magnitude of the driver in the compromised back loaded. The peaks seen at frequencies below 200 Hz are also indicative of transmission line behavior. Figures 8.21 and 8.22 contain plots of driver displacement and system impulse response respectively. Comparing these two plots against similar results in Section 7 demonstrates the transmission line like behavior at frequencies below 200 Hz.

Figure 8.19 : Back Loaded Exponential Horn Response – Compromised Configuration with a Coupling Chamber, Driver Position Ratio ξ = 0.5, Fiber Damping, and Floor Boundary Condition
SPL at 1 m for a 1 Watt Input

Far Field Back Loaded Horn System and Infinite Baffle Sound Pressure Level Responses



Woofer and Mouth Far Field Sound Pressure Level Responses

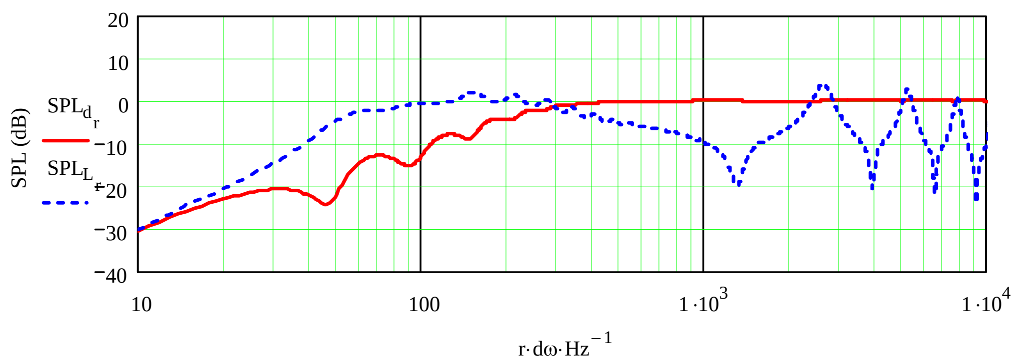


Figure 8.20 : Back Loaded Exponential Horn Response – Compromised Configuration Electrical Impedance

Back Loaded Horn System Impedance

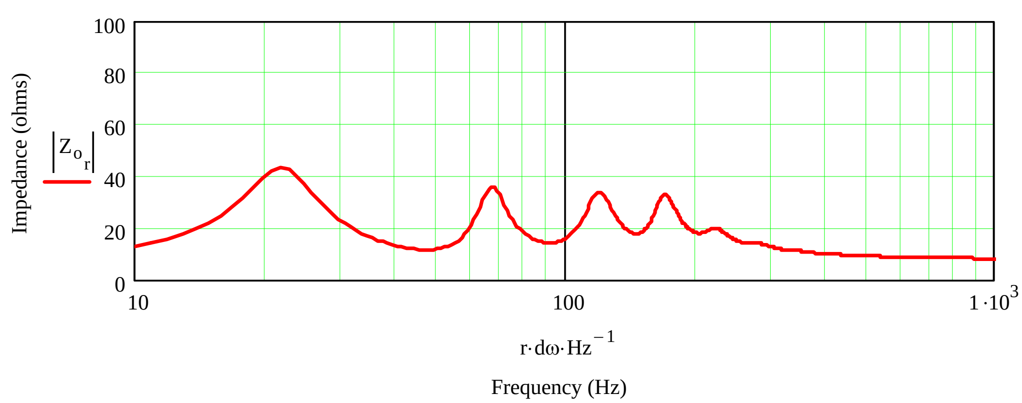


Figure 8.21 : Back Loaded Exponential Horn Response – Compromised Configuration
Driver Displacement

