Introduction:

Since I first put my MathCad transmission line models on the Internet for people to use, I have been repeatedly asked about modeling two drivers in one enclosure. Two drivers sharing one enclosure volume can be found in MTM arrangements or when one driver is mounted on the front and another on the back of an enclosure to compensate for the baffle step response. These questions started almost immediately. The frequency steadily increased until it seemed to be asked about once a week in private e-mails or publicly on one of the forums that I frequent. So I decided to put a short document together describing methods for performing these types of calculations using the current MathCad worksheets. There is no original work being derived and presented. A repeat of commonly used methods is given and hopefully an explanation that provides some understanding of the rationale behind the methods.

Method Found in the LSDC:

The first reference that I checked for guidance in modeling two drivers in a single enclosure was Vance Dickson’s The Loud Speaker Design Cookbook 3rd Edition. In Section 1.9, on page 13, some rules are given for deriving equivalent driver properties that can then be used to determine enclosure parameters from the many alignment tables presented in the text. For the standard configuration of two drivers in one enclosure, the following equivalencies are listed.

\[
\begin{align*}
  f_d &= \text{remains the same as the single driver} \\
  Q_{td} &= \text{remains the same as the single driver} \\
  V_{ad} &= \text{is twice the single driver value} \\
  Z_{VC} &= \text{impedance is half if the drivers are wired in parallel} \\
        &= \text{impedance is doubled if the drivers are wired in series} \\
  \text{SPL} &= +6 \text{ dB if the drivers are wired in parallel} \\
        &= \text{unchanged if the drivers are wired in series}
\end{align*}
\]

With these revised properties, the text’s alignment tables allow an enclosure to be sized and the SPL and electrical impedance quantified. Typically, the size of the enclosure doubles when two drivers are used.

Complete Equivalent Properties for Two Driver:

The equivalent properties provided in Vance Dickson’s The Loud Speaker Design Cookbook 3rd Edition allow you to size and enclosure but are not complete enough to run a computer simulation. Table 1 presents a complete set of equivalent driver properties for two identical drivers wired in series or in parallel as a function of the single driver’s properties. It is assumed that the input voltage is the same for each column even thought the combined electrical impedance is different for the two wiring options. These adjusted properties are entered as a single equivalent driver directly into the various MathCad worksheets to perform a simulation of two drivers in one enclosure. These equivalent properties are not unique to the MathCad worksheets and can also be used in other loudspeaker simulation/design programs.
Table 1: Equivalent Driver T/S Properties

<table>
<thead>
<tr>
<th>Single Driver T/S Parameter</th>
<th>Two Drivers Wired in Parallel</th>
<th>Two Drivers Wired in Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_d )</td>
<td>( f_d )</td>
<td>( f_d )</td>
</tr>
<tr>
<td>( Q_{ed} )</td>
<td>( Q_{ed} )</td>
<td>( Q_{ed} )</td>
</tr>
<tr>
<td>( Q_{md} )</td>
<td>( Q_{md} )</td>
<td>( Q_{md} )</td>
</tr>
<tr>
<td>( Q_{td} )</td>
<td>( Q_{td} )</td>
<td>( Q_{td} )</td>
</tr>
<tr>
<td>( R_e )</td>
<td>( R_e / 2 )</td>
<td>( 2 R_e )</td>
</tr>
<tr>
<td>( L_{vc} )</td>
<td>( L_{vc} / 2 )</td>
<td>( 2 L_{vc} )</td>
</tr>
<tr>
<td>( V_{ad} )</td>
<td>( 2 V_{ad} )</td>
<td>( 2 V_{ad} )</td>
</tr>
<tr>
<td>( B_l )</td>
<td>( B_l )</td>
<td>( 2 B_l )</td>
</tr>
<tr>
<td>( S_d )</td>
<td>( 2 S_d )</td>
<td>( 2 S_d )</td>
</tr>
</tbody>
</table>

From a mechanical point of view, when you double \( V_{ad} \) you are really specifying that the suspension stiffness of the equivalent driver is half that of the single driver. To help understand this result, the following derivation is presented considering a single driver and then a pair of identical drivers in a closed box. In both cases, the volume tuning ratio \( \alpha = V_{ad} / V_b \) will be the same.