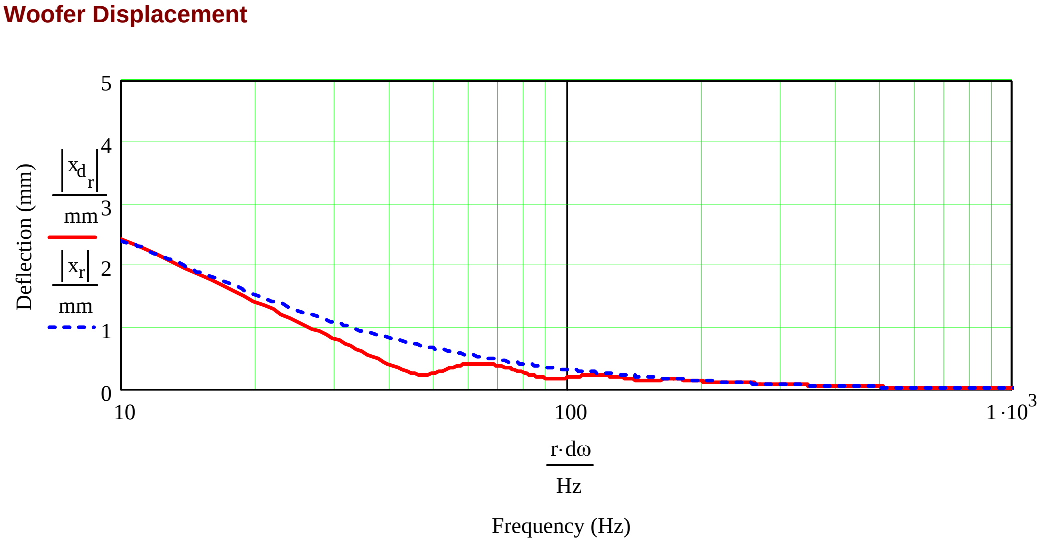
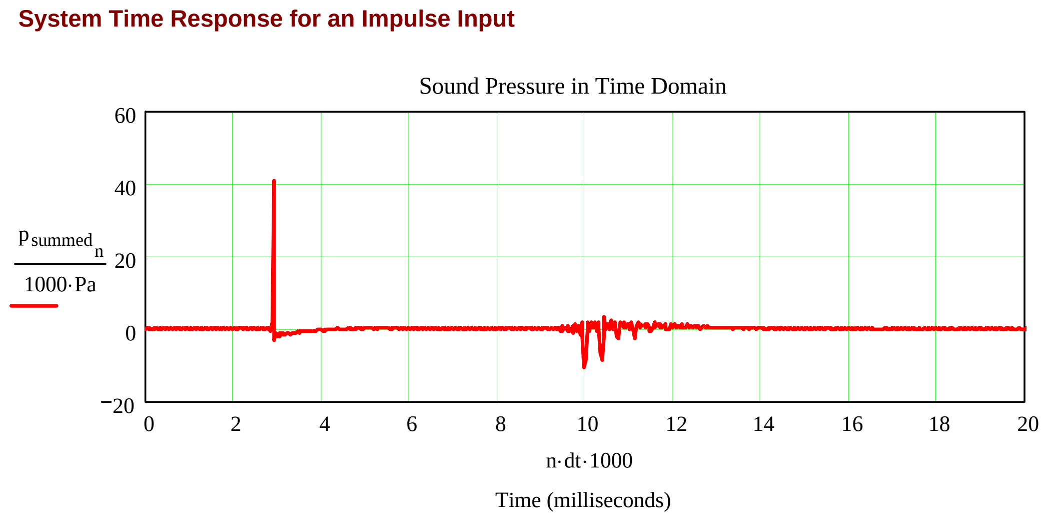


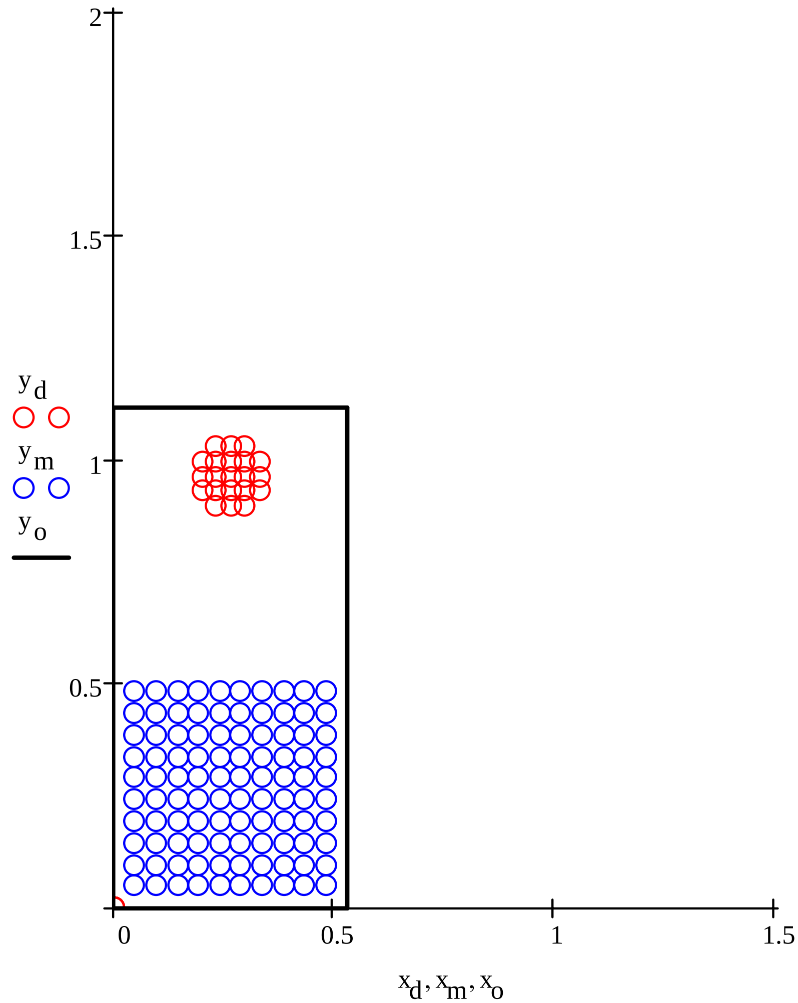
Figure 8.22 : Back Loaded Exponential Horn Response – Compromised Configuration
Impulse Response



The compromised design was modeled using the advanced MathCad worksheet. Figure 8.23 shows the front baffle used in the advance model. Remember that the technique used to perform all of the advanced calculations divides the driver and the mouth into a large number of simple sources and then sums the individual responses. The driver is represented by the small red simple sources while the mouth is broken up into the collection of small blue simple sources. The outer perimeter of the baffle is depicted as the solid black lines outlining the rectangular perimeter. The floor is represented by the x axis and the reader should visualize a mirror image collection of simple sources below the floor to account for the reflections of the sound waves. The length units on the x and y axes are again in meters.

Figure 8.23 : Driver, Horn Mouth, and Front Baffle Edge Definitions

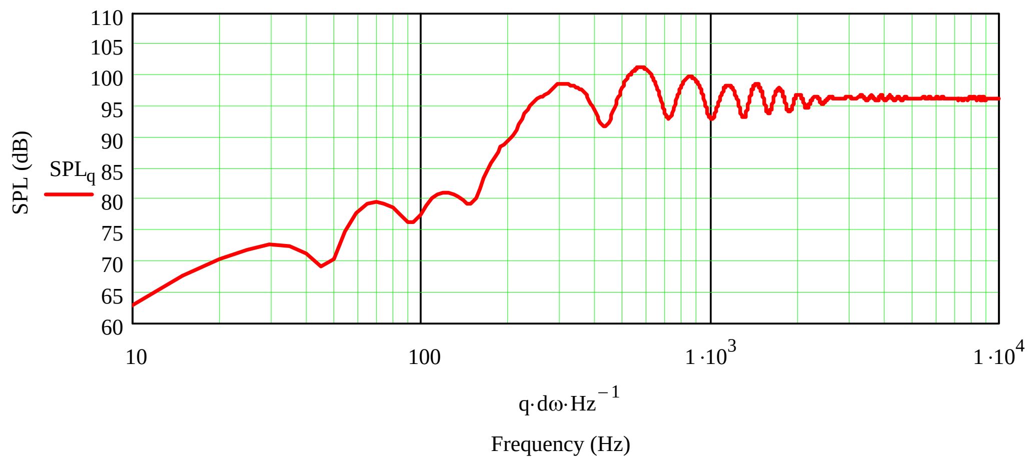
Circular Source and Rectangular Mouth Pattern with Baffle Edge Outline



The SPL results calculated by the advanced model for the driver, the mouth, and the combined compromised back loaded exponential horn system are shown in Figures 8.24 and 8.25 respectively. The listening position is at a distance of 1 m on the driver's axis. In Figures 8.26 and 8.27 and Figures 8.28 and 8.29 the plotted SPL results are repeated at listening distances of 2 m and 3 m respectively. In all plots, the contribution from the horn blends well with the rolling off driver response to produce acceptable bass output from about 50 Hz up to almost 200 Hz. Above 200 Hz all of the plots show some fairly deep nulls related to the floor bounce cancellation created by the driver position above the reflective surface. This characteristic of the system response is not a function of the back loaded horn enclosure but more a function of the driver's placement at approximately ear level. A smaller stand mounted speaker would have some of these same SPL response issues.

Figure 8.24 : Summed SPL Response for the Driver and the Horn Mouth at 1 m for a 1 Watt Input