Consider a single driver, with a cone area of \( S_d \) and a suspension compliance \( C_{ad} \), which is proportional to an equivalent air volume \( V_{ad} \), mounted in a sealed enclosure. The ratio that defines the tuning of the sealed system is \( \alpha = V_{ad} / V_b \) where \( V_b \) is the volume of air contained in the enclosure. If the air in the enclosure is pressurized, the driver's cone will move out a small distance \( x \). The change in volume and the increased pressure \( p \) are related by the driver’s suspension compliance.

\[
C_{ad} = \text{suspension compliance} = \frac{V_{ad}}{\left(\rho c^2\right)}
\]

\[
p = \frac{(S_d x)}{C_{ad}}
\]

\[
C_{ad} = \frac{(S_d x)}{p}
\]

For two drivers in a sealed enclosure, the same pressure would move each of the drivers out the same small distance \( x \). The displaced volume is double that of a single driver as shown below. A prime is used to denote the equivalent single driver property used to represent the pair of drivers.

\[
p = \frac{(S_d' x)}{C_{ad}'} = 2 \frac{(S_d x)}{C_{ad}}
\]

\[
C_{ad} = 2 \frac{(S_d x)}{p} = 2 C_{ad}
\]

Using the relationships above

\[
V_{ad}' = 2 V_{ad}
\]

and

\[
S_d' = 2 S_d
\]

which is consistent with the results defined in Table 1.
Going one step further, since the frequency of the equivalent driver does not change the equivalent moving mass can also be derived.

\[
f_d = \left( \frac{1}{2\pi} \right) \left[ \frac{1}{(C'_{ad} \cdot M'_{ad})} \right]^{1/2}
\]
\[
f_d = \left( \frac{1}{2\pi} \right) \left[ \frac{1}{(2 \cdot C_{ad} \cdot M'_{ad})} \right]^{1/2}
\]
\[
f_d = \left( \frac{1}{2\pi} \right) \left[ \frac{1}{(C_{ad} \cdot M_{ad})} \right]^{1/2}
\]
\[
M'_{ad} = \frac{M_{ad}}{2}
\]

Summarizing the mechanical relationships for the equivalent driver, the suspension stiffness and the moving mass are half that of the single driver. This is required to maintain the same resonant frequency for the equivalent driver. Looking at the impact on the enclosure size, when two drivers are used the enclosure must have twice the volume of an enclosure containing a single driver if the volume tuning ratio \( \alpha \) is held constant.

From an electrical point of view, a decision to wire the driver in series or in parallel is required. Assuming that a standard 2.8284 volts is applied to the finished speaker (1 watt into an 8 ohm resistor), the voltage across each driver is a function of the connection selected. For a parallel connection, the voltage across each driver will be the same as for a single driver. Therefore the driver displacement as a function of frequency will remain the same, the SPL will increases by 6 dB, and the impedance will be half of the single driver impedance. For a series connection, the voltage across each driver will be half of the single driver design. Therefore the driver displacement will be halved, the SPL will be unchanged, and the impedance will be double the single driver impedance.

**Sample Problem:**

To demonstrate the modeling of single and double driver designs, I have constructed a sample problem using the Radio Shack 40-1197 full range driver mounted in a ported box. The MathCad worksheet “Ported Box” has been used to perform the simulations. The measured T/S parameters of the driver are shown in Figure 1 along with the baseline enclosure geometry. Figure 2 shows the SPL response, the electrical impedance, and the driver displacement for this baseline design. For the double driver simulations, an equivalent driver is placed at the same position in the enclosure as the single driver which is assumed to be the average position of the two drivers.

**Modeling Parallel Driver in MathCad:**

To model two drivers wired in parallel, the previously defined modifications to the single driver properties were used. The enclosure volume was doubled by increasing all of the cross-sectional areas by a factor of two including the port area. The revised equivalent driver and the new enclosure geometry are shown in Figure 3.