Plotted Baffle Step and Reflection Response for the Circular Driver Source



Plotted Baffle Step and Reflection Response for the Rectangular Mouth Source

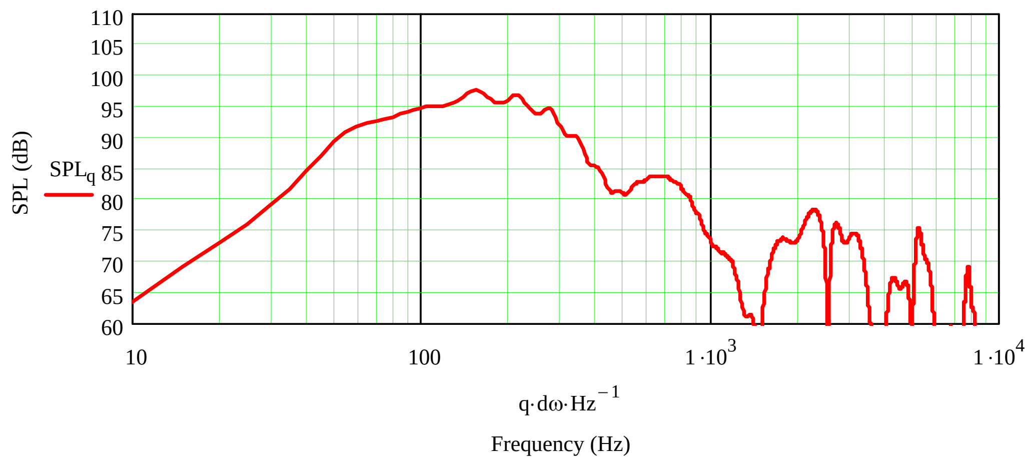


Figure 8.25 : Combined System SPL Response at 1 m for a 1 Watt Input

Plotted Response for the Back Loaded Horn System

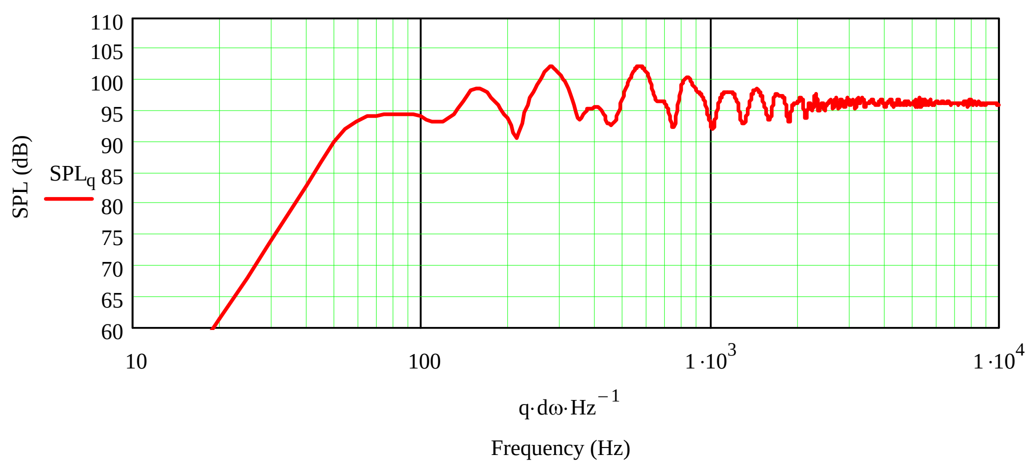
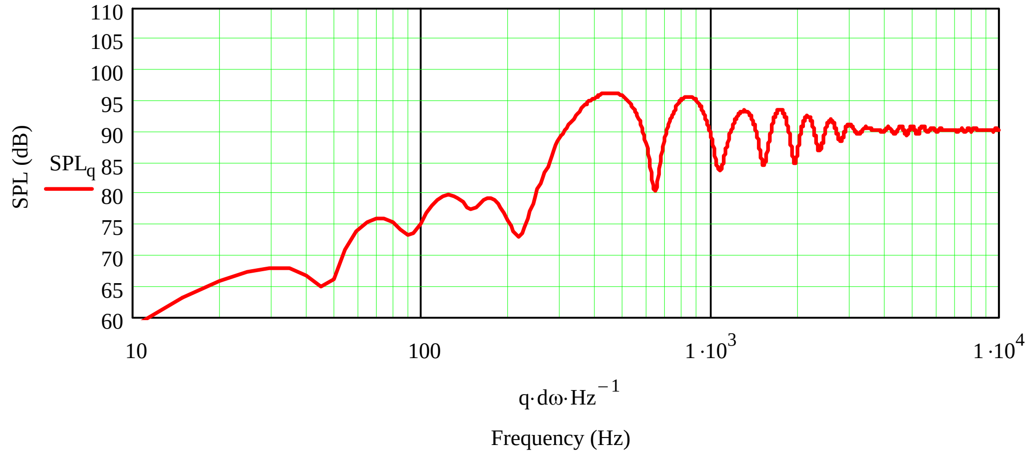


Figure 8.26 : Summed SPL Response for the Driver and the Horn Mouth at 2 m for a 1 Watt Input

Plotted Baffle Step and Reflection Response for the Circular Driver Source



Plotted Baffle Step and Reflection Response for the Rectangular Mouth Source

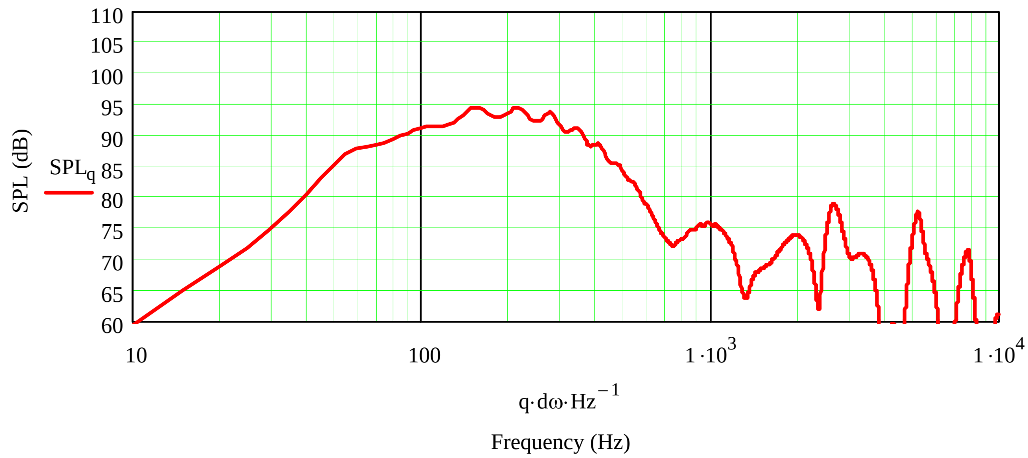


Figure 8.27 : Combined System SPL Response at 2 m for a 1 Watt Input

Plotted Response for the Back Loaded Horn System

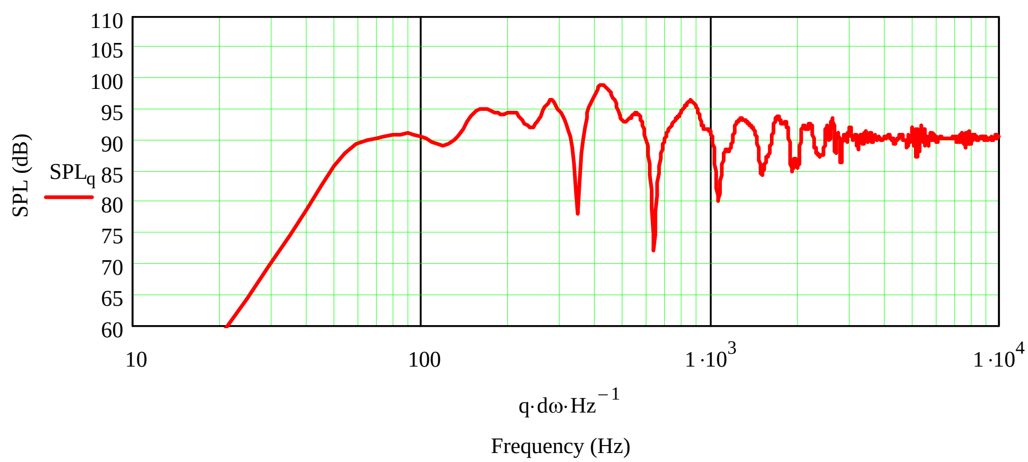
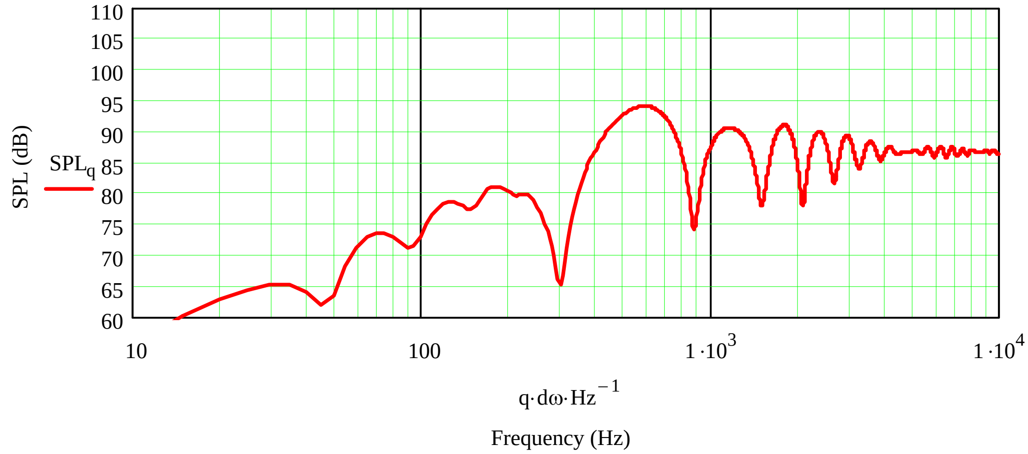


Figure 8.28 : Summed SPL Response for the Driver and the Horn Mouth at 3 m for a 1 Watt Input