The simulation results are shown in Figure 4. As expected, the SPL increased by 6 dB while the impedance dropped by a factor of two. The driver displacement is
unchanged. This modeling technique works very well and can be applied to all of the MathCad worksheets.

Modeling Series Driver in MathCad:

To model two drivers wired in series, the previously defined modifications to the driver properties were used. By default, the MathCad worksheets apply a standard 2.8284 volts to the individual driver. In a series connection only 1.4142 volts should be applied and by doubling the BI term this is accomplished. The enclosure volume was doubled by increasing all of the cross-sectional areas by a factor of two including the port area. The revised equivalent driver and the new enclosure geometry are shown in Figure 5.

The simulation results are shown in Figure 6. As expected, the SPL is essentially unchanged while the impedance increased by a factor of two. The driver displacement is reduced by a factor of two as expected. This modeling technique works very well and can be applied to all of the MathCad worksheets.

Conclusions:

An example has been presented, using the “Ported Box” MathCad worksheet, showing how to simulate multiple drivers in a single enclosure. The method works very well for drivers wired in parallel and in series. Until I write a true multi-driver worksheet, these tricks are all that is available. A number of individuals have built transmission line designs based on modeling the drivers wired in parallel and report excellent results.
Driver in a Ported Box (Bass Reflex) - Acoustic and Electrical Response

Software: by Martin J. King
email MJKing57@aol.com

Unit and Constant Definition

\[
\text{cycle} := 2 \cdot \pi \text{ rad}
\]

\[
\text{Hz} := \text{cycle} \cdot \text{sec}^{-1}
\]

\[
\text{Air Density} : \quad \rho := 1.21 \text{ kg} \cdot \text{m}^{-3}
\]

\[
\text{Speed of Sound} : \quad c := 342 \text{ m} \cdot \text{sec}^{-1}
\]

User Input (Edit This Section and Input all of the Parameters for the System to be Analyzed)

Driver Thiele / Small Parameters : RS 40-1197 (Fostex FE-103) Average Properties

\[
f_d := 83.12 \text{ Hz}
\]

\[
R_e := 7.55 \Omega
\]

\[
L_{vc} := 0 \text{ mH}
\]

\[
B_l := 4.925 \frac{\text{newton}}{\text{amp}}
\]

\[
S_d := \frac{\pi}{4} (3.125 \text{ in})^2
\]

\[
V_d := 4.95 \text{ liter}
\]

\[
Q_{ed} := 0.415
\]

\[
Q_{md} := 3.580
\]

\[
Q_{td} := \left( \frac{1}{Q_{ed}} + \frac{1}{Q_{md}} \right)^{-1}
\]

Enclosure Geometry Definition

\[
L := 14 \text{ in}
\]  
(Height)

\[
z_{\text{driver}} := 5 \text{ in}
\]  
(Driver Distance From Top < Height)

\[
z_{\text{port}} := 12 \text{ in}
\]  
(Port Distance From Top < Height)

\[
S_0 := 6.8 \text{ in}^2
\]  
(Area of the Top End)

\[
S_L := 6.8 \text{ in}^2
\]  
(Area of the Bottom End)

\[
\text{Density} := 0.25 \text{ lb} \cdot \text{ft}^{-3}
\]  
(Stuffing density : 0 lb/ft$^3$ < D < 1 lb/ft$^3$)

\[
r_{\text{port}} := 1.0 \text{ in}
\]  
(Radius of the port)

\[
L_{\text{port}} := 3.0 \text{ in}
\]  
(Length of the port)
Figure 2: SPL, Impedance, and Driver Displacement of the Single Driver Enclosure
Figure 3: Two Drivers Wired in Parallel Modified Properties and Enclosure Geometry

Driver in a Ported Box (Bass Reflex) - Acoustic and Electrical Response

Software: by Martin J. King
e-mail MJKing57@aol.com

Unit and Constant Definition

cycle := 2•π rad
Hz := cycle•sec⁻¹
Air Density : ρ := 1.21 kg•m⁻³
Speed of Sound : c := 342 m•sec⁻¹