Plotted Baffle Step and Reflection Response for the Circular Driver Source



Plotted Baffle Step and Reflection Response for the Rectangular Mouth Source

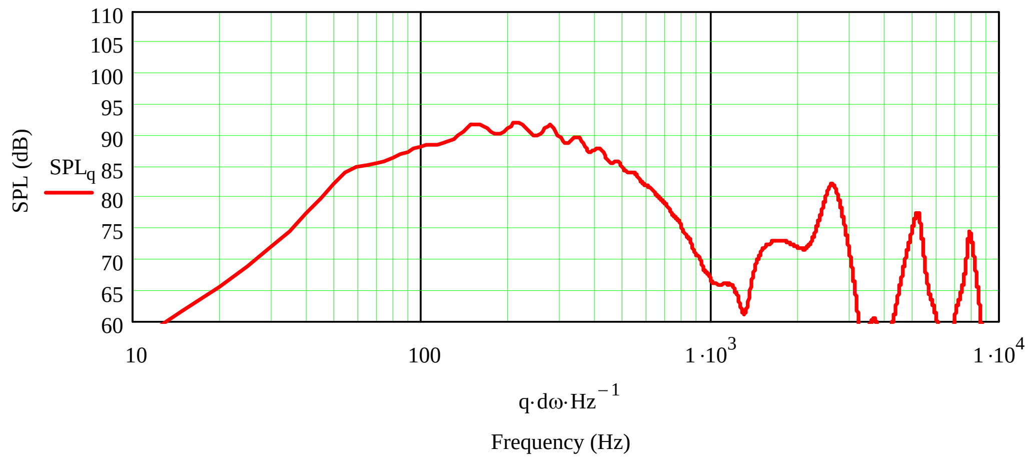
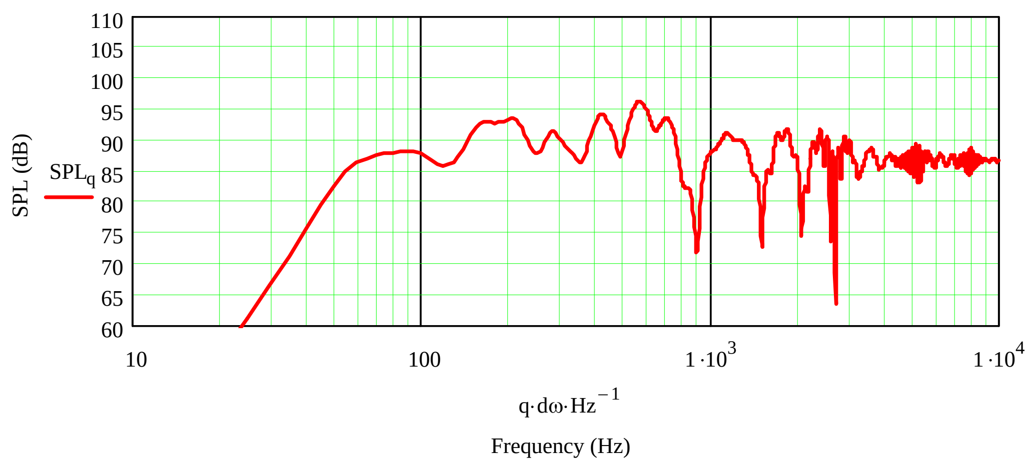


Figure 8.29 : Combined System SPL Response at 3 m for a 1 Watt Input

Plotted Response for the Back Loaded Horn System



The one thing that really stands out in Figures 8.25, 8.27, and 8.29 is the lack of any baffle step loss in the system's low frequency output. The efficiency of the horn SPL output is sufficient to cancel the theoretical 6dB loss of SPL below the baffle step frequency range. The need for any series resistance, or baffle step compensation circuit, to be added to the driver is removed. For the generic driver with a low Qts value of 0.2, this is a really remarkable result.

Calculated Response of a ML TL Speaker Enclosure :

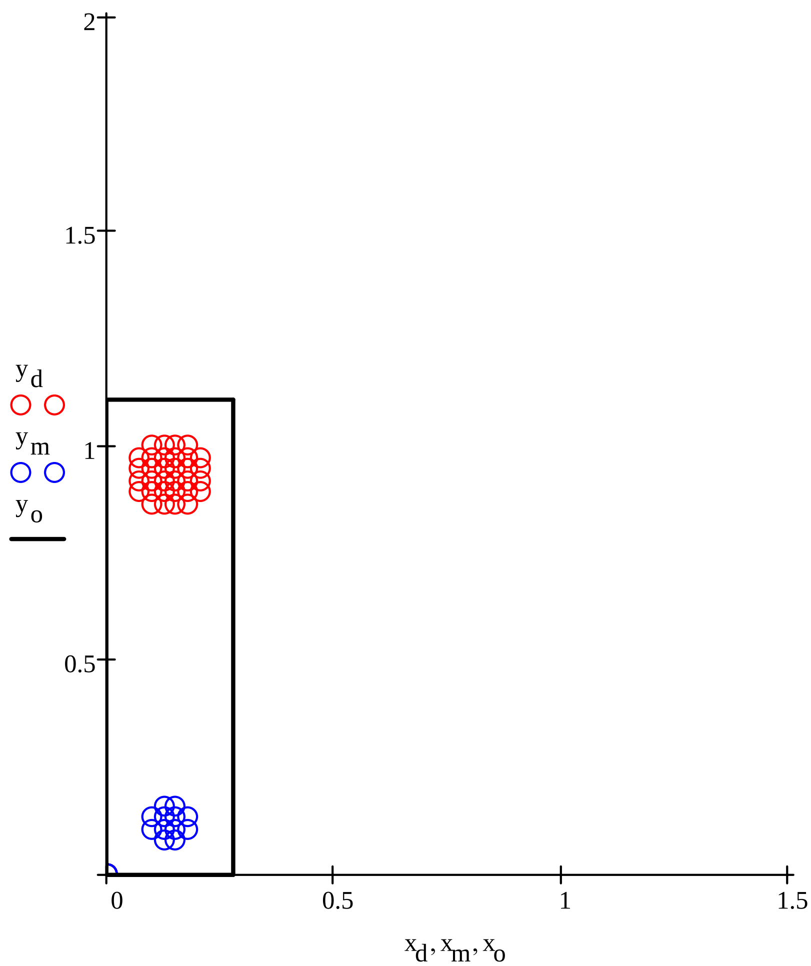
Just for completeness, the generic driver was modeled in a ML TL enclosure and the advance MathCad model was used to calculate the SPL response. No series resistance was added to the driver in the model, so examining the plotted results is an apples to apples comparison with the back loaded horn results. This is a double check since the generic driver is intended to be typical of the Lowther and Fostex drivers used in the ML TL design contained in my Projects #4 and #5. For these ML TL designs, a baffle step correction circuit is an absolute requirement to yield a balanced SPL response across the entire frequency range. Without the circuit, the bass response suffers to the point of being almost nonexistent.