User Input (Edit This Section and Input all of the Parameters for the System to be Analyzed)

Driver Thiele / Small Parameters : RS 40-1197 (Fostex FE-103) Average Properties

\[ f_D := 83.12 \text{ Hz} \]
\[ R_e := \frac{7.55 \Omega}{2} \]
\[ L_{vc} := 0 \text{ mH} \]
\[ B_l := 4.925 \frac{\text{newton}}{\text{amp}} \]
\[ S_d := 2 \frac{\pi}{4} (3.125 \text{ in})^2 \]

Enclosure Geometry Definition

\[ L := 14 \text{ in} \] (Height)
\[ z_{driver} := 5 \text{ in} \] (Driver Distance From Top < Height)
\[ z_{port} := 12 \text{ in} \] (Port Distance From Top < Height)
\[ S_0 := 2.68 \text{ in}^2 \] (Area of the Top End)
\[ S_L := 2.68 \text{ in}^2 \] (Area of the Bottom End)
\[ \text{Density} := 0.25 \text{ lb}\cdot\text{ft}^{-3} \] (Stuffing density : 0 lb/ft³ < D < 1 lb/ft³)
\[ r_{port} := 1.414 \text{ in} \] (Radius of the port)
\[ L_{port} := 3.0 \text{ in} \] (Length of the port)
Figure 4: SPL, Impedance, and Driver Displacement for Two Parallel Drivers
Figure 5: Two Drivers Wired in Series Modified Properties and Enclosure Geometry

**Driver in a Ported Box (Bass Reflex) - Acoustic and Electrical Response**

Software: by Martin J. King
e-mail MJKing57@aol.com

**Unit and Constant Definition**

cycle := 2 \cdot \pi \text{ rad}

Hz := cycle \cdot \text{sec}^{-1}

Air Density:
\rho := 1.21 \text{ kg/m}^3

Speed of Sound:
\text{c} := 342 \text{ m/sec}^{-1}

**User Input** (Edit This Section and Input all of the Parameters for the System to be Analyzed)

Driver Thiele / Small Parameters: RS 40-1197 (Fostex FE-103) Average Properties

\( f_d := 83.12 \text{ Hz} \)

\( V_d := 2.495 \text{ liter} \)

\( R_c := 2.755 \Omega \)

\( Q_{ed} := 0.415 \)

\( L_{vc} := 0 \text{ mH} \)

\( Q_{md} := 3.580 \)

\( B_l := 2.4925 \text{ newtons/amp} \)

\( Q_{td} := \left( \frac{1}{Q_{ed}} + \frac{1}{Q_{md}} \right)^{-1} \)

\( S_d := \frac{\pi}{4} (3.125 \text{ in})^2 \)

\( Q_{td} = 0.372 \)

Enclosure Geometry Definition

\( L := 14 \text{ in} \) (Height)

\( z_{\text{driver}} := 5 \text{ in} \) (Driver Distance From Top < Height)

\( z_{\text{port}} := 12 \text{ in} \) (Port Distance From Top < Height)

\( S_0 := 2.68 \text{ in}^2 \) (Area of the Top End)

\( S_L := 2.68 \text{ in}^2 \) (Area of the Bottom End)

\( \text{Density} := 0.25 \text{ lb/ft}^3 \) (Stuffing density: 0 lb/ft\(^3\) < D < 1 lb/ft\(^3\))

\( r_{\text{port}} := 1.414 \text{ in} \) (Radius of the port)

\( L_{\text{port}} := 3.0 \text{ in} \) (Length of the port)
Figure 6: SPL, Impedance, and Driver Displacement for Two Series Drivers

- SPL (dB)
  - SPL\(_r\)
  - SPL\(_l\)

- Frequency (Hz)
- Impedance (ohms)
  - |Z\(_r\)|
  - |Z\(_l\)|

- Deflection (mm)
  - |x\(_r\)|
  - |x\(_l\)|

- Frequency (Hz)