Figure 8.30 shows the geometry used in the ML TL model. The driver is represented by the small red simple sources while the port is broken up into small blue simple sources. The outer perimeter of the baffle is depicted as the solid black lines outlining the rectangular perimeter. The floor is represented by the x axis and the length units on the x and y axes are meters.

The SPL results calculated by the advanced model for the driver, the port, and the combined ML TL system are shown in Figures 8.31 and 8.32 respectively at a 1 m listening position on the axis of the driver. Comparing these plots with the simple model results presented in Projects #4 and #5 shows the influence of the floor boundary condition and the baffle step loss without introducing any passive circuitry.

Figure 8.30 : Driver, Port, and Front Baffle Edge Definitions

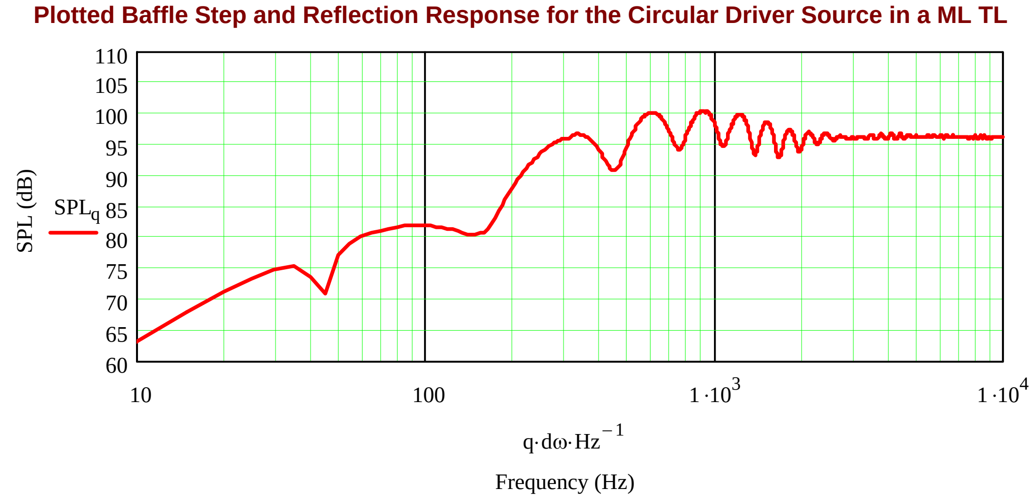
Circular Source and Circular Mouth Pattern with Baffle Edge Outline



In Figure 8.31, the floor bounce influence in the driver SPL response, upper plot, is seen just like in the previous compromised back loaded horn results. In the lower plot of Figure 8.31, the port SPL response is not really impacted by the floor boundary condition since the port is placed at the very bottom of the enclosure. The acoustic impedance acting on the port is primarily a mass loading. The floor helps to reinforce the bass output produced by the port without introducing any additional significant peaks or nulls.

The combined system response, shown in Figure 8.32, does not exhibit the desired smooth extended bass response. The response is more of a single peak at the tuning frequency followed by a deep suck out extending from the tuning frequency up to 200 Hz. A correction circuit is required for the ML TL design to perform well with a driver having such a low Qts value. In reality, a driver with a higher Qts value would yield a smoother bass SPL response at the expense of some midrange detail and higher frequency extension.

Figure 8.31 : Summed ML TL SPL Response for the Driver and the Port at 1 m for a 1 Watt Input



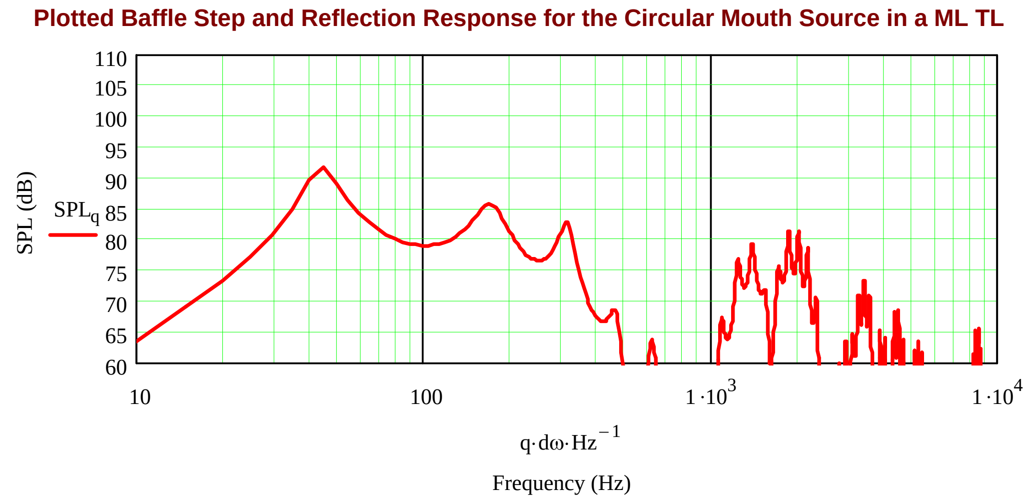
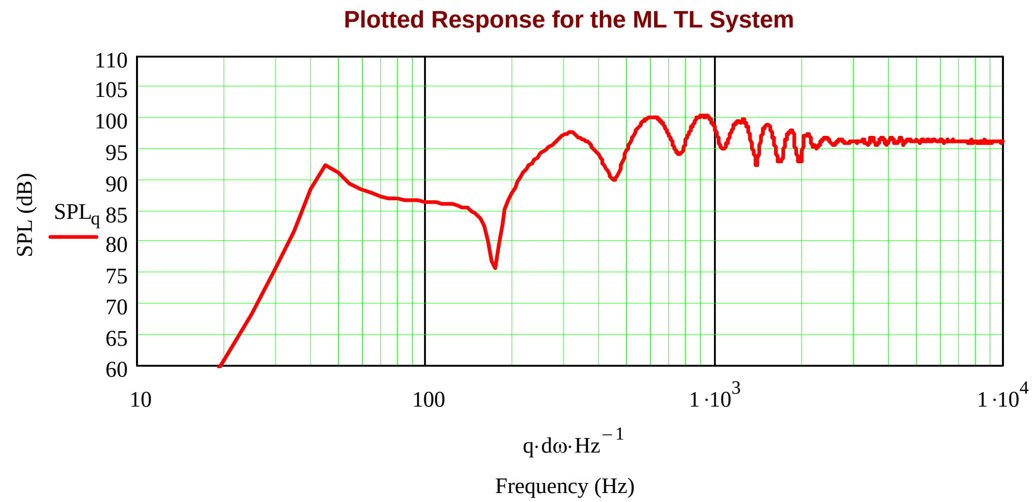


Figure 8.32 : Combined ML TL System SPL Response at 1 m for a 1 Watt Input



Summary :

Soon after I became comfortable with using the transmission line MathCad worksheets, I started to model back loaded horn enclosures. For several years, I modeled almost every back loaded horn design I could find on the Internet. With very few exceptions, the results were not very promising. Part of the problem was the lack of an advanced MathCad model worksheet, as introduced in this section, to account for the floor reinforcement and baffle step response. However, most of the problem was the designs were really trial and error attempts that just did not produce an extended and smooth bass response.

In the Section 7.0, consistent back loaded exponential horn designs were explored. To get the 50 Hz lower cut-off frequency desired a huge mouth area was required. Even when the floor boundary condition was included in the analysis, cutting the required mouth area in half, the horn was still too big for anybody but the most dedicated audiophile. An example of this type of very large back loaded horn enclosure can be found on Nelson Pass' speaker page (http://www.passdiy.com/speakers.htm). Not many DIY'ers have the room or the resources to construct and live with these types of back loaded horn designs.

This leaves all of the other back loaded horn enclosures that typically are just a little bigger then a floor standing tower speaker system to be studied. In my opinion, these back loaded horn designs should be categorized as compromised transmission line / horn enclosures. There is nothing wrong with this approach and it will be the one used when I build my first reasonably sized back loaded "horn" speaker project.

When you review a potential back loaded horn design, the first thing to check is the mouth size and the claimed lower cut-off frequency. Assuming a floor standing enclosure, as apposed to a corner loaded design, the following calculation will help determine the horn mouth's lower cut-off frequency.

$$f_c = c / [2 \times (2 \times \pi \times S_{mouth})^{1/2}]$$

If the calculated lower cut-off frequency f_c is several hundred Hz, and the claimed low frequency performance is in the 30 to 60 Hz range, chances are the design is actually behaving as a transmission line at lower frequencies transitioning to a back loaded horn at higher frequencies.

Hybrid transmission line / back loaded horn enclosures are a good compromise design that is tricky to get right by the trial and error method. It is not impossible to get a good design by trial and error and I believe that this method represents the source of the majority of back loaded horn designs found on the Internet. Unfortunately, the trial and error design method is also one of the reasons there are few solid performers available. If done correctly, a hybrid transmission line / back loaded horn enclosure will not require any baffle step correction circuit which is the real big advantage. Imagine 94 to 98 dB SPL, at a 1 m distance for 1 watt of input, over the frequency range 50 Hz to 15-20 kHz produced by a single driver enclosure that is slightly bigger then a commercial tower speaker system. No crossover, no correction circuit, just a speaker cable running directly from the amplifier to the driver. Based on the work presented in this Section, I am convinced that this is possible and even predictable with the two MathCad worksheets.